The Transfer Function of a Differential Microbarometer

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ABSTRACT

A differential microbarometer allowing the measurement of atmospheric pressure fluctuations with periods from 5 s to 18 h and resolution from 0.2 to 2.0 Pa is described. Experimental results of its calibration are in good agreement with acoustic theory provided that the transition from adiabatic to isothermal behavior is taken into account. Suggestions are made on the design of the instrument and the prediction of its transfer function from easily measurable quantities.

1. Introduction

The role of gravity waves in atmospheric processes is currently well recognized. All the atmospheric variables are influenced by them, but their presence is easily detected in barometric records because this variable is less affected by turbulence. In particular, differential microbarometers that filter out slow pressure variations identify the periodic fluctuations due to gravity waves with a resolution of few tenths of pascal. The response characteristics of a differential microbarometer constructed at the Università di Torino in Torino, Italy, will now be discussed.

2. The microbarometer and experimental setup

The instrument (Fig. 1) uses two chambers of equal volume. The “open” chamber is connected to the atmosphere by means of a pipe; the “closed” chamber is connected to the open one by means of a capillary. A differential pressure sensor (SETRA, model 239, with range ±0.01 psid = ±68.95 Pa) measures at time \( t \) the difference \( \Delta P(t) = P_1(t) - P_2(t) \) between the atmospheric pressure in the two chambers.

The microbarometer, thermally insulated, is usually placed underground in order to minimize temperature fluctuations in the closed chamber. Under these conditions and for time intervals short enough the pressure \( P_2 \) in the closed chamber can be considered constant, because the air flux through the capillary is too small to influence it. On the other hand, for longer intervals the air flux between the chambers allows pressure \( P_2 \) to follow the temporal evolution of atmospheric pressure \( P_1 \) with a certain delay. The differential microbarometer acts, therefore, as a high-pass pneumatic filter that measures the higher-frequency pressure fluctuations, filtering out the slow ones. In order to explore a frequency range as wide as possible, we have empirically chosen the length of the capillary to be as long as possible provided that diurnal pressure fluctuations are filtered enough to avoid exceeding the sensor’s range.

In order to check the filter characteristics of this instrument, a piston has been connected to a cylindrical barrel (Fig. 2) to produce sinusoidal pressure fluctuations that are measured by the microbarometer. The behavior of the system has been studied by varying the period of the oscillations. Technical data about the testing apparatus are listed in Table 1. The differential pressure between the two chambers, the absolute pressure in the barrel, the extension of the piston, and the temperatures in the barrel and in the closed chamber of the microbarometer were recorded during the tests. Both the barrel and the microbarometer were thermally insulated, and all the tests were performed in a room at constant temperature to prevent thermal effects on measured pressure.

3. Acoustic theory

The test system of Fig. 2 can be represented by a lumped-parameter model, because in the range of frequencies considered here the acoustic wavelength is much larger than the characteristic dimensions of the system. In this case (Pierce 1981) the commonly used variables are the volume velocity \( w \) across \( S \) and the (average) acoustic pressure \( p \), defined as

\[
w = \int_S v' \cdot n dS \quad \text{and} \quad p = \frac{1}{w} \int_S p v' \cdot n dS
\]

where \( S \) is any surface affected by the acoustic wave, \( n \) is the unit vector normal to \( S \), and \( v'(x, y, z) \) and

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p′(x, y, z) are the velocity and acoustic pressure. For surfaces perpendicular to the walls of the chambers and the pipes of the system, the volume velocity is related to the average displacement of fluid elements by the relation \( w = \frac{S d y}{d t} \).

In a lumped-parameter model, an acoustic system is analyzed by means of its electrical analog, in which the volume velocity corresponds to an electrical current and the acoustic pressure to a voltage. The acoustic impedance is defined as \( Z = \frac{\hat{p}}{\hat{w}} \), where \( \hat{w} \) and \( \hat{p} \) are complex quantities analogous to those used in electric circuit studies.

The electric analog of the testing system described above is shown in Fig. 3. The electric equivalents of enclosures B, 1, 2, of Fig. 2 are here the capacities \( C_B, C_1, C_2 \); acoustic capacity \( C \) of an enclosure of volume \( V \) is given by \( V/(c^2 \rho) \approx V/(\gamma P_0) \), where \( c \) is the speed of sound, \( \rho \) the air density, \( P_0 \) the ambient pressure, and \( \gamma = c_\rho/c_c \) the ratio of the specific heat at constant pressure to that at constant volume. The equivalent of tube \( T \) is the inductance \( L_T = \rho L/\left(\pi a^2\right) \approx P_0 L/\left(\frac{RT_0 \pi a^2}{2}\right) \), where \( L \) is the length of the tube, \( a \) its internal radius, \( T_0 \) the reference temperature, and \( R \) the gas constant for the air. Having a very small radius, the capillary \( C \) has an electric equivalent (Beranek 1954) in the acoustic resistance \( R = (8 \mu l)/(\pi r^4) \) where \( l \) and \( r \) are the length and radius of the capillary and \( \mu \) the dynamic viscosity of air [the inductive term \( 4 P_0 l/(3R T_0 \pi r^2) \) is negligible at frequencies studied here].

