A Raingage Evaluation of the Miami Reflectivity-Rainfall Rate Relation

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ABSTRACT

To provide a foundation for other radar studies in the Miami areas, 50 comparisons were made between shower rainfall recorded by raingages and observed with radar to evaluate the reflectivity Z, rainfall rate R relation, $Z=300R^{1.4}$, referred to here as the Miami Z-R relation. Total shower rainfalls measured by recording raingages were compared with estimates derived from the Miami Z-R relation in conjunction with radar reflectivity measurements with iso-echo contouring and the analysis scheme described. Rainfall rate comparisons were not possible because of the poor time resolution of the raingage observations. The radar and raingage estimates of shower rainfall were highly correlated (+0.93, significant at the 1% level); they had an average difference of 8% and a mean absolute difference of 30%. Stratification by shower amount revealed that the radar estimate of gage-recorded rainfall was too high for small shower amounts (<0.25 inch) and too low for large shower amounts. In terms of percentage the comparison was best for the heavy showers. Stepwise regression analysis showed that consideration of the square of the range from gage to radar, in addition to range normalization already provided in the radar receiving system, made a small (3%) but statistically significant (<1% level) reduction in the variance and improved the correlation (0.93 to 0.944) between the gage and radar estimates of precipitation. It is concluded that the Miami Z-R relation, when used with the radar system described, is an effective tool in representing point and area rainfall from South Flordia convective showers.

1. Introduction

Over the last two decades radar has been used for quantitative measurements of rainfall, based on investigations relating rainfall rate R to reflectivity Z. The Z-R relations, computed directly by measuring radar reflectivity and rainfall amount or indirectly by measuring raindrop size spectra, have been derived for various locations, seasons and storm types. Stout and Mueller (1968) give an excellent survey of these relations and a discussion of their accuracy when used for quantitative rainfall measurements.

The accuracy of Z-R relations is assuming greater importance because these relations are now being used in evaluating the results of seeding experiments designed to increase rainfall. Cloud seeding experiments in Arizona (Davis *et al.*, 1968) and in Florida (Woodley, 1970) are but two recent examples. The Florida study showed that seeding increased rainfall an average of 100–150 acre-ft per cloud by 40 min after the seeding pass, representing an increase of over 100%. If these radar-derived precipitation results are to have credibility, it is important to demonstrate that the Z-R relation and the radar system used in the analysis accurately represented rain reaching the ground during the period of experimentation.

The Miami Z-R relation is based on the work by Sims *et al.* (1963) who obtained the Z-R relation

$$Z=286R^{1.43}, \tag{1}$$

by independently calculating reflectivity (Z in $\text{mm}^6\,\text{m}^{-3}$) and rainfall rate ($R$ in mm hr^{-1}) from raindrop photographs taken during showers in Miami. Eq. (1) has been modified slightly to

$$Z=300R^{1.4}, \tag{2}$$

by Gerrish and Hiser (1965), who averaged the coefficients and exponents of the Z-R relations derived by Sims *et al.* (1963) for air mass (wet season), easterly wave, cold trough and overrunning situations, and for showers and thunderstorms for Miami. Eq. (2), referred to here as the Miami Z-R relation, is evaluated in this paper by comparing recording raingage measurements with shower rainfall estimated from the same filmed radar observations used in studying the experimental clouds during May 1968. Only total shower rainfalls were compared because the time scale on the recording raingage trace was too compressed to permit rainfall rate comparisons. Gage measured rainfall was the standard of comparison, although the accuracy might be questionable since no attempt was made to evaluate the condition and exposure of the raingages used.

2. Radar systems

The modified UM/10-cm radar of the Radar Meteorological Laboratory, Institute of Marine Sciences, University of Miami, was used in the comparison. It has a 2° conical beam, a transmitter power of 5.5×10^5 W, a

