Three-Dimensional Air Circulation in a Squall Line from Airborne Dual-Beam Doppler Radar Data: A Test of Coplane Methodology Software

MICHEL CHONG
Centre National de Recherches Météorologiques (Météo-France and CNRS), Toulouse, France

JACQUES TESTUD
Centre de Recherche en Physique de l’Environnement (CNET-CNRS), Issy-les-Moulineaux, France

(Manuscript received 14 October 1992, in final form 18 October 1993)

ABSTRACT
The detailed structure of a tropical squall line observed in central Florida was investigated from an airborne dual-beam Doppler radar, pointing respectively fore and aft. This allowed dual-Doppler observations from a straight flight path in a way similar to the coplane sampling technique proposed for two ground-based Doppler radars and for which the authors had developed an elaborate method in order to obtain a reliable wind estimation. The application of this analysis method to the airborne dual-beam radar observations is discussed, along with the basic requirements due to the specific airborne data sampling. The retrieved three-dimensional airflow structures within the convective part of the squall line are found to be quite consistent with those previously documented from ground-based radars, such as the convective-scale downdrafts sustaining a well-marked rear-to-front flow or the convective updrafts associated with the front-to-rear flow at mid-to-upper levels and that detrains into the rear part of the squall-line system. Moreover, it is found that the coplane analysis provides a more regular description of the airflow than the conventional Cartesian analysis. The suitability of the coplane approach is also examined by quantifying the uncertainty due to geometric errors that affect the accuracy of the beam pointing.

1. Introduction
In summer 1991, the Convection and Precipitation/Electrification experiment (CaPE, a cooperative program between the National Science Foundation, the Federal Aviation Administration, the National Aeronautics and Space Administration, the National Oceanic and Atmospheric Administration, the National Center for Atmospheric Research, and the U.S. Air Force) was carried out in central Florida. During this field experiment, the airborne Doppler radar on the National Oceanic and Atmospheric Administration’s WP3D 43 aircraft was equipped with the CRPE’s (Centre de Recherches en Physique de l’Environnement) dual-beam antenna that consists of a pair of antennas mounted back to back and pointing, respectively, about 20° forward and aft from a plane normal to the flight track (tilt angle). This fore/aft scanning technique first proposed by Frush et al. (1986) allows dual-Doppler observations from a straight flight path and can also be realized from a single-beam radar by changing mechanically, at each sweep, the antenna toward the fore or aft direction (Jorgensen and DuGranrut 1991). One objective of the research flights of CaPE was to test and evaluate the dual-beam radar antenna for dual-Doppler wind estimation and stereo-radar measurement of attenuation as proposed by Testud and Amayenc (1989), in the scope of the French-American ELDORA (Electra Doppler radar)/ASTRAIA (analyse stéréoscopique par radar à impulsions aéroporté) airborne Doppler radar project.

The basic procedure used to infer the Cartesian wind components combines the dual-Doppler observations with the anelastic mass continuity equation and empirical relationships between radar reflectivity and terminal fallspeed of precipitation particles. The conventional way is to do this analysis in a Cartesian space, which consists in expressing the horizontal wind components in terms of the two observed radial components corrected for the particle fallspeed and for the vertical wind component that must be a solution of the horizontal flow through the mass continuity equation. Because of this mutual dependence, an iterative process is required, correcting at each step for the contribution of the vertical velocity to the measured radial velocities (see, e.g., Ray et al. 1980). This procedure has already been applied to airborne dual-Doppler radar data (Hildebrand and Mueller 1985; Jorgensen and DuGranrut 1991, among others). Another technique, not yet applied to airborne data is the coplane analysis (Lhermitte

Corresponding author address: Dr. Michel Chong, CNRM, 42 Av. Coriolis, 31057 Toulouse, Cedex, France.

© 1996 American Meteorological Society
and Miller 1970) designed for two ground-based radars and for which elaborate techniques were developed (Testud and Chong 1983; Chong et al. 1983; Chong and Testud 1983). The obvious advantages of the coplane analysis relative to the conventional one are that (i) it is performed in a cylindrical space where the problem of determining the 3D wind field is well posed in the mathematical sense (Armijo 1969), and (ii) no iterative process is involved. In essence, two orthogonal cylindrical components are readily derived from observations, while the third orthogonal component is a solution of an ordinary partial differential equation expressing the continuity of the mass.

Three coordinated flights within the CaPE ground-based dual-Doppler network have been performed in three squall lines on 9, 11, and 12 August 1991, respectively. This paper proposes to apply the coplane methodology to the airborne dual-beam radar observations relative to a specific leg within the 9 August squall line. Section 2 describes the basic framework that leads to a coplane configuration of the dual-beam scanning. The effects of advection and the bases of the data analysis are discussed in sections 3 and 4, respectively. Section 5 briefly presents the general characteristics of the observed squall line. Section 6 investigates the 3D air circulation deduced from the coplane analysis, and a comparison with the conventional method is also presented. Geometric errors affecting the beam pointing and subsequently the coplane analysis are examined in section 7.

