Aerodynamic Heating of Miniature Bead Thermistor Thermometers in a Rarified Airstream[1]

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ABSTRACT

The recovery factors of miniature bead thermistors as used in meteorological rocketsondes were measured at subsonic Mach numbers using a whirling arm device inside a vacuum chamber. Measured values were near 0.75 at sea level pressure and remained relatively constant with pressure up to a pressure altitude of about 30 km. Above a pressure altitude of 40 km the recovery factors steadily increased with altitude and at 65 km values between 1.10 and 1.45 were recorded.

1. Introduction

When an immersion thermometer is exposed in a high-speed airstream it will, in the absence of other sources of error, record a temperature higher than a thermometer moving along with the flow. This temperature rise is due to a combination of adiabatic heating and viscous dissipation. It is usually expressed in terms of the stagnation temperature rise, $T_s - T_0$, where T_s is the temperature which the moving gas would acquire if brought adiabatically to rest, and T_0 is the free-stream temperature as recorded by the moving thermometer.

It is readily shown that

$$T_s = T_0[1+\tfrac{1}{2}(\gamma-1)M^2], \qquad (1)$$

where M is the Mach number and γ the ratio of specific heats for air.

If T_r is the temperature recorded by the thermometer, the thermal recovery factor r is defined by the relation

$$r = (T_r - T_0)/(T_s - T_0). \qquad (2)$$

In the majority of applications T_r is measured, but T_0 is the quantity desired. Use is then made of (1) and (2), together with a knowledge of r and M, to deduce T_0. Correction of aircraft temperatures for aerodynamic heating is a routine example of this procedure. The design and exposure of the thermometer is often arranged so that r is known from physical principles to be very close to unity, so that $T_r = T_s$. In other cases, for example where conventional thermometers are exposed directly to the airstream, r must usually be determined by experiment.

Recently, miniature bead thermistors of diameters between 0.005 inch (5 mils)and 0.015 inch (15 mils) have been used as atmospheric temperature sensors. An important application of these thermistors is in meteorological rocket soundings of the stratosphere and mesosphere. Temperature is measured as the rocketsonde descends by parachute. Because of the very low drag offered the parachute by the air, high fall velocities are involved in the upper levels, so that considerable aerodynamic heating occurs. In a typical sounding the correction for aerodynamic heating at 65 km may easily exceed 15C, and it increases rapidly above this level.

In the lower atmosphere the recovery factor r of any thermometer is always less than or equal to one, and remains substantially constant with pressure. However, if the air density is reduced to the point where the mean free path λ is no longer very small compared with the characteristic dimension D of the thermometer, the heat-transfer regime changes and r is no longer constant.

Oppenheim (1953) has shown that in the extreme case of free-molecule flow, where the Knudsen number $\lambda/D \gg 1$, the thermal recovery factor may be calculated theoretically for various geometrical shapes. Oppenheim's results show that the recovery factor of a spherical thermometer in a free-molecule flow is a function of Mach number but approaches a constant value of about 1.5 for values of $M < 1.0$. Under the same conditions the recovery factor of a cylindrical thermometer exposed normal to the flow approaches a value of about 1.7. Since a rocketsonde thermistor consists of a roughly spherical bead attached to cylindrical lead wires, one would expect the recovery factor in free-molecule flow to be intermediate between these values.

In the case of meteorological rocket soundings the requirements for free-molecule flow are fully met only at altitudes somewhat higher than those attained by current operational rockets. However, a very significant portion of the sounding does extend above the level where