The piston's electric equivalent is an alternate current generator, because it causes at the bottom of the barrel a known sinusoidal displacement of constant amplitude \( \hat{y}(\omega, t) = y_0 \exp(\omega t) \).

By solving for the response of the circuit of Fig. 3, it is possible to calculate the acoustic pressure at each point and hence the pressure difference between the two chambers of the microbarometer \( \Delta \hat{p}(\omega, t) = \hat{p}_1(\omega, t) - \hat{p}_2(\omega, t) = A(\omega) \exp\{i(\omega t + \phi(\omega))\} \), where the amplitude \( A \) and phase \( \phi \) are both functions of the angular frequency \( \omega \).

Experimental results described in the next section have shown that heat conducted from the walls of the enclosures plays an important role, causing the expansion and compression of the air to be more isothermal than adiabatic as the period increases. This phenomenon has been studied theoretically by Daniels (1947) and Golay (1947) for simple cavity shapes including a sphere, a narrow rectangular box, and an infinitely long cylinder. The effect can be represented by a correction factor \( \hat{\Lambda} \) in the expression of the acoustic impedance of the enclosure, that is, \( Z = \hat{\Lambda}/(i \omega C) = \gamma P_0 \hat{\Lambda}/(i \omega V) \). The parameter \( \hat{\Lambda} \) is a complex function of the nondimensional variable \( x = [(c_\rho \omega)/(2k)]^{1/2} V/S \), where \( k \) is the thermal conductivity of air, \( V \) the volume, and \( S \) the internal surface of the
enclosure. The value of $\hat{\Lambda}$ varies from 1.0 in the completely adiabatic case to $1/\gamma$ in the completely isothermal case. The absolute value and the argument of the correction factor are shown in Fig. 4 and Fig. 5 and calculations are described in the Appendix.

4. Experimental results

The amplitude $A(\omega)$ and phase $\phi(\omega)$ of the microbarometer's output have been measured for a wide range of periods (from 0.5 s to 9 h) using two different pistons, by means of which sinusoidal fluctuations of 0.65 and 29.3 Pa have been generated. The damping $A/A_2$, relative to $A_{25}$, the output amplitude for a period of 2 s, and the phase $\phi$ are shown in Figs. 6 and 7 as functions of the period of oscillation.

In Fig. 8, dampings $A/A_2$, are shown for a wider range of periods. The resonance that appears at 0.25 s is caused by the coupling between tube $T$ and the open chamber, as verified from calculations performed including and neglecting these components. This implies that in order to eliminate this high-frequency resonance it will be necessary to decrease the volume of the open chamber (or to eliminate it), to increase the radius of the tube, and/or to apply a low-pass electric filter to microbarometer output.

To simulate behavior by means of the lumped-parameter model, it is necessary to know the resistance
means of the (complex) transfer function $\hat{H}(\omega) = \Delta\hat{p}(\omega, t)/\hat{p}_0(\omega, t)$ relating each harmonic component $\Delta\hat{p}(\omega, t)$ of the input signal to its output $\hat{p}_0(\omega, t)$. When the microbarometer is used in the field to measure the atmospheric pressure fluctuations, its electric equivalent is similar to the circuit of Fig. 3, without the capacity $C_B$ and with the current generator replaced by a voltage generator.

In this case the amplitude $p_0$ of the input pressure signal $\hat{p}(\omega, t) = p_0 \exp(i\omega t)$ is used as the reference amplitude. The theoretical predictions of the absolute value and the argument of the microbarometer transfer function, determined from the lumped-parameter model with the box enclosure adiabatic–isothermal correction, are shown in Fig. 9 and Fig. 10. The results are almost the same as the transfer function $\hat{H}(\omega) = i\omega t/(1 + i\omega t)$ for the RC (resistance–capacitance) circuit, with $\tau = 2\tau_c$. If the contributions of the tube and the open chamber, which have an influence only on periods less than 5 s, are neglected, the electrical analog of the microbarometer becomes in fact an RC circuit. Its time constant is influenced by the nearly isothermal behavior of the closed chamber, which in the completely isothermal case increases its acoustic capacity by a factor $\gamma$, implying $\tau = \gamma RC = 2\gamma\tau_c$.