2. Data distribution and basic relationships

a. Coplane organization

The dual-beam sampling shown in Fig. 1 consists of two conical scans (fore and aft, respectively) around the aircraft’s longitudinal axis. Since the aircraft is moving forward, each antenna actually prescribes a helical scan. As long as a straight flight path (constant altitude and heading) is maintained, the sampling can be organized into a series of tilted half-planes about it. Figure 2 shows the spatial distribution of the individual beams from the fore and aft antenna for a particular plane at an elevation angle $\alpha$ with respect to the horizontal plane. The intersections of the various beams with this axis define the radar locations and their interspacing for constant pointing angles ($\theta_1$ or $\theta_2$) denotes the effective along-track beam spacing that is of 720 m for typical ground speed (aircraft ground-relative speed) of 120 m s$^{-1}$ and antenna rotation of 10 rounds per minute. Basically, the scanning geometry of the airborne dual-beam radar is very similar to the coplane scanning mode for ground-based radars, which consists of steering the radars to scan a series of common tilted planes about their baseline, and which ensures a complete description of the 3D air motion within a naturally generated cylindrical space. As can be seen in Fig. 2, this natural cylindrical frame is evident for airborne observations and is described by $(x, l, \alpha)$.

b. Data coordinates and beam orientation

From Fig. 2, aircraft (or more specifically radar) position, distance $r$ of an observed point from the radar, and beam orientation and accordingly geometry of the radial velocity defined by angles $\theta$ and $\alpha$ are important variables in defining data point coordinates and wind components (see section 2c). In the cylindrical coordinate system, the coordinates of an observed point $M$ on a beam axis are given by

$$x_M = a + r \sin \theta$$
$$l_M = r \cos \theta$$
$$\alpha_M = \alpha,$$

(1)

where $a$ is the location of the radar along the $x$ axis, $\theta$ is the tilt angle of the beam relative to the $l$ axis (here defined positive clockwise), and $\alpha$ is the elevation angle of the tilted plane containing the considered beam.

With respect to the Cartesian ground-relative coordinates $(x, y, z)$, the coordinates of $M$ are defined by

---

Fig. 2. Spatial distribution of the dual-beam radar observations.
\[ x_m = a + r \sin \theta \]
\[ y_m = r \cos \theta \cos \alpha \]
\[ z_m = c + r \cos \theta \sin \alpha, \quad (2) \]

where \( c \) defines the altitude of the radar.

In practice, the actual beam-pointing angles (corrected for aircraft attitude angles: heading, pitch, and roll angles) are referred to azimuth AZ and elevation EL angles (see Fig. 3) relative to the earth (AZ is relative to the north). If we assume that the \( x \) axis is at an angle \( \gamma \) from the north, \( \theta \) and \( \alpha \) can be evaluated as

\[ \sin \theta = \cos EL \cos (\gamma - AZ) \]
\[ \tan \alpha = \frac{\tan EL}{\sin (\gamma - AZ)}. \quad (3) \]

\( c. \) Basic formulation for wind synthesis

According to the coplane methodology (Armijo 1969), the wind components can be evaluated from the dual-Doppler observations in the natural cylindrical frame. Let us define \( \Gamma, \Psi, \) and \( \Phi \) as these components relative to the \((x, l, \alpha)\) coordinate system. (Evidently, \( \Gamma \) and \( \Psi \) define the coplanar Cartesian components attached to each individual tilted plane, while \( \Phi \) is the component normal to the \( \alpha \) planes.) At a specific point \((x, l)\) within an \( \alpha \) plane where are observed both radial wind velocities \( V_1 \) and \( V_2 \) from the fore and aft antennas (negative velocities are receding from the radar), the coplanar components that result from geometric combination of the measured wind vectors are given by

\[ \Gamma = \frac{-V_1 \cos \theta_2 + V_2 \cos \theta_1}{\sin (\theta_1 - \theta_2)} \]
\[ \Psi = \frac{V_1 \sin \theta_2 - V_2 \sin \theta_1}{\sin (\theta_1 - \theta_2)} + v_f \sin \alpha, \quad (4) \]

where \( v_f \), the terminal fallspeed (positive downward) of precipitating particles accounts for their contribution to the measured velocity and can be estimated from an empirical relationship with the observed radar reflectivity. Here \( \theta_1 \) and \( \theta_2 \) are the tilt angles relative to the \( l \) axis (positive clockwise), that is, positive and negative in Fig. 2, respectively. Equation (4) shows that only \( \Psi \) is contaminated by precipitation fallspeed and that uncertainty in the estimation of \( v_f \) may have substantial effect, in particular at high elevation angles. The CaPE data used in this study indicates the presence of ice and water phases with mixed phases in the melting region (see Fig. 9). Because of this, no single reflectivity—particle fallspeed relationship could be used, and to mitigate the effects of both phases, we consider two relationships specific to ice and water, respectively, with a linear combination in the melting region.

The third component \( \Phi \) is then estimated by using the anelastic continuity equation expressed in the cylindrical frame as

\[ \frac{\partial \Phi}{\partial \alpha} + l \left( \frac{\partial \Gamma}{\partial x} + \frac{\partial \Psi}{\partial l} \right) + \Psi (1 - \kappa l \sin \alpha) - \kappa l \cos \alpha \Phi = 0, \quad (5) \]

where \( \kappa = -\partial \ln \rho/\partial z \) accounts for air density decrease.

Finally, these cylindrical components are readily related to the horizontal wind components \( U \) and \( V \) (\( U \) being along \( x \) axis) and the vertical wind component \( W \) as

\[ \Gamma = U \]
\[ \Psi = V \cos \alpha + W \sin \alpha \]
\[ \Phi = -V \sin \alpha + W \cos \alpha. \quad (6) \]

\( 3. \) Implications of a storm advection

Usually, observation of precipitating systems is accompanied by advection effects due to storm motion during the sampling time even though each radial exploration can be considered as instantaneous (typically this takes a few milliseconds). It is evident from Fig. 2 that measurements from the fore and aft antennas at a common point within an \( \alpha \) plane and at a distance \( l \) from the flight track are not simultaneous, because the corresponding radar locations are dependent on time as \( a = V_a (t - t_k) \), where \( t_k \) is a reference time and \( V_a \) is the aircraft ground speed. The time lag separating these observations is given by a linear function of the distance \( l \) as

\[ \delta t = \frac{l (\tan \theta_1 - \tan \theta_2)}{V_a}. \]

It is about 3 min for equal tilt angles of 20° (namely, \( \theta_1 = -\theta_2 = 20° \)), \( l = 30 \text{ km}, \) and \( V_a = 120 \text{ m s}^{-1} \), and therefore effects of advection may have strong influence on the restored 3D wind field.