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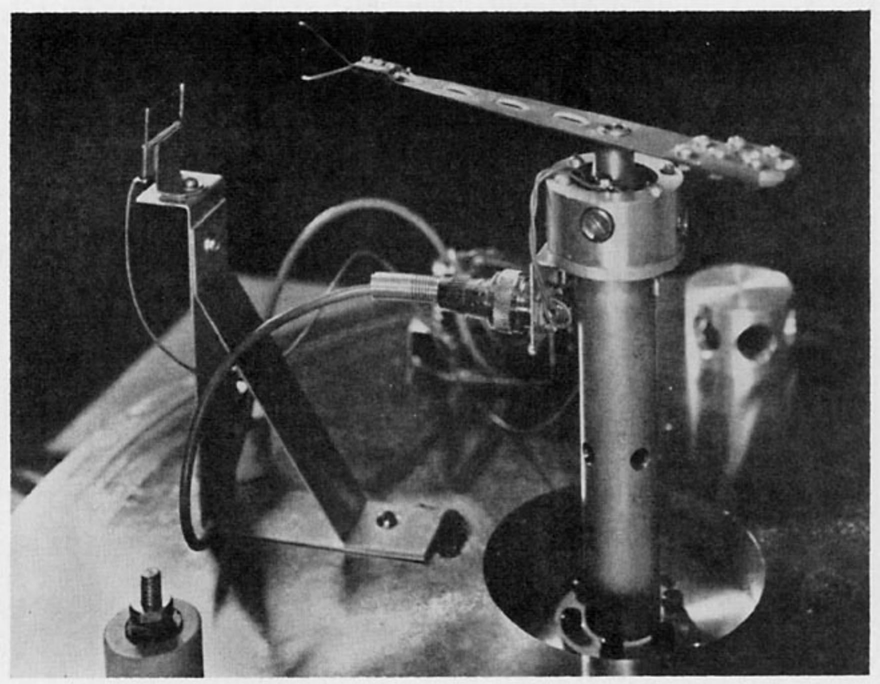

FIG. 1. Rotating arm and reference thermistor mounted on vacuum chamber pump plate.

continuum flow may be assumed. In this transition region there is no satisfactory theory for computing the recovery factor, and the amount of relevant experimental data available is extremely limited. To the author's knowledge there are no published experimental measurements of the recovery factors of miniature bead thermistors, either for high altitudes or at sea level pressures. As part of a comprehensive study of the characteristics of miniature bead thermistor thermometers (Thompson, 1966; Thompson and Keily, 1967) it was considered desirable to carry out laboratory measurements of recovery factors under conditions approaching those encountered in meteorological rocket soundings.

2. Experimental methods

The experimental procedure was to expose the thermistors on the end of a counterbalanced arm which rotated at high speed inside a vacuum chamber. A second thermistor situated within 3/16 inch of the path of the moving one provided a reference temperature with respect to which the temperature rises were measured.

Fig. 1 shows the arm, reference thermistor, etc., mounted on the vacuum chamber pump plate. The thermistors were mounted by their leads, symmetrically between two 0.05-inch diameter stainless steel supports 2.0 cm apart, which were electrically insulated from the arm. The radius of the circle described by the thermistor bead was 6 inches. With this apparatus, thermistor speeds up to 100 m $\sec^{-1}$ could be simulated at pressures ranging from atmospheric down to a few microns of mercury.

An obvious question arises as to the amount of motion induced in the air inside the vacuum chamber by the revolving arm. Devienne (1957) has made a study of this effect in connection with the use of a revolving arm for low density heat transfer studies. He found that the induced motion was quite negligible at pressures of 2 mm mercury and lower. At higher pressures the induced motion was noticeable but small.

In the present case the arm was designed so as to minimize the induced motion. The latter was measured at sea level pressure by setting up the probe of a small thermistor anemometer as close as possible to the path of the moving thermistor. A linear relation was found to hold quite closely between the thermistor speed and the induced motion, such that the latter was 4.0% of the thermistor speed. Hence, in calculating the speed of the thermistor relative to the air from the rotation speed of the arm, an allowance of -4% was made at sea level pressure but, in view of Devienne's results, this correction factor was decreased proportionally to the pressure for lower pressures.

Another effect which had to be allowed for was the increase in temperature of the air in the chamber when the arm was rotating at speed. This effect, which is presumably due to frictional heating of the air in the vicinity of the arm, was studied in detail by Devienne for his apparatus. He found that the temperature rise was least when the mean free path was not small compared with the arm thickness, and that while the temperature rise was greater at higher pressures the temperature gradients in the vicinity of the arm became small. Devienne concluded that heat-transfer measurements were valid

