Using Radar-Measured Radial Vertical Velocities to Distinguish Precipitation Scattering from Clear-Air Scattering

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ABSTRACT

Thresholds separating regimes for which Rayleigh scattering from precipitation is likely to dominate over Bragg scattering from clear air are established for several common radar wind profiler frequencies. The thresholds are first determined for radar reflectivity factor \( Z \) based on observed values of the structure parameter \( C^2_z \) in the troposphere. These thresholds for \( Z \) are then transformed to thresholds for modal terminal velocities \( V^* \) of rain and snow for exponential particle size distributions. Measurements at 915, 404, and 50 MHz in stratiform rain help substantiate the applicability of the calculated thresholds, even though fall velocities, rather than \( V^* \), are measured. Because the \( V^* \) thresholds for rain at wind profiler frequencies (i.e., \( >2.5-5.6 \) m s\(^{-1}\) at 404 MHz) are substantially greater than typical vertical air motions outside convective cells, profiler-observed radial vertical velocities are a robust indicator of the presence of rain in profiler data. Although snow can also often be identified in this manner, the \( V^* \) thresholds (i.e., \( 0.5-1.2 \) m s\(^{-1}\)) are small enough to increase the probability that mesoscale clear-air vertical motions can mask or resemble its signature. The technique developed here allows rain, and to a lesser extent snow, to be identified in radar wind profiler data under most conditions without having to examine the entire Doppler power spectrum, even when the profiler is not calibrated to measure reflectivity factor.

1. Introduction

Bragg scattering from variations in the atmospheric index of refraction is the primary scattering mechanism that allows radar wind profilers to measure winds in the clear air. However, such radars are also sensitive to Rayleigh scattering from precipitation, as established at both VHF (30–300 MHz) and UHF (300–3000 MHz) by Fukao et al. (1985), Wakasugi et al. (1985, 1986, 1987a, 1987b), Larsen and Röttger (1987), Ecklund et al. (1988), and Wuertz et al. (1988). Under certain conditions the Doppler power spectrum can simultaneously provide measurements of vertical air motion and information about precipitation terminal velocities \( V^* \), as first discussed in Fukao et al. (1985), and later shown by Wakasugi et al. (1986, 1987a) and Gossard (1988). These important results led to more recent studies by Gossard et al. (1990), Rogers et al. (1991), Gossard et al. (1992), Currier et al. (1992), Rogers et al. (1993), Chilson et al. (1993), and Gossard (1994) that are rapidly expanding the application of wind profilers to the study of microphysical processes in clouds and precipitation. Hence, information about precipitation is now available simultaneously with wind profiles and can be integrated into studies of mesoscale phenomena in which precipitation plays an important role.

Although most of these studies have not focused on the problem of distinguishing between precipitation and clear-air signals in large datasets from profiler networks, the increasing use of profiler measurements in both research and operational meteorology has created a need for data interpretation techniques that readily and accurately identify precipitation directly from spectral moment data. Such techniques could, for example, be used to increase the accuracy of radio acoustic sounding system (RASS) measurements of virtual temperature by improving the correction made for vertical air motion. This correction is not yet used operationally because it introduces large errors when precipitation is present due to the fact that reflectivity-weighted precipitation fall velocity is measured rather than the vertical air motion (Angevine et al. 1994).

The goal of this paper is to quantify the sensitivities of wind profilers to precipitation in terms of radar-observed radial vertical velocities. This is done by determining what amount of Rayleigh scattering is required to almost always exceed observed values of Bragg scattering in the troposphere (section 2) and then transforming this reflectivity threshold into a threshold for reflectivity-weighted terminal velocity (section 3). Signal power is not used directly as a threshold because many radar wind profilers are not calibrated, that is, cannot measure reflectivity factor, and because power

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returned from clear air can be difficult to distinguish from power returned from precipitation without additional information. Because background levels of Bragg scattering varies from case to case and the form of the drop size distribution affects the result, the thresholds are expressed as a range. An event characterized by stratiform rain, and observed by 915-, 404-, and 50-MHz profilers, is used to illustrate the applicability of the derived thresholds (section 4).

Henceforth the following terminology is used here: the terminal velocity \(V_t\) of a precipitation particle is its downward motion relative to the air, fall velocity is the combination of terminal velocity and vertical air motion, radial vertical velocity is the reflectivity-weighted velocity measured by a vertically pointing radar beam, and modal vertical velocity is the terminal velocity of a precipitation particle whose diameter is equivalent to the modal diameter of the reflectivity-weighted drop size distribution (see section 3).