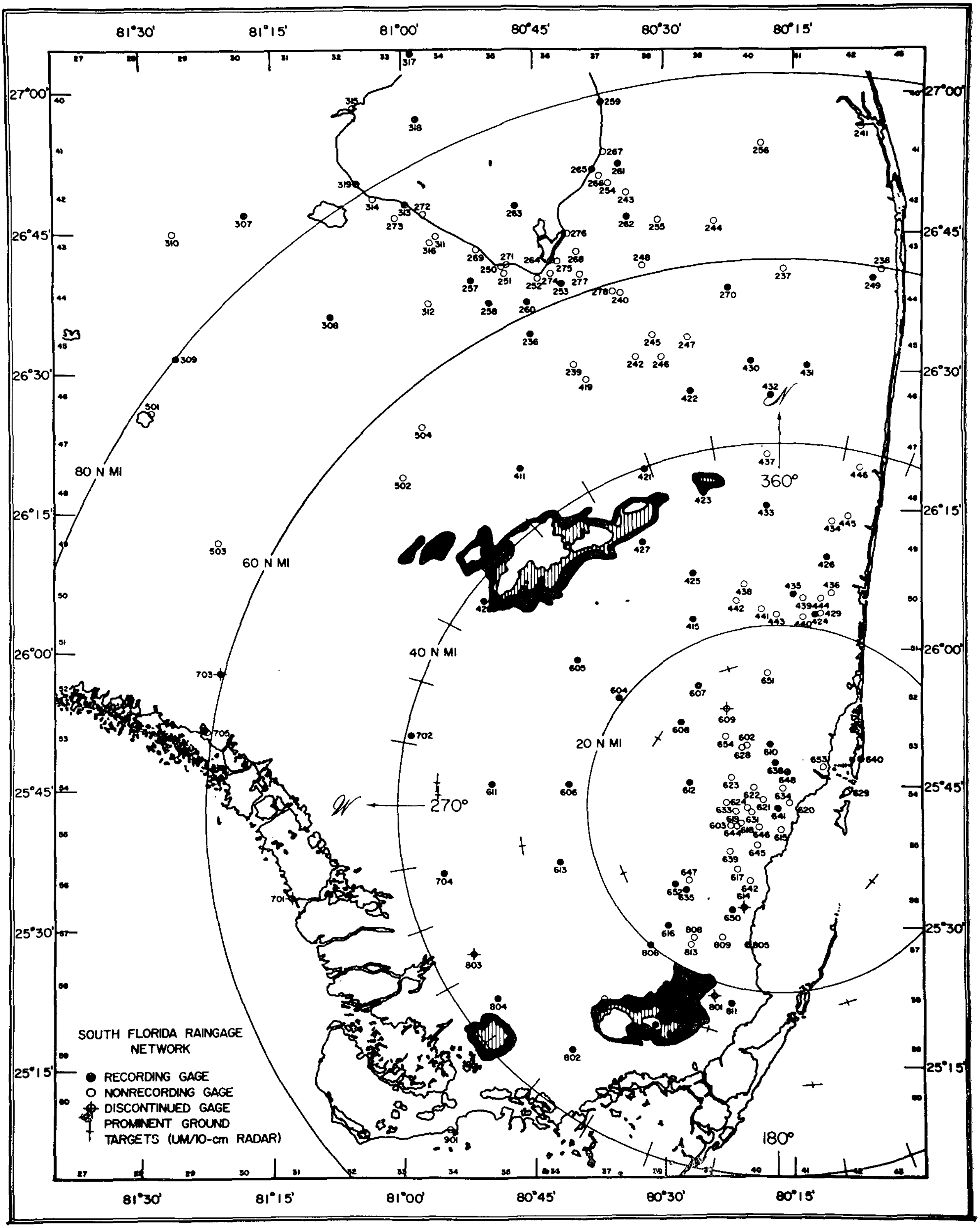

FIG. 1. Example of contoured echoes superimposed on the South Florida raingage network. The first (boundary) contour corresponds to 0.006 inch hr^{-1}, the second to 0.09 inch hr^{-1}. Inside the white area the rainfall rate >0.40 inch hr^{-1}.

TABLE 1. Comparison of radar (R_a) and raingage (G) rainfall using recording raingages and the UM/10-cm radar.