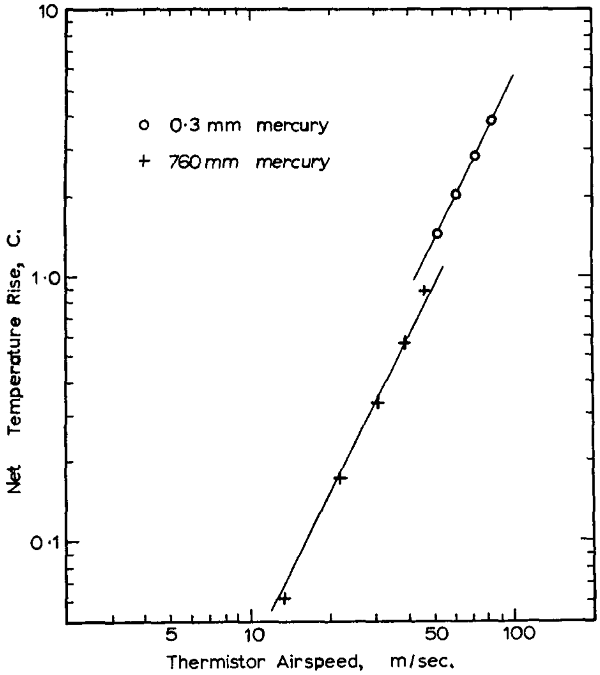

FIG. 2. Net temperature rise of moving thermistor as a function of airspeed at two different pressures.

provided temperature differences with respect to suitably placed reference thermometers were used in the calculations.

Similar effects were noted by the author although the details of the temperature distribution in the vacuum chamber were not observed, only one point very near the path of the thermistor being available. Devienne's results suggest, however, that this one point should have been reasonably representative of the free-stream temperature. The temperature rise of the reference thermistor depended only on the speed of the revolving arm and was found to be proportional to the square of the rotation speed. At a pressure of 4×10^{-2} mm mercury the reference temperature rise was about 3% of that of the moving thermistor. As the pressure was increased it was found that near 0.25 mm mercury the heating increased quite sharply to 15–20% of the moving thermistor's, and at higher pressures this percentage gradually increased, being 30–35% at sea level pressure. Changes in heating were in phase with changes in the rotation speed and there was no tendency for the heating to increase with the length of time that the rotation had been going on.

The aerodynamic heating measurements were carried out at Mach numbers between 0.18 and 0.29. Mach numbers of double these values may be reached in the upper levels of rocket soundings, but these would have exceeded the design limits of the rotating arm. A major limitation to the speed attainable was the centrifugal acceleration to which the thermistors and their mounting system were subjected during the tests. For example, it was found that while 15-mil thermistors with leads each 1 cm long were able to withstand centrifugal accelerations of 4000–5000 *g* for prolonged periods, failures of the leads were liable to occur when the acceleration exceeded about 6000 *g*. On the other hand, 5- and 10-mil thermistors were run without failures at speeds of 100 m $\sec^{-1}$ for long periods, and even faster for brief ones, with corresponding accelerations of 7000–10,000 *g*.

All of the measurements were carried out at room temperature near 22C, whereas very much colder temperatures occur in the upper atmosphere. It can be shown theoretically, however (Thompson 1966), that the errors involved in applying the results of tests of this type made at room temperature to atmospheric temperatures occurring in a meteorological rocket sounding are small.

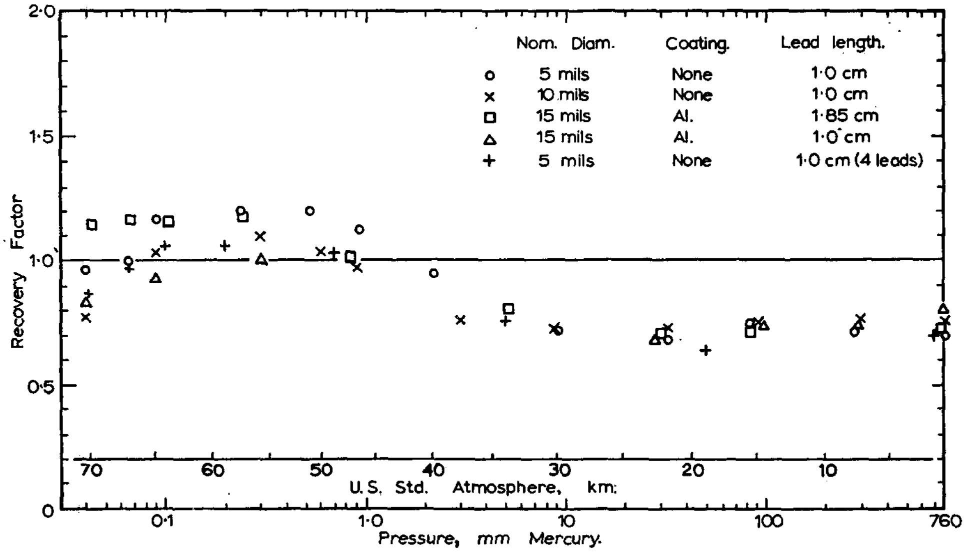

FIG. 3. Measured recovery factors of thermistors.