2. Scattering from clear air versus scattering from precipitation

At the wavelengths typically used by radar wind profilers (i.e., 0.3–6.0 m), Rayleigh scattering from precipitation can equal or exceed the Bragg scattering from clear air [see Gossard (1990) for a review of Bragg scattering]. This has been discussed in Larsen and Röttger (1987) and in Gossard (1988), both of which emphasize the variation of precipitation sensitivity with radar wavelength in terms of the strength of backscattered power. Although Bragg scattering has only a weak dependence on wavelength \(\eta \propto \lambda^{1/3}\), Rayleigh scattering is greatly enhanced at shorter wavelengths \(\eta \propto \lambda^{-2}\), where \(\eta\) is radar reflectivity and \(\lambda\) (m) is wavelength. These wavelength dependencies are combined in (1) [see Gossard (1988) and Rogers et al. (1993) for derivations], which gives the amount of Rayleigh scattering, expressed as radar reflectivity factor \(Z\) (mm\(^6\) m\(^{-3}\)), that would produce the same amount of backscattered power as a given amount of clear-air refractive index variability, which is denoted by the structure parameter \(C^2_n\) (m\(^{-2/3}\)),

\[
dBZ = 10(\log_{10}C^2_n + \log_{10}\lambda^{11/3} + 15.13),
\]

where \(dBZ = 10 \log Z\).

For this paper it is necessary to identify typical ranges of \(Z\) for which Rayleigh scattering from precipitation will most likely dominate clear-air return. This is done by determining two levels of \(C^2_n\), one representing the upper end of the range commonly observed in the troposphere and the other representing a value extreme enough that it occurs only on spatial and temporal scales much smaller than most precipitating weather systems. These are referred to as the “high” and “extreme” thresholds, respectively, and define a range over which the transition from clear air- to precipitation-dominated scattering is most likely to occur.

Data from a year of \(C^2_n\) observations at 805 m above ground level (AGL) reported by Chadwick and Moran (1980) yielded monthly averages that ranged from \(3 \times 10^{-17}\) to \(2 \times 10^{-15}\) m\(^{-2/3}\). Summaries of the vertical variation of \(C^2_n\) by Doviak et al. (1983) and Gossard and Stebbins (1990) show that on average \(C^2_n\) decreases with altitude in the troposphere, with median values ranging from \(4 \times 10^{-15}\) m\(^{-2/3}\) near sea level to \(10^{-12}\) m\(^{-2/3}\) at 12 km above sea level (MSL). The largest reported values of \(C^2_n\) include \(10^{-13} - 10^{-11}\) m\(^{-2/3}\) in the marine and convective boundary layers (Doviak and Berge 1980; Doviak et al. 1983), \(10^{-13} - 10^{-11}\) m\(^{-2/3}\) in a 25-m-thick layer in a breaking gravity wave (Gossard and Strauch 1983), and \(3 \times 10^{-16} - 3 \times 10^{-14}\) m\(^{-2/3}\) in moderate clear-air turbulence (Gossard 1988). Also, recent multiwavelength radar studies of small cumuliform clouds (Knight and Miller 1993) have made it possible to infer values as high as \(10^{-12} - 10^{-11}\) m\(^{-2/3}\) within regions roughly 200 m thick near the edges of growing cumuliform clouds.

These observations suggest that \(C^2_n = 10^{-13}\) m\(^{-2/3}\) and \(C^2_n = 10^{-15}\) m\(^{-2/3}\) can be used as the extreme and high thresholds, respectively. Based on the Chadwick and Moran (1980) climatology, the extreme threshold was exceeded only 0.5% of the time, while the high threshold was exceeded 23% of the time. Because \(C^2_n\) tends to decrease with height, these percentages should also decrease with height. The relatively large threshold values chosen here will also partially account for the enhancement of \(C^2_n\) in regions of strong water vapor gradients, such as in and near clouds. Figure 1 shows these thresholds as well as curves representing the relationship between \(C^2_n\) and \(Z\) for three common wind profiler frequencies. For example, at 404 MHz scattering from precipitation should almost always be stronger than the clear-air return when \(Z > 16.5\ dBZ\), while clear-air returns should almost always be stronger than Rayleigh scattering from precipitation or clouds when \(Z < -3.5\ dBZ\) (Fig. 1 and Eq. (1)). However, because of the natural variability of \(C^2_n\), it is possible that examples outside these thresholds could occur. The thresholds are also summarized in Table I for typical radar wind profiler frequencies, as well as for 3-, 5-, and 10-cm radars.

To better understand the implications of the threshold values chosen, empirically derived relationships between \(Z\) and rainfall rate \(R\) (mm h\(^{-1}\)) and between \(Z_{	ext{cr}}\) (the effective radar reflectivity factor) and \(R\) for snow are shown in Fig. 1:

\[
R = \left(\frac{Z}{200}\right)^{0.625},
\]

stratiform rain (Marshall and Palmer 1948) (2a)

\[
R = \left(\frac{Z_{	ext{cr}}}{610}\right)^{0.49},\quad \text{snow with density } 0.04 \text{ g cm}^{-3}
\]

(Matrosov 1992). (2b)
Fig. 1. Regime diagram for Rayleigh scattering (expressed as dBZ) vs Bragg scattering from clear air (expressed as \(\log_{10} C_n^2\)) for common radar frequencies and wavelengths. Lines for radar wind profilers (heavy bold) and for precipitation radars (bold), based on Eq. (1), show the amount of Rayleigh scattering that is equivalent to a given amount of Bragg scattering for each radar frequency. The shaded area represents the transition from conditions for which Bragg scattering usually dominates (i.e., weak Rayleigh scattering) to those for which Rayleigh scattering will almost always dominate under normal atmospheric conditions observed outside the convective boundary layer and mantle echoes. The \(Z-R\) relationships for stratiform rain (dash–dot) and snow (short dashed) are also shown to help relate the precipitation sensitivities of radars to rainfall rates.