Indeed, correction for these effects is generally considered in the analysis of Doppler radar data to mitigate temporal errors due to this nonsimultaneous nature of the observations (e.g., Heymsfield 1978; Chong et al.)
Fig. 4. Representation of the beam pointing in a reference frame moving with the precipitating system. The x and x' axes are, respectively, unadvected and advected aircraft trajectories. Their intersection point refers to the position of aircraft at a reference time \( t_0 \).

1983, for ground-based radars; Hildebrand and Mueller 1985; Ray and Stephenson 1990, for airborne radars). The procedure for such a correction consists of the use of a reference frame moving with the advection velocity, or equivalently, the storm motion is used to spatially adjust the data to the common reference time \( t_R \). To preserve the beam geometry involved in any wind synthesis, a horizontal block translation of each beam ray is required, according to the advection speed and the relative time of observation \( (t - t_R) \).

Now, how does the natural cylindrical frame or coplane organization modify through this process? The conform transformation that defines the horizontal translation is schematized in Fig. 4, which shows that in the unadvected or advected positions of the radar and defines the storm-relative aircraft path. These are

\[
\begin{align*}
x_a &= a - c_x(t - t_R) = (V_a - c_s)(t - t_R) \\
y_a &= -c_s(t - t_R),
\end{align*}
\]

where \( c_s \) and \( c_x \) are the components of the advection velocity vector.

Eliminating the time variable in (7) gives

\[
y_a = \left(-\frac{c_s}{V_a - c_s}\right) x_a,
\]

which defines a straight line as long as the ground speed is constant. If so, the advected straight path deviates from the original one by an angle \( \delta \) as

\[
\tan \delta = -\frac{c_s}{V_a - c_s}.
\]

Here \( \delta \) is either positive or negative, depending on the sign of \( c_s \) since \( V_a > c_s \). Along this advected path, the aircraft storm-relative speed is then \([ (V_a - c_s)^2 + c_s^2 ]^{1/2} \). Table 1 gives the values of \( |\delta| \) for various advection speed components and for \( V_a = 120 \text{ m s}^{-1} \).

This shows the relative importance of the use of the advected cylindrical frame \((x', t', \alpha')\) in the wind field analysis. (Here \( \delta \) is about \( 10^\circ \) for typical \( c_s = 20 \text{ m s}^{-1} \)).

As shown in Fig. 4, primed beam pointing angles similar to the unprimed ones can be used. Because of this strong similarity between the advected (primed) cylindrical frame and the nonadvected (unprimed) one, all relationships reported in section 2 strictly apply to all new primed variables and, accordingly, wind synthesis from (4) to (6) will be performed for each \( \alpha' \) plane with the respective \( \theta_1' \) and \( \theta_2' \) tilted angles. Using spherical trigonometry, the following relationships transforming \((\theta, \alpha)\) into \((\theta', \alpha')\) can be found:

\[
sin \theta' = \sin \theta \cos \delta + \cos \theta \sin \delta \cos \alpha
\]
\[
tan \alpha' = \frac{\cos \theta \cos \alpha \cos \delta - \sin \theta \sin \delta}{\cos \theta \cos \alpha \cos \delta + \sin \theta \sin \delta},
\]

where \( \theta \) and \( \theta' \) refer to either the fore or the aft antenna (with subscripts 1 or 2).

Figures 5a and 5b present an application of (8) for \( \delta = 10^\circ \) and \( -10^\circ \), respectively, and for \( \theta_1 = 20^\circ \) and \( \theta_2 = -20^\circ \). Curves labeled \( \alpha_1 \) and \( \alpha_2 \) represent the original elevation angles of \( \alpha \) planes containing the fore and aft beam rays that transform into a common \( \alpha' \) plane after correction of advection. Curves labeled \( \theta_1' \) and \( \theta_2' \) are the corresponding tilt angles in the same \( \alpha' \) plane.

Except for horizontal half-planes \((\alpha' = 0^\circ \text{ or } \pm 180^\circ)\), there exist systematic differences between \( \alpha_1 \) and \( \alpha_2 \) that can reach a maximum value of 7.4° for vertical \( \alpha' \) planes. This again highlights the role of the correction for advection; for example, combining radial velocities at \( \alpha' = 27^\circ \) requires the use of data observed at \( \alpha_1 = 25^\circ \) and \( \alpha_2 = 28.3^\circ \) in Fig. 5a. Correspondingly, the tilt angles undergo significant changes with respect to the original ones \((\theta_1' = 29^\circ \text{ and } \theta_2' = -11.1^\circ)\).

### 4. Analysis method

The analogy of airborne dual-beam radar data with those from two ground-based radars operating according to the coplane methodology can be also extended to their analysis. Three main aspects have to be re-
solved. They concern (i) the filtering and interpolation of raw data onto a regular grid attached to the cylindrical frame, (ii) the correction for the advection effects, and (iii) the integration of the anelastic continuity equation.