DATE (MAY)	STN. NO.	RAIN GAGE MEASUREMENTS RAINFALL (G) (IN.)	RADAR MEASUREMENTS			$G-R_a$ (IN.)	$G-G_p$ $G_p=-0.1488+1.1428R_a+.000153d^2$	DIST. OF GAGE FROM RADAR (N. MI.)
			MAX RAINFALL RATE (IN HR^{-1})	SHOWER DURATION (MIN.)	RAINFALL (R_a) (IN.)			
16	433	.00	.20	11	.01	-.01	-.03	33.3
16	423	.07	.45	37	.10	-.03	-.09	35.8
16	415	.05	.10	35	.03	.02	.05	27.5
19	422	1.22	3.40	52	.71	.51	.22	47.0
19	423	.22	2.00	44	.29	-.07	-.16	35.8
19	425	.15	.80	70	.22	-.07	-.07	27.4
19	430	.18	1.50	56	.37	-.19	-.18	23.0
19	415	.05	1.50	62	.18	-.13	-.12	27.5
19	425	.08	.60	30	.10	-.02	.00	27.4
19	424	.03	.20	29	.02	.01	.08	21.6
19	424	.20	1.00	34	.13	.07	.13	21.6
20	812	.78	1.50	62	.41	.37	.35	27.0
20	426	1.41	3.60	56	1.19	.22	.08	28.1
20	428	.10	.40	15	.03	.07	-.01	38.2
20	604	.21	4.00	35	.46	-.25	-.23	20.4
20	605	.19	2.00	22	.21	-.02	-.01	26.5
20	423	.11	.50	21	.09	.02	-.04	35.8
20	435	.39	3.50	28	.40	-.01	.00	23.5
20	807	.99	3.50	65	.92	.07	.01	22.5
20	613	.18	.60	48	.11	.07	.12	23.5
21	435	.10	1.00	35	.13	-.03	.02	23.5
21	605	.01	.50	16	.02	-.01	.03	26.5
25	807	.48	.65	88	.39	.09	.11	22.5
26	702	.20	1.50	32	.13	.07	-.03	38.7
26	433	1.06	3.00	35	.77	.29	.16	33.3
27	426	.11	1.50	18	.14	-.03	-.02	28.1
28	428	.30	.55	35	.18	.12	.02	38.2
28	812	.17	.50	50	.16	.01	.02	27.0
28	812	.47	.90	55	.33	.14	.13	27.0
28	811	.15	.50	48	.16	-.01	.14	22.1
28	432	.02	.40	22	.05	-.03	-.21	45.6
28	422	.43	.45	17	.06	.37	.17	47.0
28	807	.13	.45	50	.11	.02	.08	22.5
28	807	.03	.60	20	.06	-.03	.03	22.5
28	804	.15	.60	22	.07	.08	.02	36.1
30	613	.00	.40	14	.04	-.04	.02	23.5
30	423	.06	.60	28	.10	-.04	-.10	35.8
30	605	.68	4.00	64	.81	-.13	-.21	26.5
30	423	.90	4.00	60	.77	.13	-.03	35.8
30	423	.98	4.00	73	1.04	-.06	-.26	35.8
30	605	.22	5.00	17	.62	-.40	-.45	26.5
30	424	.38	5.00	32	.55	-.17	-.17	21.6
30	426	.18	1.00	21	.17	.01	.01	28.1
30	807	.22	2.45	21	.30	-.08	-.05	22.5
30	807	.60	2.00	38	.46	.14	-.15	22.5
30	811	.04	.55	26	.07	-.03	.03	22.1
30	812	.05	.60	20	.09	-.04	-.02	27.0
30	811	.48	3.55	30	.38	.10	.12	22.1
30	702	1.40	4.00	113	1.05	.35	.12	38.7
30	435	1.59	5.00	75	1.30	.29	.17	23.5

minimum detectable signal of 10^{-14} W, a pulse length of 2 μsec, and a pulse repetition rate of 300 pulses sec^{-1}. Included are special logarithmic and linear radar receivers, an rf range attenuation corrector, and an iso-echo contour (IEC) unit. The UM/10-cm radar was calibrated twice daily during the experiment. For more details on this radar unit, including a discussion of its calibration, the reader is referred to Senn and Courtright (1968) and Woodley (1970).

The comparisons between radar and raingage rainfall observations were made in the annulus 20–50 n mi from the radar. The antenna tilt was 0.5°, which means that the center of the radar beam was within 1500 ft of cloud base (~2500 ft) for clouds within this annulus. All

TABLE 2. Statistical summary of comparison between rainfall recorded by rain gages (G) and observed by radar (R_a). Columns 1–5 are in inches, columns 6 and 8 percentages, the others dimensionless.