This comparison reveals that only heavy rain is likely to appear regularly in VHF radar spectral moment data, where heavy rain is defined following the National Weather Service criteria as \(R > 8.4\) mm h\(^{-1}\), moderate rain as \(2.8 < R < 8.4\) mm h\(^{-1}\), and light rain as \(R < 2.8\) mm h\(^{-1}\). At UHF, however, Rayleigh scattering from precipitation is likely to exceed the clear-air return under conditions where rainfall rates are greater than

<table>
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<th>(f) (MHz)</th>
<th>(\lambda) (m)</th>
<th>(\log_{10} C_n^2)</th>
<th>(Z) (dBZ)</th>
<th>(V^*) in rain (m s(^{-1}))</th>
<th>(V^*) in snow (m s(^{-1}))</th>
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<td>(P = 1000) mb</td>
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<td>−54.6</td>
<td>0.5</td>
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</table>
those characteristic of light rain or even drizzle. Cloud particles are unlikely to appear at either VHF or UHF because of the small radar reflectivity factors that characterize them [i.e., typically less than −10 dBZ (see Gossard and Strauch 1983)]. Conversely, the much greater sensitivity of 3-, 5-, and 10-cm radars to clouds and precipitation is also apparent in Fig. 1 and Table 1.

3. Derivation of relationships between \( Z \) and \( V^* \) in rain and snow

Having established thresholds for precipitation sensitivity in terms of radar reflectivity factor, a parameter that is not always measured by wind profilers, it is possible to transform these into thresholds that are applicable to radial vertical velocities, a parameter that is usually measured by wind profilers. This is done by first using empirical fits to drop size distributions measured in precipitation to calculate both \( Z \) and the particle diameter that corresponds to the peak in the spectrum of \( Z \) (referred to as the modal diameter \( D_m \)). Well-established relationships between terminal velocity \( V_t \) and particle diameter can then be used to calculate the terminal velocity of a particle with diameter \( D_m \), referred to as the modal terminal velocity \( V^* \). Although the reflectivity-weighted mean of the Doppler power spectrum \( V_Z \) is normally used to represent the radial velocity, \( V^* \) is used here because it offers some advantages in terms of analytical and conceptual simplicity and, as shown in the appendix and as described by Atlas et al. (1973), typically differs by just a few percent from \( V_Z \). This approach ultimately yields the desired relationships between \( Z \) and \( V^* \), relationships that are similar to those used by Gossard et al. (1992) to determine precipitation type using data from a calibrated 915-MHz wind profiler. However, unlike previous studies, this study focuses on establishing the typical limits for which such measurements are likely to be possible as a function of radar wavelength.

a. Rain

By assuming the Marshall–Palmer (1948) drop size distribution,

\[ N(D) = N_0 e^{-\Delta D}, \]  

(3)

it is possible to calculate the radar reflectivity factor from

\[ Z = \int_0^\infty D^6N(D)dD, \]  

(4)

where \( D \) is the drop diameter (mm), \( N\Delta D \) is the number of drops per cubic meter between the sizes \( D \) and \( D + \Delta D \), \( N_0 = 0.08 \text{ cm}^{-4} \), and \( \Delta (\text{m}^{-1}) = 4.1 R^{-0.21} \) [\( N_0 \) and \( \Delta \) are from Marshall and Palmer (1948)]. Integration of (4) using the form of \( N(D) \) given by (3), yields

\[ Z = 7.2 \times 10^7 N_0 \Delta^{-7}, \]  

(5)

where \( N_0 \) and \( \Delta \) are as in (3). Sekhon and Srivastava (1970) showed that truncation of the exponential drop size distributions at diameters less than infinity reduces \( Z \) from the value calculated from (5). Although the truncation point is poorly defined, they concluded from previously published results that \( Z \) from (5) should be multiplied by a correction factor of 0.8 for rain and 0.51 for snow, which reduces the radar reflectivity factor by 1 dBZ in rain and 3 dBZ in snow. This correction is incorporated henceforth using \( C_r = 0.8 \) for rain and \( C_r = 0.51 \) for snow.

