Synoptic Density Maps of the Upper Atmosphere[1]

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ABSTRACT

Knowledge of the horizontal variation of density is required in a number of problems, such as the calculation of aerodynamic heating rates of lifting reentry vehicles in near-horizontal flight. On constant-pressure surfaces the density field is given by the isotherms; but since the pressure surfaces may have great inclination in the stratosphere and mesosphere, compared with the troposphere, constant-height analyses are needed for a direct measure of the horizontal density variations. With suitable assumptions, previously analyzed constant-pressure maps can be converted hydrostatically to yield accurate constant-height maps. A computerized procedure is described for obtaining such maps from high-level constant-pressure analyses of the National Meteorological Center. Examples of synoptic density maps obtained for 30 and 40 km are shown and are discussed in relation to standard atmosphere values of the density. Good agreement is found between average values based on the maps and 1966 Supplemental Atmosphere values for 30 and 60N. The synoptic variability, however, is considerable, with strong horizontal gradients a common feature in high latitudes in winter. The summer atmosphere at the heights examined is found to be nearly barotropic, with relatively weak density gradients. For a hypothetical reentry trajectory, sample data for a specific day in winter are used to calculate the density changes along the trajectory.

1. Introduction

In meteorological calculations requiring knowledge of air density, two practices are often followed; that is, either 1) pressure is used as a vertical coordinate and the density is implicitly taken into account through variations in temperature, or 2) it is assumed that the density variation on a constant-pressure surface, given by the isotherms, gives a satisfactory approximation of the horizontal density variation. In the stratosphere and mesosphere the pressure surfaces may have great inclination, compared with the troposphere, and other means must be found for obtaining the horizontal distributions of density. Excellent climatological data at constant altitude are available, as in the *U. S. Standard Atmosphere Supplements, 1966*, but little is known concerning the *synoptic* variations of density.

Aside from their basic value for depicting the atmospheric structure, synoptic data are useful in a variety of applied problems. The fuel consumption of supersonic aircraft, for example, can be shown to be an implicit function of the air density. In actual practice the temperature or density may be used, according to the conditions of flight (Duvergé, 1967). If the aircraft flies at pressure altitude, temperature data may be used, but if radio altimeters are employed and the aircraft flies at true altitude, the density is required.

Another problem is the calculation of aerodynamic heating of lifting reentry vehicles (Martin, 1966), which is a function of the density changes along the reentry trajectory. For certain vehicles the region of the mesopause is most critical, as here large *vertical* density gradients are expected in association with low temperatures and temperature inversions (Fig. 1), in accordance with the equation of state, differentiated with respect to height, i.e.,

$$-(1/\rho)\partial\rho/\partial z = g/(RT) + (1/T)\partial T/\partial z.$$

Another region of interest is the upper stratosphere and lower mesosphere, where large *horizontal* gradients of the density are expected in certain synoptic situations, primarily in winter, in connection with differential heating of the stratosphere. It is in this altitude region that the trajectory of certain reentry vehicles undergoes a quasi-horizontal damped oscillation (Fig. 2), and the wings, so to speak, must be designed to withstand the aerodynamic heating due to the horizontal density increases.

It is the purpose of this paper to describe the derivation of a pilot series of stratospheric constant-height density maps and to provide preliminary information on the horizontal gradients encountered. Sample data will then be used to calculate the density along the trajectory of a hypothetical reentering vehicle.

2. Procedure

a. General considerations

Constant-height density maps can be constructed in a number of ways. Faust and Rogall (1962) took