1. ΣG	2. $\bar{G}$	3. ΣR_a	4. $\bar{R}_a$	5. $\overline{G-R_a}$	6. $\frac{\overline{G-R_a}}{\bar{G}}\times 100$	7. $\overline{\lvert G-R_a\rvert}$	8. $\frac{\overline{\lvert G-R_a\rvert}}{\bar{G}}\times 100$	9.[a] r_{G,R_a}	10.[b] b	11.[c] R_{a_0}
17.51	0.36	16.58	0.33	0.03	8	0.11	30	0.93	0.80	0.06

[a] Correlation of G and R_a.

[b] Slope of least-squares best fit line.

[c] Intercept on R_a axis of least-squares best fit line.

cloud echoes were contoured with the IEC unit built at the radar laboratory (Senn and Andrews, 1968).

3. Method

The photographs of the radarscope were projected frame by frame onto the raingage map as shown in Fig. 1. The map projection represents true distances to within 50 ft. The ground targets on the map were aligned with the corresponding ground targets on the radar film. Several areas of contoured echoes are shown schematically on the raingage map. Each contour corresponds to a reflectivity threshold and rainfall rate; the former was obtained with the calibration systems described by Andrews and Senn (1968), the latter from the Miami Z-R relation (2).

Readings were made from the radar photographs taken at 1–2 min intervals at the location of a recording raingage, plotted vs time, integrated with a planimeter to provide total shower rainfall, and then compared with the shower rainfall recorded by the raingage. Evaporation of the raindrops in falling from the level of scan to the ground was neglected. The person reading rainfall rate from the film did not know the gage amount until after the reading.

A linear interpolation scheme was used to determine the radar rainfall rate at a gage location between two known contours. The rainfall rate for a point bounded by only one contour, A for example, was obtained by linearly interpolating between A and the next higher contour permitted by the system, which was assumed to exist as a point value at the center of the area contained by A. The rainfall rate for a point within the highest contour permitted by the system was linearly interpolated between the boundary value and a rainfall rate of 6.00 inches hr^{-1}, which was assumed to exist as a point value at the center of the contoured area.

4. Analysis problems

Comparing a radar estimate of shower rainfall with a raingage estimate is difficult. The vertical separation (1000–4000 ft) between the gage and the level of the radar scan, and the drift of the precipitation while falling this distance, are decided problems. Also degrading the comparison are the tremendous differences between the size of the radar and raingage samples and the likelihood that the radar beam at 0.5° elevation is never uniformly filled with precipitation. The most serious obstacles are the convective rains in Florida; the observation that it often rains heavily on one side of the street and not on the other, is certainly true here. Unfortunately, the 10-cm radar does not have resolution to this distance scale, and even if it did, the 0.5 n mi diameter of the dot representing the raingage precludes accurate rainfall rate interpolation on a scale less than one-half the dot's diameter. Because a gage might easily be placed too close or too far by 0.25 n mi with respect to an echo, there will be errors in representing rainfall with a radar at the location of a gage even if the Z-R relation, the map projection, and the raingage locations on the map are perfect. However, the resulting error in estimating point rainfall should not be systematic.

5. Results

The raingage (G) and radar observations (R_a) permitted 50 comparisons, none of which involved seeded clouds. These comparisons are tabulated in Table 1 and summarized in Table 2. Using the raingage results as the standard, the mean absolute difference is about 30% while the mean difference is an 8% underestimate by the radar. The mean difference is defined here as the average difference divided by the average gage-recorded rainfall, rather than the mean of the individual percentage differences, in order not to give undue weight to the few comparisons with the small absolute differences but

TABLE 3. Stratification by shower amount of G and R_a comparison.