The spectrum of radar reflectivity factor, denoted by \( S_Z \), as a function of \( D \) is given by

\[ S_Z(D) = N(D)D^6. \]  

(6)

For the Marshall–Palmer drop size distribution [Eq. (3)], it can be shown that the maximum in \( S_Z \) occurs at a diameter \( D_m \) that is related to \( \Delta \) through

\[ D_m = \frac{6}{\Delta}. \]  

(7)

Inserting (7) into (5), and incorporating the correction factor \( C_r \), yields a relationship between \( Z \) and \( D_m \), where units are as above:

\[ Z = C_r 257N_0 D_m^7. \]  

(8)

Because there are well-defined relationships between \( V_t \) and \( D \), as a function of air density \( \rho \), it is straightforward to transform (8) into a relationship between \( Z \) and \( V^* \) (m s\(^{-1}\)), where \( V^* \) is the terminal velocity corresponding to the peak of the Rayleigh-scattering part of Doppler power spectrum. The equations, which apply to individual particles (from Atlas et al. 1973; Gossard et al. 1992),

\[ D = 0.225 \left( \frac{\rho}{\rho_0} \right)^{0.5} V_t^{1.05}, \quad \text{for } 0.40 \leq V_t < 2.5 \text{ m s}^{-1}, \]  

(9a)

\[ D = -1.67 \ln[0.937 - 0.097 \left( \frac{\rho}{\rho_0} \right)^{0.4}] V_t, \]  

(9b)

where \( \rho \) and \( \rho_0 \) are the air density at the measurement level and at 1000 mb, respectively, can be inserted into (8) yielding

\[ Z = 0.0075 C_r N_0 \left( \frac{\rho}{\rho_0} \right)^{3.5} (V^*)^{7.63}, \]  

(10a)

for \( 0.40 \leq V^* < 2.5 \text{ m s}^{-1} \),

\[ Z = 257 C_r N_0 \left( -1.67 \ln[0.937 - 0.097 \left( \frac{\rho}{\rho_0} \right)^{0.4}] V^* \right)^7, \]  

(10b)

for \( V^* > 2.5 \text{ m s}^{-1} \).
These equations [(10a) and (10b)] can be used either to calculate $V^*$ for a given $Z$ or to predict $Z$ [or its corresponding $C_s^2$ through Eq. (1)] for a measured $V^*$. The curves for (10a) and (10b) are shown in Fig. 2, which relates $Z$ to $V^*$ at approximately 1000, 700, and 500 mb. By using an alternative formulation of the $V_0(D)$ relation given by Lhermitte (1990), it can be shown that the choice of a $V_0(D)$ relation introduces an approximately 0.1 m s$^{-1}$ uncertainty in the $V^*$--$Z$ relationship in rain, that is, the Lhermitte (1990) formulation increases $V^*$ by 0.1 m s$^{-1}$ over a wide range of $Z$. The values of $V^*$ that correspond to the radar reflectivity factor thresholds discussed in section 2 can be read from Fig. 2 or calculated from (10a) and (10b). For example, at 404 MHz and 500 mb, observations of downward radial vertical velocities greater than 3 m s$^{-1}$ indicate that Rayleigh scattering from rain is most likely dominating the clear-air return. This conclusion can be made with even greater confidence if the measured downward velocities exceed 5 m s$^{-1}$. Although it is possible for vertical air motions to take on these values, such conditions are relatively unusual and are limited to very small regions such as within convective cells and mountain waves. This example illustrates how the $V^*$ thresholds can be used and that these thresholds are large enough that measured radial vertical velocities in rain should usually be distinguishable from the vertical air motion. It should be noted that while these thresholds will often apply to single measurements, they are of most use when the radial vertical velocities exceed the thresholds in regions that have vertical and temporal continuity and are correlated with appropriate signatures in other moments of the Doppler power spectrum (see section 4). These patterns are especially evident in time–height cross sections of radial vertical velocity data, as shown in Ralph et al. (1995). The calculated thresholds are summarized in Table 1. This analysis also shows that it is very unlikely that Rayleigh scattering from cloud particles, which have values of $V^*$ of roughly 1–10 cm s$^{-1}$ [based on observed cloud drop size distributions having median diameters of 15–50 µm (see Gossard and Strauch 1983)] would dominate the clear-air return for VHF or UHF radar wind profilers (Fig. 2).

![Fig. 2. The Z-V* diagram based on exponential drop size distributions in rain [Eqs. (10a) and (10b)] for three pressure levels (broken curves) and in snow [Eq. (16)] for a range of snow densities (solid lines). The curves for rain are broken at the transition from the domain of validity of (10a) to that of (10b). Dashed lines represent the average transition zone from Bragg-dominated to Rayleigh-dominated scattering for 404-MHz radar wind profilers (see Fig. 1 and Table 1), while the arrow indicates that larger radar reflectivity factors are in the regime in which Rayleigh scattering dominates over Bragg scattering. The scattering regimes are also noted schematically for 915- and 50-MHz radars. Marshall–Palmer (1948) and Matrosov (1992) particle size distributions are assumed.](image-url)
b. Snow