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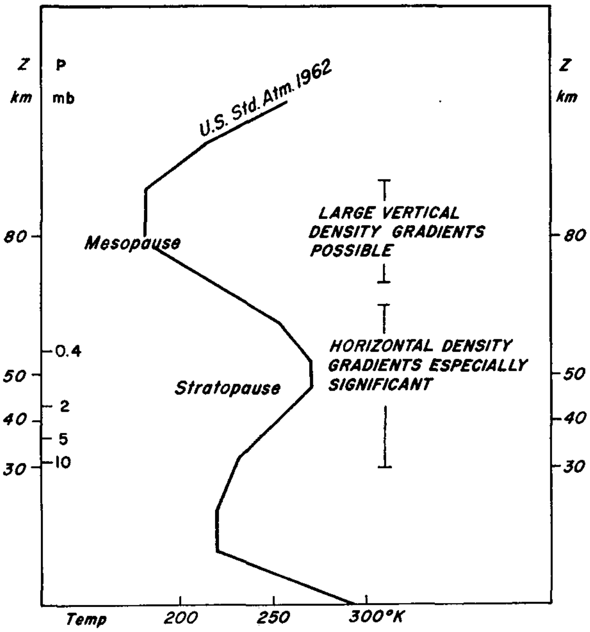

FIG. 1. Altitude regions with density gradients of potential significance to vehicle reentry. Constant-height density maps described in article are derived from NMC constant-pressure analyses for pressure levels shown at left.

radiosonde observations for a large network of stations and from pressure-altitude curves of the data, obtained densities at kilometric heights. These served as the basis for a short series of constant-height analyses at levels up to 20 km. A practical alternative is the use of previously analyzed constant-pressure maps as a base from which to extrapolate the pressure or density to some fixed height. For hydrostatic extrapolation, the temperature through the layer of extrapolation must be known. Wehry (1966) constructed density maps to 30 km by this method, assuming that the base temperature on a constant-pressure surface prevailed throughout the layer. Wehry was interested in evaluating density effects on supersonic transport performance in the lower stratosphere. In this part of the atmosphere departures from an isothermal lapse rate are not ordinarily great, and the error due to neglect of the actual temperature structure may not be large. Our procedure takes into account the actual temperature conditions, with certain simplications noted below.

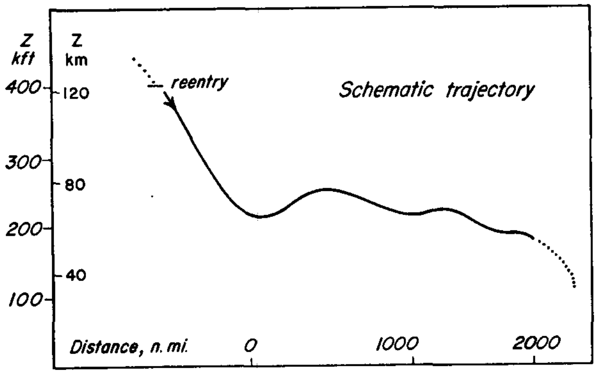

FIG. 2. Schematic reentry trajectory with quasi-horizontal oscillatory leg, adapted from Fig. 1, Air Weather Service (1965). For other examples of such trajectories, see Sissenwine (1968).

For extrapolation we use the integrated hydrostatic equation

$$p=p_0(T/T_0)^{-g/R\gamma},\quad \gamma=\partial T/\partial z, \tag{1}$$

or, on substituting for p from the equation of state,

$$\rho=\rho_0(T_0/T)(T/T_0)^{-g/R\gamma}. \tag{2}$$

For the special case of an isothermal layer, the corresponding equations are

$$p=p_0\exp\{-g\Delta z/(RT)\}, \tag{1a}$$

$$\rho=\rho_0\exp\{-g\Delta z/(RT)\}. \tag{2a}$$

In the above, g, the acceleration of gravity, is assumed constant, and p, ρ, and T and R have their usual meteorological meanings. The zero subscript refers to some starting level, and p, ρ and T without subscript are at an upper or lower level, depending on whether upward or downward integration is performed. Eqs. (1a) and (2a), exact for an isothermal layer, may be used for a non-isothermal layer with a high degree of accuracy if for T the mean temperature of the layer is employed. This, indeed, is the form of the equation used by the Meteorological Rocket Network (Webb *et al.*, 1966) for deriving pressures and densities from rocket temperature profiles.

It may be noted that while either pressures or densities can be extrapolated with the same facility, it would make sense to derive first the density field at constant pressure, which is merely a function of the temperature, and then by extrapolation to find the field of density at constant height. Inspection of the more general forms of the hydrostatic equation (1) and (2), however, shows that extrapolating the pressure should be slightly more accurate in the case of uncertain lapse rates, owing to the presence of an additional term, T_0/T, in the density relationship. For this reason, we have preferred to use (1a) rather than (2a).