For these three points, Testud and Chong (1983), Chong et al. (1983), and Chong and Testud (1983), respectively, proposed original and efficient solutions widely and successfully applied to dual-Doppler radar observations of tropical squall lines (see, e.g., Chong et al. 1987; Roux 1988). Except for the correction for the advection effects that can be easily handled by referring all data to an advected cylindrical frame prior to any data processing (see section 3), the solutions proposed for the first and last points are used here, with minimum modifications due to the specific nature of airborne observations.

a. Filtering and interpolation scheme

As in the case of ground-based radars, the recourse to this process is necessary because the raw data (reflectivity, radial velocity, and pointing angles) used in (4) are subject to random errors; the samplings from the fore and aft antennas do not coincide in space, and finally a sufficient accuracy is required in estimating coplane derivatives involved in (5). One solution examined in Testud and Chong (1983) is the use of the Cressman (1959) weighting function expressed in the 2D coplanar space defined by $x$ and $l$, since the sampling is performed in common tilted planes. In the case of airborne measurements, the measured pointing angles have to be corrected for aircraft attitude so that the regular distribution of beams, as shown in Fig. 2, is no longer respected. Therefore an interpolation to common tilted planes should be also realized by extending the Cressman scheme to the 3D space as follows. The weight applied to an observation point $M(x_M, l_M, \alpha_M)$ to account for its contribution to a grid point $(i, j, k)$ is defined as

$$\omega_M = \frac{1 - D^2}{1 + D^2},$$

where

$$D^2 = \frac{(x_M - x_{ijk})^2 + (l_M - l_{ijk})^2}{R_0^2} + \frac{(\alpha_M - \alpha_{ijk})^2}{\alpha_0^2} < 1,$$

$x_{ijk}, l_{ijk}, \alpha_{ijk}$ being the cylindrical coordinates of the grid point. Parameters $R_0$ and $\alpha_0$ control the filtering within the $\alpha$ plane and perpendicularly to it, respectively. The equation $D^2 = 1$ defines a pseudoellipsoid centered on the grid point and within which all available data account for the determination of the filtered gridpoint value. The use of this cylindrical form of the Cressman function is preferred to the Cartesian one because it ensures the same filtering for any $\alpha$ plane.

b. Integration method

At constant $x$ and $l$, the general integrated form of (5) may be written as

---

**Fig. 5.** Graphic representation of relationships (8) with $\theta_i = -\theta_j = 20^\circ$, and (a) $\delta = 10^\circ$ and (b) $\delta = -10^\circ$. 

---
\[ \Phi(\alpha) = \exp[kz(\alpha)] \int_{\alpha_0}^{\infty} \chi(\alpha) \exp[-kz(\alpha)] d\alpha + \Phi(\alpha_0) \exp\{k[z(\alpha) - z(\alpha_0)]\} \tag{9} \]

where

\[ z(\alpha) = c + l \sin\alpha \tag{10} \]

\[ \chi(\alpha) = -l \left( \frac{\partial \Gamma}{\partial x} + \frac{\partial \Psi}{\partial l} \right) - (1 - kl \sin\alpha) \Psi \tag{11} \]

where \( \alpha_0 \) denotes the elevation angle from which the integration starts with a specified boundary condition \( \Phi(\alpha_0) \) at the corresponding level \( z(\alpha_0) \). Expression (9) is similar to that developed in Chong and Testud (1983) except that \( \alpha \) is not restricted to positive values since radar positions are above the earth's surface: from (10), negative \( \alpha \) occur for \( z < c \).

It is well known that the choice of the boundary condition is a major problem because the accumulation of statistical errors differs dramatically for an upward or downward integration process. Due to the density stratification, an upward integration with the physical condition \( W = 0 \) at the surface provides an exponential amplification of the statistical error or bias if it exists. (Bias may be introduced by the nonobservation of low-level features by the radar.) On the contrary, a downward integration induces an error that tends to damp but may generate nonnegligible bias at lower levels because of the arbitrary character of the upper boundary condition.

To mitigate the error/bias amplification from an upward integration and to account for the damping effect of the downward integration, Chong and Testud (1983) proposed a solution that is based on a variational concept. (Its performances are widely described in their paper.) The interest of their approach resides in the fact that the physical boundary condition at the surface is respected while maintaining a statistical error similar to that obtained from a downward integration. We recall briefly the basic considerations in the context of airborne radar data.

(i) If an error \( \delta \Phi_\theta \) exists at the surface, a correction should be applied to the estimate \( \Phi^* \) from an upward integration with \( W = 0 \) at the surface, as [see (9)]

\[ \Phi[z(\alpha)] = \Phi^*[z(\alpha)] + \delta \Phi_\theta \exp[kz(\alpha)] \tag{12} \]

where \( \Phi \) is the corrected component. [Note that this correction does not introduce additional terms in the basic continuity equation (5); this means that the divergence profile \( \chi \) in (9) is not modified.]

(ii) This correction should be consistent with the physical consideration of \( W = 0 \) at \( z(\alpha) = 0 \) corresponding to a specific \( \Phi^* = -\Psi \tan\alpha_0 \), where \( \alpha_0 = -\arcsin(c/l) \); see (6) and (10). Equation (12) shows that this condition can be respected only in the statistical sense; that is, \( \langle \delta \Phi_\theta \rangle = 0 \).

(iii) The above correction reduces as much as possible the huge errors introduced at higher levels by the upward integration. If they are actually canceled at the top, the corrected \( \Phi_{\text{top}} \) reflects the actual one and the corrected profile may be viewed as a result of a downward integration with \( \Phi_{\text{top}} \) as an upper boundary condition. Therefore, \( \delta \Phi_\theta \) may be interpreted as the error \( \sigma_\delta \) at the surface induced by the downward integration. [This error can be readily derived from (9)]. This gives an additional condition that, combined with the requirement specified in (ii), can be written as

\[ \frac{1}{N} \sum \frac{\delta \Phi^2_\theta}{\sigma_\delta^2} = 1 \tag{13} \]

where \( N \) is the number of boundary conditions.