Rainfall category (inches)	n*	$\bar{G}$ (inches)	$\overline{G-R_a}$	$\frac{\overline{G-R_a}}{\bar{G}}$	σ (inches)	$\frac{\sigma}{\overline{G-R_a}}$
$0\leq G\leq 0.10$	15	0.046	−0.023	−0.49	0.04	−1.74
$0.11\leq G\leq 0.25$	17	0.175	−0.046	−0.27	0.13	−2.83
$0.26\leq G\leq 0.50$	7	0.418	0.091	0.22	0.18	1.98
$G>0.50$	11	1.055	0.200	0.20	0.20	1.00

$$\sigma=\left[\frac{\sum[(G_i-R_{a_i})-(\overline{G-R_a})]^2}{n-2}\right]^{1/2}$$

* Number of observations.

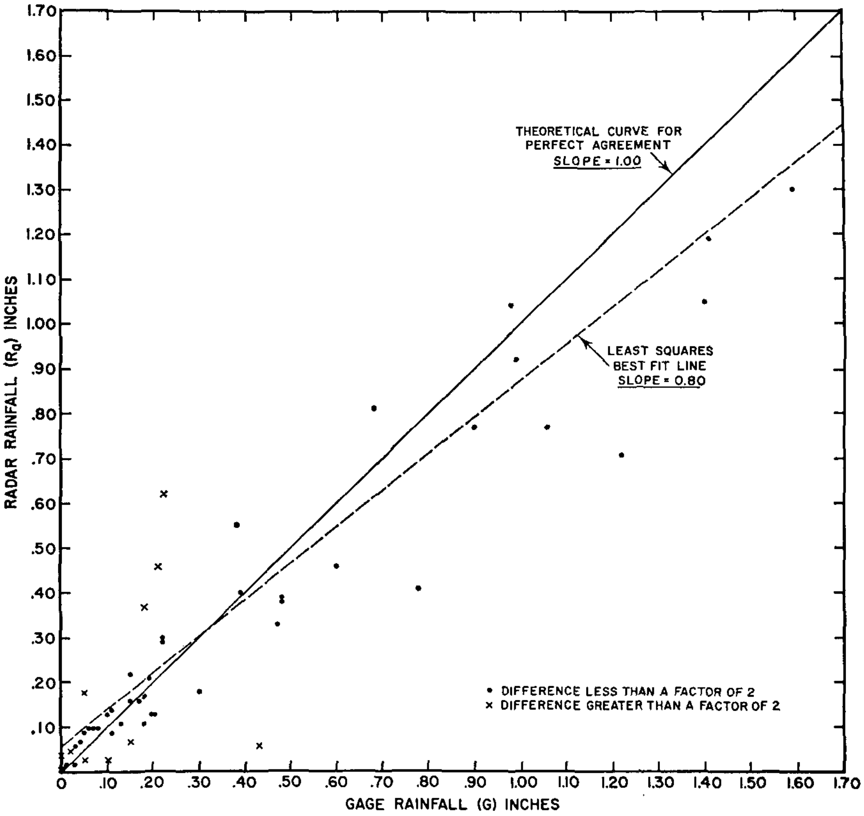

FIG. 2. Comparison between G and R_a. The solid line is the line of perfect agreement, and the dashed line the least-squares best fit line for the plotted points.

large percentage differences. The correlation coefficient between G and R_a was found to be 0.93, significant at better than the 1% level.

A more meaningful analysis of the comparison between G and R_a is stratification of the observations by shower amount, as shown in Table 3. The mean difference $(\overline{G-R_a})$ suggests that the radar overestimated the gage-recorded rainfall for shower amounts <0.25 inch and underestimated it for larger amounts. The radar did best for the heavy showers, as suggested by the ratio of the mean difference to the mean gage-recorded rainfall and the ratio of the standard error to the mean difference. The better radar-raingage agreement with increasing shower amount is a fortunate circumstance because the heavy rainfall is the one of most concern.

The stratification by shower amount of the radar-raingage comparison suggests that a different Z-R relation might work better in the Miami area. However, an alternate interpretation, just as satisfactory, centers on limitations inherent to the comparison and not to the Z-R relation. The radar overestimate of light showers might be explained by radar illumination of precipitation that evaporated before reaching the ground, while the radar underestimate of the heavy showers might be explained by the failure of the intense precipitation cores to uniformly fill the radar beam. It is not known which of the two interpretations of the stratification by shower amount is the more valid.