From the extrapolated pressures and observed temperatures at constant height, the density may then be computed by the equation of state. The computations must be made at a large number of points in order to define adequately the fields of pressure, temperature and density at constant height. With the aid of a computer, however, only one time-consuming task is required; namely, the preparation of the basic temperature and height data taken from constant-pressure maps.

b. Constant-pressure maps utilized

Weekly constant-pressure maps for the 10-, 5- and 2-mb surfaces have been used for the derivation at constant height of maps for the 30-, 40- and 50-km levels. The constant-pressure maps, based on radiosonde data at the lower levels and on radiosonde and rocket data at the upper levels, were analyzed by the Upper Air Branch, National Meteorological Center (NMC), according to procedures described by Finger *et al.* (1965, 1966). The rocket-level charts have been constructed by an iterative process which takes into account both the observational data at the levels of analysis, which are often sparse, and the contour field at 10 mb, based on a relatively large number of radiosonde observations. Moreover, the geostrophic and thermal wind equations have been used to insure consistency between the wind and the temperature and height fields.

c. *Computer program*

A program was written for steps 2 through 6 below, step 1 being carried out manually.

1) Extract and punch geopotential height and temperature of the 10-, 5- and 2-mb surfaces for a pre-defined network of grid points.
2) Compute the lapse rate (linear) between pairs of pressure levels.
3) Determine the thickness of the layer over which the pressure is to be extrapolated, making an appropriate conversion from geopotential to geometric height. (Layer thicknesses were usually less than 4 km.)
4) Calculate the temperature of a given constant-height level (30, 40 or 50 km) and then the mean temperature of the layer of extrapolation.
5) Extrapolate the pressure hydrostatically, according to Eq. (1a), to the nearest constant-height level.
6) Calculate the density from the pressure and temperature at constant height.

The calculations were carried out for each of 285 grid points primarily in the Western Hemisphere. The grid used has been adapted from the NMC numerical weather prediction grid. Under conditions of reduced atmospheric variability (summer and low latitudes in winter), input data were provided only at alternate grid points. The output data consist of grid values of constant height, according to the following model:

Pressure	(10^{-2} mb)
Temperature	(°C)
Density	(10^{-2} gm m^{-3})

The printout is designed so that isopleths of the three variables can be drawn directly on composite sheets making up the entire map area.

d. *Errors*

Error in the calculated pressures and densities is expected from several sources. Primarily, these include 1) observational error in the rocket and radiosonde temperatures, 2) error due to the use of an approximate form of the integrated hydrostatic equation, and 3) error in determining the layer thickness.

The differentiated equation of state,

$$\frac{d\rho}{\rho}=\frac{dp}{p}-\frac{dT}{T}, \tag{3}$$

is helpful for evaluating errors in the density. A principal error source is the rocket thermistor temperatures, which are believed to be 2–5C too warm at 50 km, the error decreasing as the rocket descends, according to Ballard (1967). In the construction of the basic constant-pressure maps, rocket temperatures were adjusted according to correction information given by Ballard. Nevertheless, assume that the temperature were too high by, say, 5C over a 4-km layer. The pressure extrapolated by means of the approximate hydrostatic equation (1a) would then be in error by 1.5% (pressure too high). At altitude, the temperature error would be approximately: $\Delta T/T \cong 5/250 = 2\%$. Since at altitude the pressure and temperature errors are of opposite sign [Eq. (3)], the net error in density should be less than 1%. This analysis in intended merely to give an order of magnitude for the basic component errors. Other factors must be considered. The extrapolated pressure is subject to further error, negligibly small for thicknesses of only a few kilometers, due to the use of Eq. (1a). Moreover, an error occurring in the *base* pressure will also appear in the derived density at altitude. A more precise analysis, however, would be deceptive, since temperature errors have to some degree been compensated for in the process of adjusting the pressure contour fields to achieve consistency with the wind. In general, it is expected that the error in the pressure and density due to uncertain knowledge of the temperature structure should not exceed 2% at 50 km and below.