(iv) The arbitrary character of \( \delta \Phi_\theta \) may lead to highly varying fields of the final estimate \( \Phi \) at any level of integration (say, any \( \alpha \) plane). So, an integral such as

\[ \int \int \int \left[ \left( \frac{\partial \Phi}{\partial x} \right)^2 + \left( \frac{\partial \Phi}{\partial l} \right)^2 \right] dxdl \tag{14} \]

should be minimized.

Equations (13) and (14) specify the variational problem to be solved to determine the so-called floating boundary condition \( \delta \Phi_\theta \). Note that this applies to integration paths that intersect the surface, that is, for distance \( l \) higher than the radar level [10].

5. The 9 August 1991 squall line

a. Radar observations

Airborne dual-beam radar observations were obtained for several flight tracks at 13 000 ft (~4000 m) between 2000 and 2130 UTC as the squall line moved from west to east toward the CaPE network. Figures 6a and 6b visualize the reflectivity fields (shaded areas) at 1-km altitude for two flight tracks (dotted lines) at the time periods 2028–2036 and 2118–2129, respectively. Origin (0, 0) of the coordinate system refers to (28.89ºN, 80.81ºW) for the two legs (hereafter referred to as legs 1 and 2) that were mainly along the north–south-oriented squall line. Leg 1 was at 10º clockwise from north, along the eastern flank of the system. Leg 2 was at about 28.5º and along the western flank of the leading convective cells, allowing observations of both convective and stratiform regions of the squall line. This much more rectilinear leg 2 will be used to infer the 3D wind field within the convective cells, according to the plane methodology. Note that this will be restricted to the first 50 km (from south) because the ending part of leg 2 was associated with an ascending portion transferring the aircraft from 13 000 to 21 000 ft.
b. Thermodynamic structure

From the Kennedy Space Center (KSC) site (28.47°N, 80.55°W), three soundings were launched on 9 August 1991, at 1015, 1500, and 2225, respectively. The two first soundings were performed well before the arrival of the squall line at KSC, while the third was performed shortly thereafter (see Fig. 6 reporting the relative position of KSC). Using the eastward propagation speed of 10 m s⁻¹, the 1500 and 2225 soundings can be equivalently replaced approximately 240 km ahead of the leading edge of the squall line and 25 km behind it, respectively. Therefore they can represent the "preconvective" and "postconvective" thermodynamic conditions.

Figure 7 reports the temperature and dewpoint profiles on a skew T–logp thermodynamic diagram at 1500 and 2225, respectively. They present strong similarities with previous observations of the tropical convective atmosphere. The preconvective profile (Fig. 7a) is marked by a potentially unstable low-level (<850 hPa) air. This low-level relatively moist (60%–90%) air was subject to condensation above 950 hPa and to free convection up to 130 hPa from the 900-hPa level. The maximum temperature difference between the environment and the lifted air parcel along the 25°C wet adiabat reported in Fig. 7a is of 8.5 K at 400 hPa. On the other hand, air behind the convective cells (Fig. 7b) was mostly stable. Its differences with the preconvective air are typical of the redistribution of the moist static energy and are mainly dominated by moisture changes. (Only a relative humidity of 60% could be observed in the subsaturated layer 1000–800 hPa.) At the surface, the passage of the convective line was accompanied by a strong temperature drop of 7.3°C, with, however, a relative moistening (relative humidity of 85% against 60% for the 1500 sounding).

6. Coplane-derived kinematic structure

a. Data preprocessing

As mentioned in the previous section, the coplane analysis described in section 4 has been applied to the data obtained from leg 2 because they satisfied the basic requirement for a straight flight path (constant altitude and heading). Indeed, statistics of the discrete radar positions within the portion of interest (the first 50 km of leg 2, see section 5) reveal that the aircraft flew northeastward at a mean level of 4150 m with a standard deviation of 7 m. These positions were fitting well a straight line oriented at 28.5° from north, within a standard cross-line deviation of 300 m. They still remain about a straight line at 26° with the same deviation when advection is considered.

To perform the coplane analysis in the cylindrical frame attached to this mean advected (storm relative) flight path, we need to do some approximations when
processing data that are associated with cross-line and/or vertical deviations of the flight track, such as the following.

(i) We suppose that the deviations can be neglected, while preserving the beam pointing angles. In other words, the beams are shifted toward the straight-line path (shifted analysis).

(ii) No assumption is made about the positions of the aircraft. The beam pointing angles and the data coordinates are reevaluated according to the cylindrical frame (unshifted analysis).

These two approximations will be analyzed in the next section.

b. Wind field structure

The coplane-derived kinematic structure, presented in Figs. 8 and 9, accounts for the unshifted analysis. It concerns the convective part of the squall line and has been obtained by considering a domain of 50 km × 35 km × 15 km with a grid resolution of 1 km × 1 km × 1° in the cylindrical frame and 1 km × 1 km × 0.5 km in the Cartesian frame. The x axis (defined by the mean advected flight path and common to both frames) and y axis are positively defined in the southwestward and southeastward (toward the leading edge of the system) directions, respectively. Since the x axis is closely parallel to the squall line, the y axis will denote the line-transverse direction. Moreover, in this application, the proposed filtering (section 4a) was achieved with $R_0 = 3$ km and $a_0 = 2°$ and restricted to elevation angles between $-35°$ and $45°$ to limit the effects of precipitation fallspeed. We considered the two reflectivity—particle fallspeed relationships for water and lump graupel particles, as used in Hauser and Amayenc (1986) in their study of the microphysics of a tropical squall line. The optimal determination of the floating boundary conditions in solving the continuity equation (section 4b) was performed by assuming a statistical error of 2 m s$^{-1}$ for radial velocity measurements. [In their study of various sources of errors, Testud and Hilbrecht (1991) found that such an error may occur.]