In Fig. 2, G and R_a are plotted on a scatter diagram, where the solid line is a theoretical line of perfect agreement (slope 1, intercept 0) between the two measures of shower rainfall and the dashed line is the least-squares best fit line (Guttman and Wilks, 1965) for the plotted points. Gage-recorded rainfall G was the independent variable in this analysis. The values of the slope and the R_a intercept of the best fit line are given in Table 2.

The scatter of points about the best fit line (Fig. 2) is probably the result of the nonsystematic, inexact placement of the raingages with respect to the echoes, especially in regions of intense rainfall rate gradient. The point scatter appears random, in contrast to the systematic error that would be generated by shortcomings in the Miami Z-R relation or the UM/10-cm radar system.

Regression analysis with G as the dependent variable was performed to derive a prediction equation that might then be used to estimate gage-recorded rainfall from rainfall measurements by radar. The best fit equation

$$G = -0.0128 + 1.1424R_a, \tag{3}$$

provides a better estimate of G than R_a itself.

Stepwise regression analysis revealed that consideration of the square of the distance (d^2) from the gage to the radar (in addition to range normalization already provided in the radar receiving system) would improve the radar estimate of gage measurements. With R_a and d^2 as predictors and G as the predictand, stepwise regression gave the equation

$$G_p = -0.1488 + 1.1428 R_a + 0.000153 d^2. \quad (4)$$

The multiple correlation of R_a and d^2 with G is 0.944, and the variance between the estimated and actual gage-recorded rainfall is reduced by 3% over what it was with a simple linear regression between G and R_a, a reduction significant at the 1% level. The multiple correlation was higher with d^2 than with d itself.

Eq. (4) indicates that the radar underestimated the rainfall with increasing shower to radar range. This is not surprising in view of the difficulty in uniformly filling the radar beam with precipitation at increased range.

Eq. (4) was used as a prediction equation to provide a new estimate G_p of the raingage rainfall; R_a and the ranges of the radar to the raingages tabulated in Table 2 were substituted into (4) and G_p calculated. The gage-recorded rainfalls G were then differenced with the corresponding predicted rainfalls G_p and tabulated in column 8 of Table 2. Eq. (4) generated estimates of G that are improved over the original estimates R_a. As examples, the average error is 1% if the differences $(G-G_p)$ are summed algebraically and 10% if their absolute values are summed.

6. Conclusions

Based on the preceding discussion the following conclusions seem justified:

1) The Miami Z-R relation in conjunction with the UM/10-cm radar system gave estimates of point rainfall that averaged within 30% of the values provided by raingages.

2) The radar tended to underestimate the gage-recorded rainfall with increasing shower amount and increasing distance of the gage from the radar.

3) In terms of percentage, the radar did best for large shower amounts.

4) Consideration of range effects in addition to range normalization made a small (3%), but statistically significant (<1%), reduction in the variance between the gage and radar estimates of precipitation.

5) The precipitation increases from Florida seeded clouds found by Woodley (1970), using the Miami Z-R relation, represent real increases on the ground, provided the Miami Z-R relation is equally valid for the seeded clouds.

6) The UM/10-cm radar in conjunction with the Miami Z-R relation should represent area rainfall better than point rainfall because in the former case the exact position of an echo with respect to a particular ground feature is not a critical consideration.

If one can extrapolate the above conclusions to other months during the South Florida wet season, this study has the following important implications:

1) The UM/10-cm radar with iso-echo contouring is an effective tool in evaluating cumulus seeding experiments in South Florida, probably the best tool for clouds at ranges no greater than 100 n mi.

2) Combined with the Miami Z-R relation, the radar should be of more use to hydrologists in South Florida than the current network of raingages because the radar i) generally comes within 30% of the rainfall measured by a gage and ii) provides the equivalent of a raingage network of infinite density when used to estimate area rainfall. This is vitally important in South Florida, where a meaningful interpolation between rainfall measured by raingages separated by more than a few miles is very difficult, if not impossible, to obtain.

If the Miami Z-R relation and the UM/10-cm radar systems are to be used in the future to estimate rainfall, it is important that the whole system be automated to circumvent the tedious analysis that was necessary in this study. Also, additional comparisons of this nature should be made in different months and under differing synoptic conditions to clarify the situation under which the Miami Z-R relation provides an accurate estimate of shower rainfall.

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