Error of a different kind may stem from the determination of the layer thickness. Slight inaccuracies may be introduced in reading contour values from the constant-pressure maps, but the main error occurs in converting from geopotential to geometric altitude. Standard values of the geometric increment have been used corresponding to 45N latitude and the mean heights of the pressure surfaces. The error involved is about 70 m at 30 km, increasing to 120 m at 50 km, in very low and high latitudes. The combined error in the derived pressures and densities due to interpolation of contour values and use of standard geopotential corrections is expected not to exceed 1.5%. (If desired, this error can be reduced by use of geopotential corrections appropriate to the latitudes involved.)

A conservative estimate of the maximum total error in pressure and density is therefore about 4%. The average error probably lies in the vicinity of 2%.

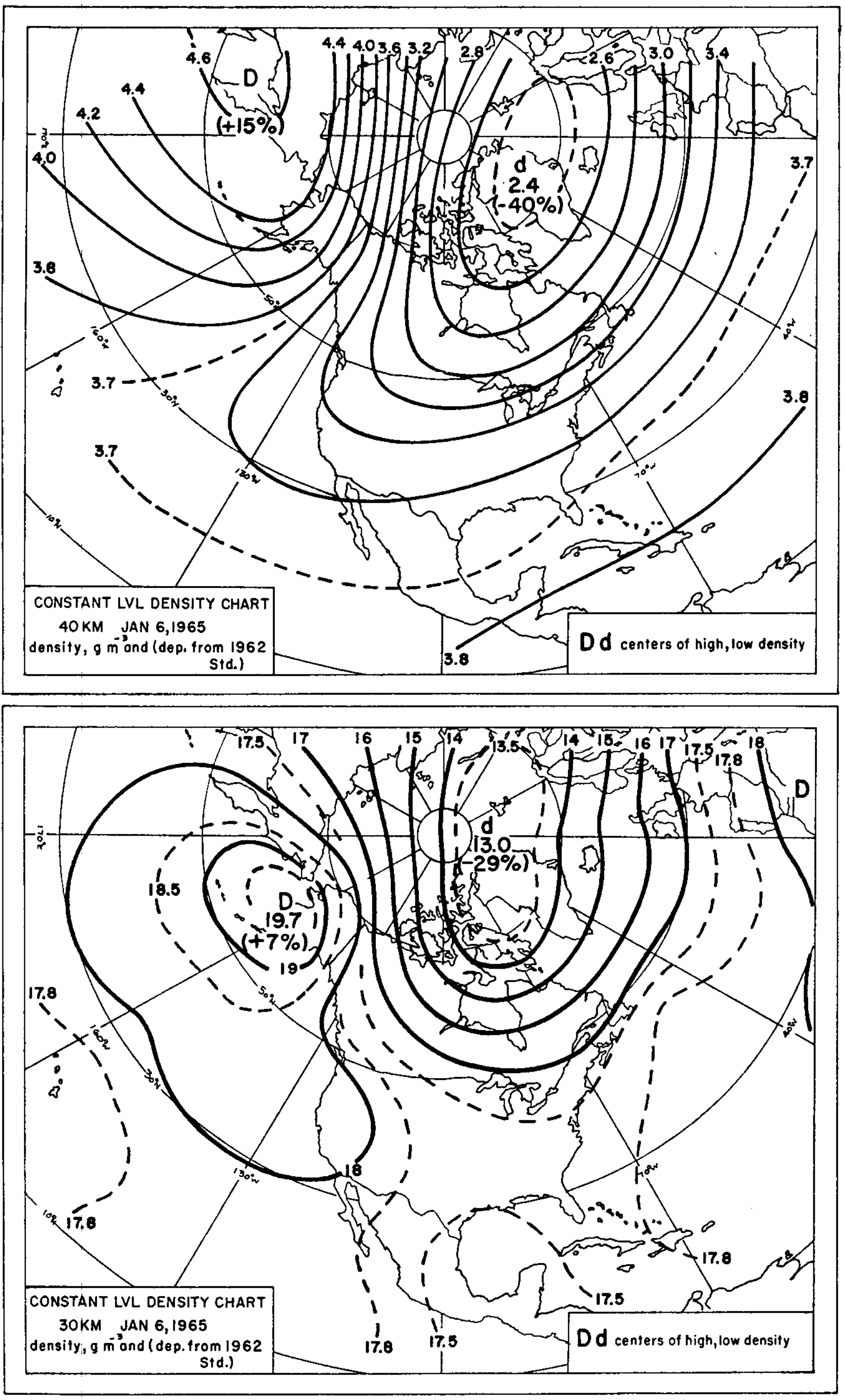

FIG. 3. Constant-level density maps for 6 January 1965 at 40 and 30 km with maximum percentage departures (in parentheses) from 1962 Standard Atmosphere. Basic isopycnic interval at both levels corresponds approximately to 5% changes in density. Note intense gradient between pole and Kamchatka-Aleutian region.

3. Results

With the aid of the computer program described, a pilot series of constant-height maps has been obtained at weekly intervals in January of 1964 and 1965 (i.e., on eight January dates) and for a single date near mid-month in July of each of these years. Only the isopycnics have been drawn, although analyses of the pressure and temperature fields can be obtained with