Figures 8a–c present the horizontal cross sections of the 3D system-relative wind and reflectivity fields at the altitudes 1.5, 3.5, and 8 km, respectively. The cellular structure of the convective line clearly appears at lower levels and is accompanied by significant perturbations of the horizontal flows. The following two basic flows characterize the system-relative circulation.

(i) A mid-to-low northerly flow that progresses toward the leading edge, with a well-established rear-to-front component ($x > 10$ km in Figs. 8a,b). This rear-to-front flow is mostly associated with cell downdrafts originating at midlevels. Their close position with the reflectivity cores strongly suggests that they may result from falling heavy precipitation.

(ii) A northeasterly inflow that incorporates the system from front to rear. At intermediate levels (3.5 km, Fig. 8b) it interacts with the rear-to-front flow and par-
participates in the feeding of updrafts or downdrafts. At higher levels, the front-to-rear inflow extends over the whole system and is connected to updrafts organized along $y = 20$ km.

The existence of a rear-to-front flow at low levels and a front-to-rear flow at higher levels is well consistent with previous studies (e.g., Zipser 1969; Houze and Betts 1981; Chong et al. 1987). However, a non-observed feature in the present study is the low-level front-to-rear inflow at the leading edge, which is of importance since it contributes to feed a main and strong low-level updraft. In fact, this is probably due to the absence of available data that were eliminated
either in the initial data editing process or by the rejection test we systematically applied to low-level data contaminated by surface echoes. (This was necessary to obtain a good quality wind field.)

The vertical structure of the convective line is illustrated in Figs. 9a,b that represent the line-transverse relative flow and precipitation on the one hand and the vertical velocity pattern on the other hand, in vertical cross sections at $x = 15$ and 24 km, respectively. Note the well-marked radar brightband region around 4-km altitude in Fig. 9a, which justifies the use of two relationships for precipitation fallspeed. The organization of the low-level rear-to-front and mid-to-upper front-to-rear flows, even in the absence of observed low-level inflow, depicts a high resemblance with previous observations of tropical squall lines (see, e.g., Fig. 13 in Chong et al. 1987), such as the following.

- The midlevel updraft cores lying above 6-km altitude and approximately 10–25 km behind the “boundary layer leading updrafts” as a visual extrapolation of the leading flow can define. These updrafts are likely induced by the release of the water loading through the precipitation process (see their relative position with the precipitation cores), allowing air parcels to be positively buoyant. They contribute to expand over a deep layer inflow air lifted up from low levels, before its rearward transfer. Maximum values of 9 and 11.5 m s$^{-1}$ can be found in Figs. 9a and 9b, respectively.

- The relatively deep rear-to-front flow (about 4–5 km) associated with the convective-scale downdrafts that reach 3.2 m s$^{-1}$ for $x = 15$ km (Fig. 9a) and 5.4 m s$^{-1}$ for $x = 24$ km (Fig. 9b). One can also note that the highest downdrafts occur in regions of high precipitation and that they tend to spread out near the surface, inducing stronger horizontal flows toward the front of the system. Midlevel air at 5-km altitude originating from the front appears to incorporate them.

In fact, the consistency of the updraft and downdraft cores with the horizontal flows mainly results from the variational approach used to solve the continuity equation. This case study illustrates how floating boundary conditions overcome the problem of the absence of low-level divergence field that may induce significant deterministic errors, in particular in regions of downdraft cores and in general for $y > 15$ km where divergence data were limited to levels higher than 1 km (see also Chong and Testud 1983).

c. Comparison with conventional dual-Doppler analysis

To better assess the veracity of the coplane approach, the conventional dual-Doppler analysis that expresses the observations in a Cartesian space has been applied to the same advected dataset and Cartesian domain as in the coplane analysis. From the notations of Fig. 3 for beam pointing angles and defining $\phi = \gamma - AZ$, the Cartesian wind components are described by

$$U = \frac{-1}{\sin(\phi_2 - \phi_1)} \left[ \frac{V_1 + (W - u_\tau) \sin EL_1}{\cos EL_1} \sin \phi_2 - \frac{V_2 + (W - u_\tau) \sin EL_2}{\cos EL_2} \sin \phi_1 \right]$$
Fig. 9. Vertical cross sections of the relative line–transverse wind and reflectivity (left), and vertical velocity (right) at (a) $x = 15$ km and (b) $x = 24$ km.

$$V = \frac{1}{\sin(\phi_2 - \phi_1)} \left[ \frac{V_1 + (W - v_r) \sin EL_1}{\cos EL_1} \cos \phi_2 - \frac{V_2 + (W - v_r) \sin EL_2}{\cos EL_2} \cos \phi_1 \right]$$  \hspace{1cm} (15)

and

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} - kW = 0.$$  \hspace{1cm} (16)

Subscripts 1 and 2 in (15) refer to fore and aft radar data that are interpolated onto the Cartesian grid using the Cressman scheme with influence radii of 3 km in the horizontal and 1.5 km in the vertical. Note that the contribution of the vertical motions in the $U$ component is negligible in the present application because the $x$ axis is along the quasi-linear flight path. As in previous works (e.g., Ray et al. 1980), (15) and (16) are iteratively solved, assuming $W = 0$ for the initial step. The O’Brien (1970) technique is applied to integrate (16) by imposing that each vertical column is mass balanced with $W = 0$ at the surface and top.