To extend the previous analysis to snow it is necessary to account for the bulk density $\rho_s$ of the snow (i.e., the overall density including air pockets within the particles) in the calculations of $Z_e$ and $V^*$. Although both these parameters are influenced by variations in crystal type, aggregation, and riming, the present analysis examines only dry aggregates (referred to henceforth simply as snow or snow particles), for which the problem is tractable and the results are well defined. In addition, this is a common form of snow. For dry aggregates, $\rho_s$ typically ranges from 0.02 to 0.06 g cm$^{-3}$ (Fujishiro et al. 1990). In contrast, for moist or wet snow, $\rho_s > 0.08$ g cm$^{-3}$ (Ihara et al. 1982). Although early observations of particle size distributions were first reported in terms of melted diameter (Gunn and Marshall 1958; Sekhon and Srivastava 1970), more recent work (Ihara et al. 1982; Matrosov 1992) describes the size distributions in terms of the snow particle diameter $D_s$, a parameter that is required for accurate calculation of $Z_e$ and $V^*$. Diameter $D_s$ is defined as the diameter of a snow particle of density $\rho_s$ that contains the same amount of water as a liquid drop of diameter $D$ (Matrosov 1992). The size distribution for snow is given by

$$N_s(D_s) = N_{0s}e^{-\Lambda_s D_s},$$  \hspace{1cm} (11)

where $D_s$ is expressed in millimeters, and $N_s(D_s)$ is the number of snow particles. The parameters $N_{0s}$ (cm$^{-4}$) and $\Lambda_s$ (mm$^{-1}$), summarized by Matrosov (1992), depend on the snow density and on the snowfall rate, where $R$ is the snowfall rate (in millimeters per hour of melted snow) and $\rho_w$ is the density of liquid water:

$$N_{0s} = \left( \frac{\rho_s}{\rho_w} \right)^{0.33} 0.025 R^{-0.94}$$ \hspace{1cm} (12a)

$$\Lambda_s = \left( \frac{\rho_s}{\rho_w} \right)^{0.33} 2.29 R^{-0.45}.$$ \hspace{1cm} (12b)

To calculate the radar reflectivity factor in snow, it is also necessary to account for the difference between the dielectric constants of ice ($K_i$) and liquid water ($K_w$). The effect of this dependence is to reduce the amount of power backscattered from an ice particle relative to that from a liquid drop of the same diameter. This dependence is given by (Gossard et al. 1992)

$$\left| \frac{K_i}{K_w} \right| = \left( \frac{\rho_i}{2.00} \right)^2,$$ \hspace{1cm} (13)

where $\rho_i$ is the density of ice, which for our purposes is given by the bulk density of the snow particle, that is, $\rho_s$ (g cm$^{-3}$). Multiplying (8) by this factor accounts for the difference in dielectric constant and yields

$$Z_e = 257 C_i \left| \frac{K_i}{K_w} \right|^2 N_{0s}(D_{ms})^7,$$ \hspace{1cm} (14)

where $D_{ms}$ is the modal diameter in the snow particle distribution given by $D_{ms} = 6/\Lambda_s$, as in (7) for rain, and $C_i = (0.51)$ results from truncation at large sizes (see section 3a).

This can be transformed into a $Z_e$-$V^*$ relationship through

$$D_s = \frac{0.13 V^2}{\rho_s - \rho},$$ \hspace{1cm} (15)

which is based on observations of dry snow aggregates by Magono and Nakamura (1965). Neglecting air density $\rho$ in (15) introduces less than a 5% error in $D_s$ for typical values of $\rho_s$ and is excluded from the analyses. The dependence of $N_{0s}$ on $V^*$ and $\rho_s$ can be determined by inserting (12b) and (15) into $D_{ms} = 6/\Lambda_s$. Now $Z_e$ can be expressed in terms of $\rho_s$ and $V^*$:

$$Z_e = 5.36 \times 10^{-4} C_i(\rho_s)^{-3.29}(V^*)^{0.83}.$$ \hspace{1cm} (16)

Curves representing this function are shown on Fig. 2 for a range of $\rho_s$ (i.e., 0.01–0.08 g cm$^{-3}$) that encompasses most observed values in dry aggregates but that also extends to somewhat smaller and larger values. As with rain, the threshold for sensitivity to snow can be expressed in terms of a threshold $V^*$, which for 404 MHz ranges from 0.5 to 1.3 m s$^{-1}$ depending on $\rho_s$ and $C_i^2$ (Fig. 2 and Table 1).

c. Uncertainties in the $Z$-$V^*$ relationships

When scattering occurs from both clear air and precipitation, it creates a bimodal Doppler power spectrum that can cause the observed radial vertical velocity to lie between the two spectral peaks when the two scattering mechanisms create power returns that are of comparable strength. This will inevitably occur under some conditions in almost every event, for example, at a particular altitude or time. However, it appears to affect a relatively small portion of the data, most likely because the measured radial velocities are significantly affected only when the signal powers are very similar. Nonetheless, it should be anticipated that this behavior will make the transition between regimes less abrupt in real data and that the most appropriate thresholds may be slightly smaller than those calculated above.

To test the sensitivity of these results to the type of drop size distribution used, we also examined the distribution developed by Best (1950, 1951) and the gamma distribution (see Gossard and Strauch 1983). The primary difference between the Best and the exponential distributions is that the Best distribution includes a greater number of smaller drops and fewer larger drops for the same total liquid water content. This difference causes the peak in the radar reflectivity spectrum to shift to smaller drops, that is, $D_m$ (Best) = 0.70$D_m$ (exponential), as shown in Gossard and Strauch (1983). Thus, for the same value of $Z$, the Best distribution yields a smaller $V^*$. As calculated
Table 2. Radar wind profiler locations and characteristics.