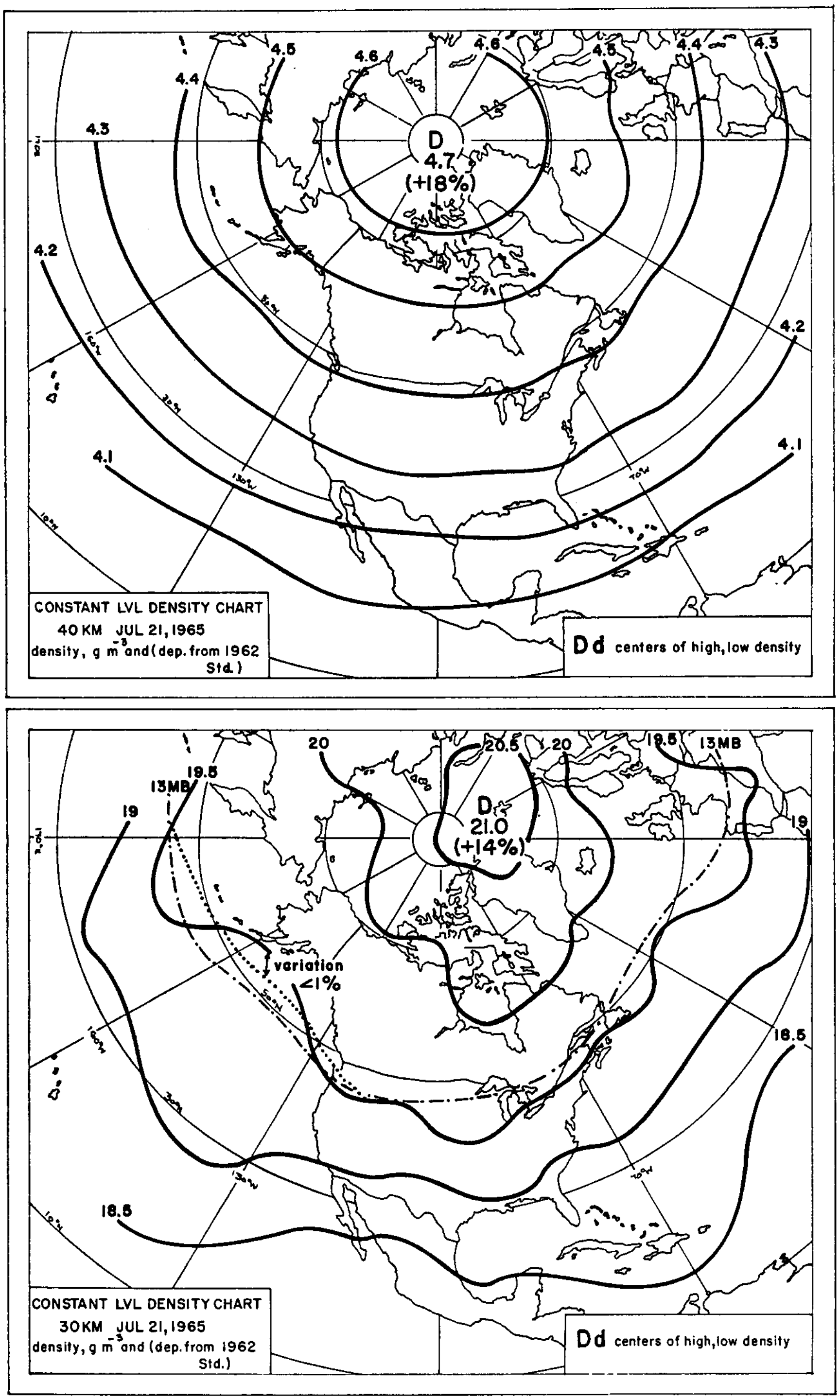

Fig. 4. Constant-level density maps for 21 July 1965 at 40 and 30 km with maximum percentage departures (in parentheses) from 1962 Standard Atmosphere. Isopycnic interval corresponds approximately to 2.5% changes in density. The 13.0-mb isobar has been entered on the map for 30 km; dotted line denotes smoothed isopycnic (see text).

little additional labor. The charts are for the 30- and 40-km levels. The layered structure at the stratopause requires that a special routine be introduced for deriving data for 50 km; charts for this level will be produced subsequently.

The maps obtained thus far reveal a number of interesting features regarding the synoptic variation of density and, moreover, can be used for preliminary climatological estimates of the density variability. Some of these features will be illustrated with the aid of selected maps for a day in winter (6 January 1965, Fig. 3) and a day in summer (21 July 1965, Fig. 4).

1) In winter the density over a wide span of longitude falls off sharply from middle latitudes to extremely low values near the pole. The center of low density is usually displaced some 10° from the pole, at longitudes between 40W and 80E. On the day shown (Fig. 3), the central value analyzed is 40% below standard at 40 km. The average value for January 1964–1965 was found to be 2.3 gm m^{-3} (43% below standard) at 40 km and 12.5 gm m^{-3} (33% below standard) at 30 km.

2) A special feature on the winter maps is a persistent area of high density, often shown as a closed synoptic system, in the Kamchatka-Aleutian region of the North Pacific. It can be shown from hydrostatic considerations (Quiroz and Miller, 1968) that the time changes of density at some altitude may be specified by the changes in pressure at an altitude one scale-height lower. Accordingly, the high density area of the North Pacific can be shown to be related to the well-known Aleutian anticyclone, a common winter feature on stratospheric constant-pressure charts. On the day of the map shown (Fig. 3), the density in the region of the Aleutian high is about 15% greater than standard at 40 km. The average value based on all the January maps is 11% greater at 40 km, 6% greater than standard at 30 km.