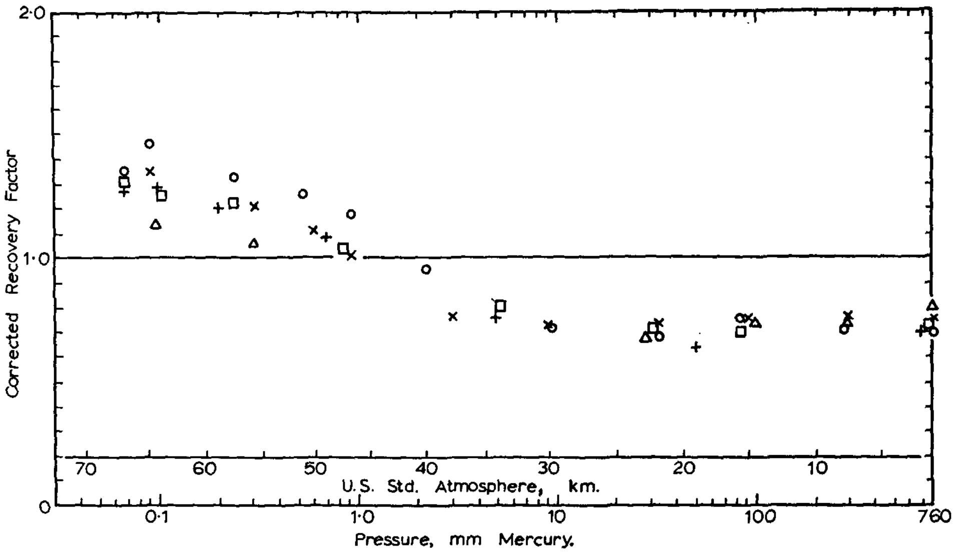

FIG. 4. Recovery factors of thermistors, corrected for conduction and radiation losses in the experiments. Symbols have the same meanings as in Fig. 3.

The current used in determining the thermistor bead temperatures was always sufficiently small that there was no significant electrical heating at any pressure.

3. Results

Fig. 2 shows the net temperature rise (i.e., the difference between that of the moving thermistor and that of the stationary thermistor) as a function of relative speed, plotted on logarithmic scales for a 10-mil thermistor. The slope of the lines is 2.0, indicating the validity of (1) and (2) for a constant r.

Fig. 3 shows the measured recovery factors of five miniature bead thermistors as a function of pressure. The nominal diameter of the beads is indicated in mils. The 10- and 15-mil thermistors had 0.001-inch diameter platinum-iridium leads and the 5-mil thermistors had 0.0007-inch diameter leads also of platinum-iridium.

At pressures greater than 10 mm mercury the measured values are fairly constant with pressure and lie near 0.75. These results are somewhat lower than the value of 0.9 assumed for these pressures by other authors (e.g., Barr, 1961; Wagner, 1963) in evaluating the aerodynamic heating correction for rocketsonde thermistors. However, reference to the original papers showed that these authors had based their estimates on published experimental data taken at Mach numbers between 2 and 4. Moffat (1962) has summarized results of measurements of the recovery factors of butt-welded thermocouples of various wire diameters at sea level pressure and subsonic Mach numbers. For wires normal to the airflow, measured values of r were near 0.68 and for wires parallel to the flow r was near 0.86. Hottel and Kalitinsky (1945) made tests on a thermocouple consisting of 0.010-inch diameter wires whose junction was encased in a 0.07-inch diameter solder ball. Although much larger, this configuration is similar to a bead thermistor. With airstream parallel to the wire, r was about 0.78 and for flow normal to the wire r was 0.73.