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<th>Platteville (demonstration network)</th>
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<td>7°</td>
<td>4°</td>
</tr>
<tr>
<td>Peak transmitted power (kW)</td>
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<td>1.4</td>
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<tr>
<td>Effective antenna area (m²)</td>
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<td>8.2</td>
<td>8000</td>
<td>70</td>
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<tr>
<td>Calibrated</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
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from (9) and (15), this translates to a 15%–30% decrease in the threshold values of $V^*$ from those based on the exponential distribution for observations of rain and a 16% decrease for snow. It should also be noted that for the same total liquid water content the Best distribution yields a value of $Z$ that is 4 dBZ less than for the exponential distribution. In the case of the gamma distribution, the summary presented in Table 4-1 of Gossard and Strauch (1983) and the results of Willis (1984) have been used. The transformation $D_m = [(6 + \alpha)/(\gamma G)]^{1/\gamma} D_0$, from median diameter $D_0$ to $D_m$, derived following the method used in section 3a, uses the parameters of the modified gamma distribution ($G$, $\alpha$, $\gamma$) given in these two references. By using the reflectivity factors from Table 4-1 of Gossard and Strauch (1983) for “maritime” and “continental” rain, 42.6 and 31.0 dBZ, respectively, it was found that the gamma distribution predicted $V^* = 9.1$ and 7.0 m s$^{-1}$, respectively. These compared to $V^* = 7.5$ and 6.2 m s$^{-1}$ predicted for the same reflectivity factors using Eq. (10b), which is based on the Marshall–Palmer distribution. Also, using conditions representative of results shown in Willis (1984), for which $D_0$ was mostly between 2.0 and 2.5 mm, the gamma distribution approach produced $V^* = 8.3$–8.9 m s$^{-1}$, compared to 8.0–8.6 m s$^{-1}$ from Eq. (10b), which corresponded to 50 and 58 dBZ, respectively, as determined from a $D_0–Z$ relationship derived from Table A1 of Willis (1984). It should be noted that these comparisons apply to large rainfall rates where the differences between the gamma and the Marshall–Palmer size distributions are the greatest. Thus, it appears that the maximum difference of 21% determined here is likely to be larger than under most circumstances.

This analysis serves to demonstrate that the choice of drop size distribution introduces an uncertainty in the threshold values of $V^*$ that is on the order of 15%–30% of the values derived for the Marshall–Palmer-type drop size distribution that are given in Table 1. This conclusion is also consistent with earlier analyses by Gossard et al. (1992) for log normal and modified Cauchy size distributions in rain for which $V_Z$ varies by roughly 5%–30% from the values calculated for the Marshall–Palmer-type distribution.

4. A comparison of calculated $V^*$ thresholds with observations in stratiform rain

Spectral moment data from 915- and 404-MHz wind profiling radars that are calibrated to within ±3 dBZ and from an uncalibrated 50-MHz radar are shown for a case characterized by moderate to heavy stratiform rain along the Front Range of northern Colorado. The Stapleton 915-MHz profiler is at Denver's Stapleton International Airport, the Platteville 50-MHz profiler is 47 km north of Stapleton, and the Erie 404-MHz profiler is 3 km northeast of the 50-MHz profiler (see Table 2 for more precise profiler locations and characteristics). This region is at the base of the eastern slope of the Colorado Rocky Mountains, which exceed 4 km in altitude, and at the western edge of the Great Plains.

On the larger scale, the conditions during this event are related to 1) the southward passage of a cold front at the surface 12–24 h before the interval studied here, 2) the presence of a lower-tropospheric east–west-oriented baroclinic zone, and 3) a strong midtropospheric shortwave trough with its associated vorticity advection in the southwest of the United States above upslope flow. Between 1400 and 2300 UTC 24 August 1992, National Weather Service observers at Stapleton International Airport (collocated with the 915-MHz profiler) reported continuous moderate to heavy rain, steady temperatures of 11°–13°C, and northerly to easterly winds at the surface (Fig. 3). Precipitation amounts, reported every 3 h at Stapleton airport and totalized for every hour at three special mesonet sites (see Table 3 for their locations) in the vicinity of the radars, indicate that the rain was relatively steady and was spread over the entire area containing the radars. Vertical profiles of hourly averaged winds from an uncalibrated 404-MHz profiler collocated with the 50-MHz radar clearly show that the upslope flow extended up to 4.0 km and that greater than 30 m s$^{-1}$ south-southwesterly flow
Fig. 3. Standard surface weather observations from the National Weather Service office at Denver’s Stapleton International Airport, including hourly and special rainfall rate estimates (R−, light rain; R+, heavy rain) at the observation times marked, temperature (top) and dewpoint temperature (bottom) (°C), wind direction and speed (half barb—2.5 m s⁻¹, full barb—5.0 m s⁻¹). Rainfall rates are plotted from Stapleton (3-h totals), and from three mesonet sites (hourly rates are from the total hourly accumulation calculated from measurements made every 5 min) near the profilers. An interval during which mesonet data was missing is marked (m).

existed in the upper troposphere (Fig. 4). Virtual temperature profiles from radio acoustic sounding systems (RASS) operated in combination with the 915-MHz Stapleton profiler and the 50-MHz Platteville profiler show the freezing level at 3.9–4.1 km at 1534, 2006, and 2104 UTC (Fig. 4). Additional virtual temperature profiles between these times (not shown) indicate that the altitude of the freezing level did not change by more than 100–200 m during this interval. Because the observed radial vertical velocities actually represent precipitation fall velocities in this study (as shown below), the standard correction of the observed acoustic velocity for the effect of vertical air motion is not made in these profiles.