Figure 10 presents the vertical section of the resulting airflow at $x = 24$ km. Globally this compares well with the flow structure derived from the coplane approach (see Fig. 9b), in the sense that both Cartesian and coplane methods describe the mean descending rear-to-front and ascending front-to-rear branches. However, strong local differences are evident as well on the horizontal ($V$) components as on the vertical ($W$) ones (see $y = 8$ and 26 km). The airflow structure from the coplane method appears quite regular, while
that derived from the Cartesian analysis presents large variations from one grid point to another. The profiles of the standard deviations of $V$ and $W$ (Fig. 11) for the cross section of Figs. 9b and 10 clearly show the high variability of the Cartesian-derived motions. The major differences in the $V$ component (Fig. 11b) between coplane and Cartesian analyses at high levels are mainly caused by uncertainties in estimating the $W$ component since its contribution increases with increasing elevation angles. This is confirmed by the standard deviations of the $U$ component (Fig. 11a), for which $W$ should have a little contribution: both methods provide similar results.

This comparative study demonstrates the significant improvement in obtaining 3D wind vectors using a coplane analysis. Although the integration method may have certain effects on the final results, the advantage of this approach resides in the fact that it is the optimal and straightforward way of using dual-Doppler radar data: estimation of two orthogonal cylindrical components and associated divergence from observations only, and determination of the third component from this divergence. On the contrary, the iterative procedure in the conventional Cartesian method is strongly dependent on the accuracy in determining the wind components from one step to another. In essence, an initial divergence field needs to be estimated, which is then corrected at each step for the contribution of the corresponding vertical velocities. This strong feedback does not ensure reliable divergence field and wind components.

### 7. Error analysis

Sources of errors are various when processing airborne radar data and their effects on the synthesized wind fields have been investigated by several authors (Jorgensen et al. 1983; Hildebrand and Mueller 1985; Ray and Stephenson 1990). In particular, geometric effects may lead to significant uncertainties (Ray and Stephenson 1990). In the context of the coplane application, it seems important to quantify these effects because the analysis is performed in a cylindrical frame.
with at least a resolution of 1°. This section addresses the following questions that are directly related to the geometry of the radar beams: (i) How does the aircraft’s deviation from the supposed straight-line path affect the wind field? and (ii) What are the effects of random errors in aircraft heading, pitch, and roll angles, due to turbulent oscillations? The first question is inherently associated with the approximations used to overcome the problem of cross-line and/or vertical deviations of the flight track (see section 6a). In the following we focus on the cross-line deviations. In the second question, errors in aircraft angles have influence on the beam pointing and subsequently on the locations of the measurements in the cylindrical frame.

Our approach to analyze these sources of errors is based on numerical tests close to the experimental conditions and defined as follows:

1) We consider the dual-beam sampling from the leg analyzed in section 6 and replace the actual radial velocities by those derived from an analytical form of the 3D wind field. (We do not consider any contribution of particle fallspeed.) Random errors with an rms value of 2 m s⁻¹ simulating the radar statistical error are then added to the radial components. All numerical tests hereafter described account for these errors. We also assume that the radar positions are on the x axis (no cross-line deviation). Wind field obtained from this dataset will be the reference one.

2) To simulate the aircraft’s deviation, a sine function with a wavelength of 80 km is superposed to the straight-line path. Data on each beam refer to the modified positions of the aircraft. The approximations proposed in section 6a (i.e., the shifted and unshifted analyses) are then applied to this new dataset and results are compared to the above-mentioned reference wind field.

3) The errors in the aircraft heading, pitch, and roll angles are successively investigated by introducing random errors on the considered angle, while maintaining the radial components simulated in 1. The modified pointing angles [relationships between aircraft angles and azimuth and elevation angles can be found in Hildebrand and Mueller (1985)] are assumed to be the measured ones. No aircraft’s deviation is added here to allow for inspection of individual contribution of the aircraft angle errors.

a. Reference wind field

A 2D (y–z plane) wind field has been simulated, which derives from a description of the vertical velocity in terms of a wave with an amplitude of 10 m s⁻¹, horizontal and vertical wavelengths of 50 and 30 km, respectively, and a phase shift varying linearly with height. This field that both verifies the boundary condition \( W = 0 \) at the surface and the mass continuity equation is shown in Fig. 12a. In the x direction, a constant U component of 3 m s⁻¹ was assumed.

Results of the coplane analysis of dual-beam measurements of such a field are presented in Fig. 12b for \( x = 24 \) km. The consistency of the retrieved wind field relative to the actual one is evident. Only a little damping of the components is observed, due to the filtering and interpolation scheme applied to the simulated noisy data. The corresponding divergence field is depicted in Fig. 13, showing that the wavy character is well represented with some perturbations due to residual random errors. The various results hereafter presented will be relative to the particular plane of Fig. 12b.
b. Effects of aircraft's deviation

To test the validity of the shifted and unshifted analyses, a maximum deviation of 1 km (amplitude of the deviation function) has been assumed. The results of both analyses are shown in Figs. 14 and 15, respectively, representing the differences between the obtained and reference fields. The following remarks can be formulated from the comparison of Figs. 14 and 15.

- The unshifted analysis (Fig. 15) provides the best results since its differences with the reference winds are always smaller.
- It leads to less-organized uncertainties, while the shifted analysis (Fig. 14) can produce large systematic (or bias) errors, and also contributes to introduce a phase shift in the wave structure (compare Fig. 14b with Fig. 12b), increasing the vertical motions (Fig. 14c).
- A limited region of increased $W$ is also produced by the unshifted analysis. It corresponds to the region of maximized convergence/divergence (see Fig. 13). This means that only amplification or damping of the divergence field is introduced without any bias.