At pressures below 7 mm mercury the measured recovery factors shown in Fig. 3 increase with decreasing pressures, and in most cases exceed unity below 1 mm mercury. A maximum is reached at about 0.2 mm mercury, at which pressure the Knudsen number is ~1 for the thermistor beads and ~10 for the lead wires. The reason for this maximum, and the fact that the measured recovery factors do not achieve the value expected for free-molecule flow at very low pressure, is that in these experiments both radiation to the cooler walls of the chamber and thermal conduction of heat to the arm through the lead wires prevent the thermistor from achieving the full recovery temperature at low pressures. Thermal inertia and conduction of heat to the main body of the rotating arm prevent the relatively heavy thermistor support posts from undergoing significant aerodynamic heating during the relatively brief duration of the tests. Thus, during the time required to make a measurement, we can assume that the thermistor supports and the aluminum walls of the vacuum chamber remain at a constant temperature, less than the recovery temperature of the thermistor.

It is possible to estimate a correction for these effects. Consider any element dA of the thermistor bead or lead-wire surface. Then for small temperature differences the net heat lost in unit time by radiation can be written as

$$dQ_r = 4\epsilon_l \sigma T^3 \theta_c dA,$$

where θ_c is the temperature difference between the element and the chamber walls, ϵ_l the emissivity and T the ambient temperature. We can therefore define heat-transfer coefficients

$$h_T'=4\epsilon_{lT}\sigma T^3 \quad \text{and} \quad h_w'=4\epsilon_{lw}\sigma T^3$$

for the heat exchange by radiation of the thermistor bead and lead wires which are analogous to those (h_T,h_w) describing the heat exchange with the air. If we introduce these and assume that the recovery temperatures of the thermistor bead and lead-wires individually are not greatly different from that of the thermistor as a whole, then reasoning similar to that used by Thompson and Keily (1967) in computing the response of a thermistor to solar radiation results in the following expression for the difference between the temperature rise expected in the absence of conduction and radiation and that actually measured:

$$(T_r-T_0)-\Delta T=(T_r-T_0)K^{-1}\{2ka_wp[\alpha \coth pd + (1-\alpha)\,\mathrm{csch}\, pd]+h_T'A_T\}, \quad (3)$$

where

$$K=2ka_wp \coth pd+(h_T+h_T')A_T.$$

In this equation, a_w is the cross-sectional area of a thermistor lead-wire of thermal conductivity k, radius r_w and length d,

$$p=\{2(h_w+h_w')/kr_w\}^{\frac{1}{2}}, \quad \alpha=h_w'/(h_w+h_w'),$$

and A_T is the surface area of the thermistor bead. The quantity K is the "dissipation rate" of the thermistor as mounted in the chamber, and can be readily measured independently (Thompson, 1966). At subsonic Mach numbers both K and h_w are functions of pressure but are substantially independent of airspeed for pressures less than 1 mm mercury.

The recovery factors of the five thermistors, corrected for conduction and radiation with the aid of (3), are shown in Fig. 4. Measured values of K and A_T for the actual thermistors were used, while values of h_w for the thermistor lead wires were taken from Thompson (1966). For uncoated thermistors ϵ_{lT} was taken as 1.0 and for the two aluminized thermistors ϵ_{lT} was taken as 0.13 on the basis of experimental results given in the same report. The emissivity of the thermistor lead wires was assumed to be 0.10. No corrections were attempted for pressures lower than 0.05 mm mercury.

It is seen that the corrected recovery factors continue to increase with decreasing pressures, and approach values more consistent with the free-molecule theory. Because it is difficult to assess the accuracy of the individual measurements, it is perhaps not wise to draw conclusions about individual thermistor types. However, the results show a tendency for the smaller thermistors to have higher recovery factors at low pressures as would be expected, since at a given pressure the Knudsen number would be higher. The "four-lead" thermistor was an experimental type having four lead wires instead of the usual two and since the bead shape was consequently more deformed it cannot be compared directly with the others.

It should be noted that before applying these results to actual rocket soundings it is necessary to ascertain that the thermistors are exposed sufficiently clear of the rocketsonde body so that the airflow over the thermistor is not disturbed. It should also be noted that these results cannot necessarily be applied directly to rocket soundings in which the recently-developed "thin-film" method of mounting the thermistor is used. As pointed out by Morrissey and Carten (1967), there is considerable coupling between the film and the thermistor so that the aerodynamic heating error of the sensor as a whole must depend on the degree of aerodynamic heating of the film as well as on that of the thermistor. This coupling has yet to be fully studied.

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