The similarity of the RC curve to the theoretical prediction of acoustic theory shows that the departure of the closed chamber from isothermal behavior is negligible. Figures 9 and 10 show that the microbarometer correctly measures pressure fluctuations with a period ranging from 5 s to 20 min. Fluctuations with longer periods (up to 18 h, when damping is reduced to about 0.1) can be reconstructed from sensor output by means of the aforementioned transfer function. The sensor resolution, however, about 0.2 Pa, will be degraded in this case. For a period of 18 h, the resolution

R, which is very sensitive to the radius of the capillary. It is difficult to be sure that radius is constant along the length of the capillary, however, because the presence of some impurity can influence it. Therefore, a measure of capillary flow resistance was obtained from the time constant $\tau_c = RC/2$ of the exponential decay of differential pressure between the two chambers when the “open” chamber is sealed. From the experimental value $\tau_c = 429$ s and from the known value of capacity $C = C_0 = C_1$ of the chambers, the values of resistance $R$ and the radius of the capillary have been derived.

Theoretical predictions from the lumped-parameter model neglecting and including the adiabatic–isothermal correction factor for the aforementioned types of enclosures have been plotted in Figs. 5, 6, and 7. Without the correction, the theoretical predictions are far from the experimental data, but with the correction the differences between theoretical predictions and measurements are always less than the experimental errors. The correspondence between experiment and theory is very good when the formula for a box enclosure is used.

It appears, therefore, that with the Daniels (1947) correction factors, the acoustic theory correctly predicts measured damping. In particular, the resonance at lower periods is very well reproduced. The influence of the transition from adiabatic to isothermal behavior is mainly due to the presence of the barrel, because its dimensions cause the transition to occur for periods of 6–100 min. In this range the behavior is already nearly isothermal in the chambers.

5. The Transfer Function

If the microbarometer response is linear, the reconstruction of the input pressure signal is possible by

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**Fig. 8.** As Fig. 6 but in a wider range of periods to show the resonance at shorter periods.

**Fig. 9.** Absolute values of the microbarometer’s transfer function $\hat{H}$ versus periods: — from acoustic theory (with box enclosure correction factor); ---- RC circuit transfer function ($\tau = 2\tau_c$).
is about 2 Pa, or a factor of 10 worse than the resolution at 20 min.

As seen above, transition from adiabatic to isothermal behavior can influence the transfer function of a pneumatic system at low frequencies, and resonance at high frequencies can appear. In order to minimize these problems, it is therefore suggested that the ratio \( V_1/S \) between the volume and the internal surface of the closed chamber be selected to ensure isothermal behavior (an internally finned surface can help) and that the capillary be connected directly to the inlet pipe \( T \), including chamber 1 only for calibration purposes. In this case the transfer function of the instrument is that of an RC circuit whose time constant can be very easily derived using the relationship

\[
\tau = \gamma \left( 1 + \frac{s}{s} \right) \tau_e,
\]

where \( \tau_e \) is the time constant of the exponential decay of differential pressure when the (sealed) calibration chamber 1 is connected to chamber 2, and \( s \) is the ratio \( V_1/V_2 \) between the two chamber's volumes.

6. Conclusions

The experimental setup shown in Fig. 2 was used to test the performance of a differential microbarometer over a wide range of frequencies. The results show the importance of the transition from adiabatic to isothermal behavior. This phenomenon was accounted for by introducing the correction factor derived by Daniels (1947) in the lumped-parameter model of the system. With this correction, the agreement with experimental data was very good. The microbarometer transfer function can be determined from the exponential decay of differential pressure when it is connected to a calibration chamber of known volume. By means of this transfer function, atmospheric pressure fluctuations with periods from 5 s to 18 h can be reconstructed from the microbarometer output, although resolution decreases from 0.2 to 2.0 Pa for periods of 20 min to 18 h.

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APPENDIX

The Adiabatic–Isothermal Correction

Calculations of adiabatic–isothermal correction factor \( \Delta = |A| e^{\theta} \) for three types of enclosures (Daniels 1947) are listed here:

\[
|A| = (C^2 + D^2)^{-1/2} \quad \text{and} \quad \theta = \frac{\pi}{2} + \cot \frac{C}{D},
\]

where \( C = A(\gamma - 1) - \gamma \) and \( D = B(\gamma - 1) \). Coefficients \( A \) and \( B \) are defined as the real and imaginary parts of the following expressions, depending on the enclosure's type.

Narrow rectangular box:

\[
A + iB = 1 - \frac{\tanh y}{y};
\]

Sphere:

\[
A + iB = 1 - \left( \frac{3}{y} \right) \coth y + \left( \frac{3}{y^2} \right);
\]

Infinitely long cylinder:

\[
A + iB = 1 - \frac{2J_0(z)}{zJ_0(z)}.
\]

In these expressions \( y = (1 + i) x \), \( z = x(-2i)^{1/2} \) and \( x = \left( \frac{c_p \rho \omega}{(2k)} \right)^{1/2} V/S \), where \( \omega \) is the angular frequency, \( V \) the volume of the enclosure, \( S \) its internal surface, \( \rho \) the air density, \( c_p \) the specific heat at constant pressure, and \( k \) the thermal conductivity of air. Terms \( J_0 \) and \( J_1 \) are the first kind, order 0, and order 1 Bessel functions.

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