Similar results are found with negative aircraft deviation amplitude. For higher amplitudes, the uncertainties are increasing accordingly. The various tests we performed show that a maximum deviation amplitude of 1 km (the grid resolution) is a good compromise in using the coplane analysis in its unshifted version. The rms values of $U$, $V$, and $W$ uncertainty in the presented plane are 0.37, 0.36, and 0.33 m s\(^{-1}\), respectively, which are acceptable compared to the rms values of the reference $U$, $V$, and $W$ components of 3, 9.8, and 3.65 m s\(^{-1}\).

Fig. 14. Vertical section of deviations of (a) $U$, (b) $V$, and (c) $W$ wind components from the reference ones at the particular plane $x = 24$ km, in the case of a maximum aircraft's deviation of 1 km from a straight-line path and using the shifted correction procedure.
c. Errors due to uncertainty in heading, pitch, and roll angles

A series of numerical tests has been performed following the procedure described in 3, by considering various standard deviations of the errors affecting the heading, pitch, and roll angles, respectively. On the basis of the acceptable errors as found in the previous subsection (i.e., rms of the wind component uncertainty is less than 0.4 m s\(^{-1}\)), we could establish the limits for which the coplane analysis remains suitable for deriving reliable wind fields. We found that errors in the heading, pitch, and roll angles should have standard deviations less than 3°, 1°, and 0.5°, respectively.

Examples of the contamination of the wind field by the uncertainties in the aircraft’s heading, pitch and roll angles are presented in Figs. 16, 17, and 18 for the respective standard deviations of 2°, 1°, and 0.6°. All cases reveal that these uncertainties mainly degrade the accuracy of the \( U \) component, while they little affect the \( V \) component. This is consistent with an error analysis applied to (4). Mostly, the perturbations in \( U \) and \( V \) appear to be randomly distributed, and their rms values for the considered plane in Figs. (16)–(18) are, respectively, 0.40 and 0.17 m s\(^{-1}\) in the case of heading and pitch errors, and 0.60 and 0.17 m s\(^{-1}\) in the case of roll errors. The perturbations in the vertical component \( W \) are dependent on the accuracy in the horizontal components. For lower \( U \) perturbations (Figs. 16 and 17), they appear quite small, while for higher perturbations (Fig. 18) they may become nonnegligible. The corresponding rms values are 0.24, 0.35, and 0.55 m s\(^{-1}\) for Figs. 16–18, respectively.

8. Conclusions

This paper has examined the 3D airflow within a squall line from the airborne dual-beam Doppler radar observations, gathered during the CaPE experiment. The organization of the basic sampling of the fore and aft antennas into a series of tilted planes around a straight flight path has been used to test the ability of the coplane methodology analysis we initially developed for ground-based dual-Doppler measurements.

This analysis has been adjusted to account for the specific nature airborne data such as the nonsimultaneous character of observations within each plane or the downward-looking angles, and includes the three main aspects that ensure a reliable wind synthesis: (i) the filtering and interpolation of raw observations using the classical Cressman scheme; (ii) the correction for the advection effects by defining a new reference frame about an advected flight path; and (iii) the choice of the boundary conditions in solving the necessary continuity equation, using a variational method.

The general features of the airflow within the convective region of the squall line were found to be similar to those previously documented. In essence, it was

Fig. 15. As in Fig. 14 but using the unshifted correction procedure.
Fig. 16. Vertical section of deviations of (a) $U$, (b) $V$, and (c) $W$ wind components from the reference ones, in the presence of an aircraft heading angle error of 2°.

Fig. 17. As in Fig. 16 but for an aircraft pitch angle error of 1°.
characterized by the classical rear-to-front and front-to-rear flows. The front-to-rear flow at mid-to-upper levels consisted of ascending air originating from the boundary layer and that subsequently detrained rearward. The rear-to-front flow lying behind the leading edge of the squall line was transporting drier air originating from convective-scale downdrafts associated with the heavy precipitation cells.

Finally, this study has clearly provided a convincing proof of the airborne dual-beam antenna technique and of the coplane methodology to derive high quality dual-Doppler wind estimation. A comparison with the conventional analysis done in a Cartesian space has been performed to demonstrate a real advantage of the coplane approach for this case study. The coplane-derived airflow structure was seen to be quite regular in contrast to the Cartesian-deduced flow that presented strong small-scale fluctuations. In fact, the basic difference resides in the use of a cylindrical frame for the coplane approach, where the problem of determining the 3D wind field is well posed and the solution is straightforward. On the contrary, the solution for the Cartesian analysis involves an iterative procedure that may be subject to errors in the divergence field at each step. Numerical tests have also been performed to investigate the geometric errors that may lead to a questionable use of the coplane analysis. One of these was the aircraft’s deviation from a straight-line path for which we proposed two correction procedures that we called shifted and unshifted analysis. The unshifted analysis consisting in a reevaluation of coplane coordinates from the actual positions of the measurements provided the best results compared to the shifted analysis based on a shift of the observed beam toward the straight-line path. For the unshifted procedure, it was found that aircraft’s deviations should be less than the grid resolution of the analysis. The other errors have concerned fluctuations in the aircraft heading, pitch, and roll angles that should remain within 3°, 1°, and 0.5°, respectively.

Acknowledgments. ELDORA/ASTRAIA is a French–American project of airborne dual-beam Doppler radar, jointly developed by the Centre de Recherches en Physique de l’Environnement and the National Center for Atmospheric Research with the financial support, for the French part, of the Centre National d’Etudes Spatiales, the Centre National d’Etudes des Télécommunications, the Institut National des Sciences de l’Univers, and Méteo-France. Thanks are also due to D. Jorgensen who provided us with the sounding data from the Kennedy Space Center site.

REFERENCES
Chong, M., and J. Testud, 1983: Three-dimensional wind field analysis from dual-Doppler radar data. Part III: The boundary con-