Having established that relatively steady stratiform precipitation was occurring at all the Front Range profiler sites between 1400 and 2300 UTC on 24 August 1992, it is now possible to illustrate the precipitation sensitivity of UHF, and to a lesser extent VHF, radar wind profilers. Several vertical profiles of the moments of the Doppler power spectrum are shown in Fig. 4 for times that are representative of conditions over many hours. Two profiles from the 915-MHz profiler for times 5.5 h apart (each made over 135 s), one profile from the 404-MHz profiler (made over 94 s), and one profile from the 50-MHz radar (a consensus average of six 58-s measurements between 2001 and 2007 UTC) are shown. The profiles from the calibrated 915- and 404-MHz radars contain the characteristic signatures of the bright band, which are not only evident as an enhancement in Z, and as an increase in downward velocities from those characteristic of snow to those of rain, but also as an increase in the spectral width as snow turns to rain below the freezing level. The location of the bright band roughly 0.4–0.8 km below the freezing level (Fig. 4) is in good agreement with the 0.6–0.8-km distance measured by aircraft within the anvil of a mesoscale convective system (Willis and Heymsfield 1989). Comparison of the V* thresholds derived earlier (Table 1) with the observed radar reflectivity factors and radial vertical velocities in rain (interpreted here as due primarily to V*) at both 915 and 404 MHz clearly demonstrates the applicability of those thresholds to this case. In snow the reflectivities and radial vertical velocities also show agreement with the thresholds in Table 1. However, at 1534 UTC the observed radial vertical velocity decreases to below the largest threshold velocity for snow at 915 MHz (i.e., 1.0 m s⁻¹) when the reflectivity factor decreases below 0 dBZ (Fig. 4), a value that lies within the predicted transition range (Table 1). Further evidence of the applicability of these thresholds has been presented in Gossard et al. (1992), where the minimum reflectivity factors associated with the signature of precipitation at 915 MHz is −2 dBZ.

Unlike the 915-MHz profiler, which throughout the 6-h interval showed distinct indications of precipitation similar to those at the particular times shown in Fig. 4, the 50-MHz profiler showed evidence of downward radial vertical velocities indicative of snowfall rates that would be observable at 50 MHz only during the 1–2-h interval for which the observers at Stapleton International Airport reported heavy rain, and even then only near the melting level. It should be noted that the lowest gate of the 50-MHz profiler is centered just 300 m below the freezing level, and its highest gate is at only 7.5 km for the mode being used to simultaneously measure the RASS and radial vertical velocity data shown in Fig. 4. These observations suggest that the event was on the lower threshold of precipitation sensitivity at VHF. This conclusion is supported more quantitatively by the fact that the radial vertical velocities and radar reflectivity factors measured by the 915-MHz profiler (i.e., 1.2–1.8 m s⁻¹ and 25–35 dBZ, respectively) in the same layer are in the range that should mark the transition to precipitation-dominated scattering at 50 MHz (Table 1).

5. Conclusions

The analysis presented here quantifies the sensitivity of radar wind profilers to rain and snow in terms of radar reflectivity factor and, most important, in terms of modal terminal velocity V*; a parameter that is nearly equivalent to radial vertical velocity measured by profilers in precipitation when vertical air motion is small. For conditions where C₉ is unusually large, it

<table>
<thead>
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<th>Table 3. Mesonet locations.</th>
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<table>
<thead>
<tr>
<th>Aurora</th>
<th>Erie</th>
<th>Platteville</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>Longitude (°W)</td>
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<td>105.01</td>
</tr>
<tr>
<td>Altitude (m MSL)</td>
<td>1607</td>
<td>1584</td>
</tr>
</tbody>
</table>
was found that only heavy precipitation is likely to regularly appear in spectral moment data from VHF radars, whereas even light rain or drizzle are likely to be apparent at UHF (Table 1), conclusions consistent with observations presented in several earlier papers summarized in the introduction. Although no special analysis was made for graupel or hail, it is likely that under most circumstances their reflectivity factors and $V^*$ values will exceed the thresholds derived for rain and hence will be readily detectable. In contrast, clouds have radar reflectivities that are too small to be regularly apparent in radar wind profiler spectral moment data and are associated with $V^*$ values that are much smaller than the threshold velocities derived here. In particular, the $V^o$ values associated with precipitation rates sufficiently large to normally dominate the clear-air return are large enough that they are readily distinguishable from vertical air motions, except under extreme conditions such as those associated with convection and mountain waves. This implies that vertical radial velocities from radar wind profilers can be used as a key parameter in determining when and where precipitation appears in such data. Because the $V^*$ thresholds are smaller in snow than in rain, there is an increased probability that mesoscale clear-air vertical motions could mask or resemble the signature of snow. However, as illustrated by the case shown here and by three additional cases covering a broad range of weather systems shown in Ralph et al. (1995), it appears that the $V^*$ thresholds in snow are useful and are actually somewhat conservative.