3) The maximum horizontal density gradients found on the maps may be of interest. Gradients in low latitudes are relatively uninteresting. A radical change of slope occurs in winter northward of 40–50N. The largest gradients are often oriented at some angle to the meridians and are usually found between the polar low-density center and the Aleutian high. In our map series, the maximum horizontal density change amounted to 2.6% per degree latitude sustained over a distance exceeding 1000 n mi (43% increase in 1000 n mi measured along 170E between the pole and 70N, on 6 January 1965).

4) In summer the latitudinal gradient is reversed at most longitudes and is much weaker. High density occurs in the vicinity of the pole. The maps for July indicate a central value close to 21.0 gm m^{-3} (14% greater than standard) at 30 km, and 4.7 gm m^{-3} (18% greater than standard) at 40 km. The summertime atmosphere is nearly barotropic, with only minor perturbations associated with a non-smooth temperature field. This point will be discussed further.

5) The essentially barotropic character of the summer atmosphere has been mentioned. The wavelike perturbations in the 30-km density on 21 July 1965, not evident in the pressure field (illustrated by the 13.0-mb isobar), are believed to be real and explainable by corresponding variations in the temperature. Such perturbations are not found on the chart for 40 km, which is based on a temperature structure in which small irregularities have been smoothed out. The reality of the temperature oscillations influencing the 30-km analysis may be questioned, and it is tempting to attribute these to observational error. A recent analysis of fine-scale structure revealed in specially reduced radiosonde and rocket data (Miller *et al.*, 1968) suggests that such temperature oscillations are indeed real. In any event, smoothing of the isopycnics can easily be done, if desired, and would involve an error of usually less than 1% in the density.