Table 1 summarizes the radar reflectivity factors and $V^*$ values that can be used as approximate thresholds for identifying precipitation events when the vertical air motion is believed to be small relative to the threshold velocities appropriate to that radar frequency. These thresholds were shown to be consistent with observations made by 915-, 404-, and 50-MHz radar wind profilers in stratiform rain. However, statistical analyses based on numerous cases in a variety of locations and weather conditions could refine these thresholds, and objective techniques could be developed that may determine the threshold on a case-by-case basis. It must be emphasized, however, that the natural variability of $C_n^2$ will inevitably lead to circumstances in which the signatures of precipitation will appear much below these conservative thresholds. For example, the tendency for $C_n^2$ to be smaller in the upper troposphere
suggests that wind profilers will be somewhat more sensitive to precipitation at those altitudes than at lower altitudes. It may also be expected that the larger values of $C_n^2$ found in the atmospheric boundary layer, in mantle echoes, and in strong clear-air turbulence may mask some of the precipitation that would otherwise appear in the data. In addition, the development of automated techniques that could simultaneously identify significant multiple peaks in Doppler power spectra (e.g., Wakasugi et al. 1986) could extend the thresholds derived here to smaller values because it should then be possible to identify the Rayleigh scattering portion of the spectra even though it has substantially less power than does the clear-air part of the spectra.

Overall, it appears that precipitation produces a relatively unambiguous signature in the radial vertical velocities observed by radar wind profilers. As seen in the case presented here, enhanced velocity variance in rain, as well as enhanced signal power in both rain and snow, provide additional signatures that can be used to identify intervals of precipitation, especially when a bright band is present. Ralph et al. (1995) extends the results of this paper by documenting the signatures of precipitation in signal power, radial vertical velocity, and velocity variance in three types of weather systems, that is, a mesoscale convective system, a winter storm, and jet stream cirrus. The analysis of time–height cross sections presented in Ralph et al. (1995), rather than simple vertical profiles, adds yet more certainty to the interpretations because temporal continuity can also be examined.

Acknowledgments. The author is indebted to Dr. Earl Gossard and Dr. Richard Strauch for numerous substantive discussions on this topic, for their encouragement, and for providing the data used in this study. Helpful discussions with Dr. Sergey Matrosov and comments by Dr. Dave Parsons are also gratefully acknowledged. Comments from Dr. S. Fukao and two anonymous reviewers refined the text.

APPENDIX

Assessing the Impact of Using the Mode of the Doppler Power Spectrum Rather than the Reflectivity-Weighted Mean

Although earlier calculations made by Atlas et al. (1973) suggest that the reflectivity-weighted mean of the Doppler power spectrum $V_Z$, which is the parameter normally adopted as the radial velocity in meteorological applications, differs only slightly from the mode of the Doppler power spectrum $V^*$, it is useful here to quantify this difference as a function of modal diameter. This is done by using the results of Gossard et al. (1992) in which the terminal velocity $V_t$ is related to the droplet diameter through a simplified, but only approximate, form of (9a) and (9b) that applies to all values of $V_t$:

$$ V_t = 10.5 \left( \frac{\rho_0}{\rho} \right)^{0.5} (1 - e^{-D/C}), \quad (A1) $$

where $C = 2$ yields the best overall fit in terms of reflectivity-based calculations such as $V_Z$ (Gossard et al. 1992). This can then be inserted, with (3) and (6), into

$$ V_Z = \frac{\int_0^{\infty} V_t S_Z(V_t) dV_t}{\int_0^{\infty} S_Z(V_t) dV_t}. \quad (A2) $$

After integration, this yields the following relationship between $D_m$ and $V_Z$:

$$ V_Z = 10.5 \left( \frac{\rho_0}{\rho} \right)^{0.5} \left[ 1 - \left( \frac{\Lambda}{\Lambda + C^{-1}} \right)^2 \right], \quad (A3) $$

where $\Lambda = 6/D_m$. Because $V^*$ is defined as the velocity of a drop that has diameter $D_m$, which is determined from the peak of the reflectivity-weighted drop size spectrum (6), then it is appropriate to compare (A3) with (9a) and (9b) for various values of $D_m$. This is shown in Fig. A1, where it is evident that the differences between $V_Z$ and $V^*$ are relatively small (i.e., $<10\%$). Although differences may also be expected in snow, Atlas et al. (1973) showed that the differences should be even smaller than in rain. In addition, because the results are sensitive to snow density and ice crystal habit, both of which can vary substantially in the atmosphere, the differences between $V_Z$ and $V^*$ in snow are not calculated here.

REFERENCES