4. Climatological comparison

Despite the brief sampling period, the density charts appear to give a good idea of climatological tendencies. Moreover, they provide previously unavailable information on longitudinal differences and on horizontal gradients computed over variable distances. (Earlier information of this type has necessarily been inferred from widely separated observations in time and space.) It is emphasized that the map-based statistics discussed below are presented only as preliminary data; a longer sampling period is required for climatological stability.

Table 1 gives density means at longitude intervals of 30°, at 30 and 60N. The last three columns give the latitudinal averages, the 1966 Supplemental Atmosphere values, and the per cent departures of the latitudinal averages from the 1966 Supplemental Atmosphere values. The second line of each data group gives the per cent departure from the latitudinal average computed from the maps.

Excellent agreement can be seen with the Supplemental Atmospheres, the departure being at most 5% for 60N in January. The longitudinal variation in the averages is at most 5% at 30N (40 km), but amounts to as much as 40% at 60N (40 km), from −20% at 10W to +20% at 170E. It is interesting to note that zero departure from the latitudinal averages occurs at or near 100W longitude. In view of the small overall differences from the Supplemental Atmospheres, the latter may be judged to be most valid in the vicinity of this longitude. The Supplemental Atmospheres also provide for "warm regime" and "cold regime" values at the northerly latitude. Again, there is good agreement with the map values at the Pacific (170E) and Atlantic (10W) extremes of data in Table 1, respectively. A major point emerging from this analysis is that the longitudinal variation indicated above is a *characteristic* feature of the high-latitude winter. Density charts for a major stratospheric warming in late December 1967, to be discussed in a paper in preparation, reveal patterns which may be radically different from the characteristic pattern described here.

The longitudinal differences in Table 1 are shown quite dramatically in Fig. 5. Also provided, from values of the actual densities on each of the eight weekly maps

TABLE 1. Mean density (gm m^{-3}), 1964–1965, along 60N and 30N at various longitudes for January and July at 30 and 40 km, with per cent departure from latitudinal average (170E—10W); and departure of latitudinal averages from values in 1966 Supplemental Atmospheres (S.A.).

	Longitude 170E	160W	130W	100W	70W	40W	10W	Latitudinal average (170E—10W)	S.A.	Departure from S.A.
40 km										
					60N latitude					
Jan.	4.19 +20%	4.13 +18%	3.88 +11%	3.49 0%	3.10 −11%	2.88 −17%	2.80 −20%	3.49	3.33	+5%
Jul.			(longitudinal variation <1%)					4.52	4.53	0%
					30N latitude					
Jan.	4.04 +3%	3.97 +2%	3.91 0%	3.84 −2%	3.84 −2%	3.87 −1%	3.89 −1%	3.91	3.96	−1%
Jul.			(longitudinal variation <1%)					4.27	4.33	−1%
30 km										
					60N latitude					
Jan.	17.8 +6%	18.3 +9%	17.7 +5%	16.7 −1%	15.9 −5%	15.5 −8%	15.4 −8%	16.8	16.5	+2%
Jul.			(longitudinal variation <2%)					19.7	19.8	−1%
					30N latitude					
Jan.	18.0 +1%	18.1 +1%	17.9 0%	17.8 −1%	17.7 −1%	17.9 0%	18.1 +1%	17.9	17.8	+1%
Jul.			(longitudinal variation <2%)					18.9	19.1	−1%

(January, 1964–1965), is a preliminary indication of the dispersion as a function of latitude and longitude. The larger range at 60N in relation to 30N is consistent with variability estimates published in earlier studies. With respect to longitude, the appreciable dispersion at the western and eastern extremes of the data, along 60N, can be shown to be due to shifts in the position and intensity of the high and low density systems. The greatest variability, however, occurs between these systems, at 70–100W.

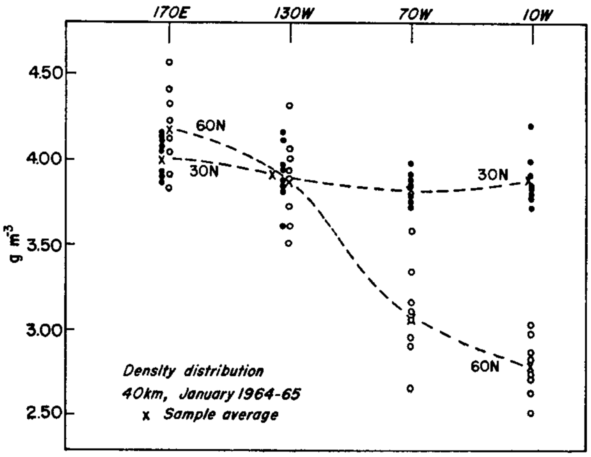

FIG. 5. Plot of weekly map values of density at selected grid points, illustrating latitudinal and longitudinal variations and dispersion of weekly values.

5. The density along a hypothetical reentry trajectory

Constant-level synoptic density maps provide a realistic means for evaluating the density changes along a given trajectory. Logarithmic interpolation is required to obtain values at points of the trajectory, and indeed this could be achieved between levels of constant-pressure maps. The constant-level maps, however, are more convenient from a computational standpoint, and they are eminently useful for directly isolating areas of potentially significant density effects.

Fig. 6 depicts a portion of hypothetical reentry trajectory, similar in form to that in Fig. 2 though at a lower range of altitude (35-40 km). The situation of 6 January 1965, characterized by a large density gradient along 170E in in high latitudes (Fig. 3), has been chosen for illustration. If one imagines the reentry vehicle moving southward along this meridian, then the

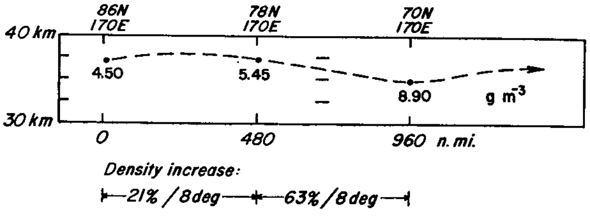

FIG. 6. Density values along a hypothetical trajectory along 170E on 6 January 1965. Trajectory is similar to, but not in scale with, trajectory in Fig. 2 and is depicted at lower altitudes.

densities along the trajectory would be as shown in Fig. 6. In the first segment, a density increase of 21% occurs in 480 n mi (8° latitude). The aerodynamic heating itself is, of course, a function of the time in flight, among other factors (Martin, 1966). In the second segment the density increase is 63%, of which two-thirds is explainable by the normal increase with decreasing height. The significant result here is that, owing to the presence of a large horizontal gradient, the total density change is 50% greater than would be experienced in a standard atmosphere.

6. Summary

A computerized procedure for obtaining constant-level density analyses from stratospheric constant-pressure charts has been described. The initial series of maps produced is for 30 and 40 km and for dates in January and July of 1964 and 1965. The average density error below 50 km is estimated at 2%. Maps for 50 km, which involve special consideration of the temperature structure about the stratopause, will be obtained subsequently.

The maps provide previously unavailable information on the synoptic variation of density. At 40 km, departures exceeding 40% below and 15% above standard may occur at centers of low and high density, respectively. (Variations at 30 km are similar but with reduced amplitude.) Horizontal gradients in winter are typically slight in low latitudes, with a radical change in slope often occurring north of 40–50N. Maximum gradients are commonly found at some inclination to the meridians, between a quasi-polar low and a high in the Aleutian area. The largest horizontal gradient found on the maps involved a density change of almost 3% per degree latitude, sustained over more than 15° latitude.

Good agreement was found between latitudinal averages computed from the maps and density values for 30 and 60N given in the *U. S. Supplemental Atmospheres, 1966*. Along a given latitude circle the longitudinal variation in high latitudes (winter) may reach 40% or more.

The summer atmosphere in the upper stratosphere is essentially barotropic, with a relatively weak density gradient between the pole (high density) and low latitudes.

Finally, it was shown, for a hypothetical reentry trajectory along a density ascendant on a specific day, that the horizontal density change contributes significantly to the total density change along the trajectory.

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