NOTES AND CORRESPONDENCE

Spatial Correlation of Beam-Filling Error in Microwave Rain-Rate Retrievals

GERALD R. NORTH AND ILYA POLYAK

Climate System Research Program, College of Geosciences and Maritime Studies, Texas A&M University, College Station, Texas

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ABSTRACT

In this paper the authors consider the possibility of correlations between the random part of the so-called beam-filling error between neighboring fields of view in the microwave retrieval of rain rate over oceans. The study is based upon the GARP (Global Atmospheric Research Program) Atlantic Tropical Experiment (GATE) rain-rate dataset, and it is found that there is a correlation of between 0.35 and 0.50 between the errors in adjacent rainy fields of view. The net effect of this correlation is reducing the number of statistically independent terms accumulated in forming area and time averages of rain-rate estimates. In GATE-like rain areas, this reduction can be of the order of a factor of 3, making accumulated standard error percentages increase by a factor of the order of $\sqrt{3}$. For the Tropical Rainfall Measuring Mission using the microwave radiometer alone, this could increase the accumulated random part of the beam-filling error for month-long 5° × 5° boxes from about 1.2% to 2%. The effect is larger for less rainy areas away from the equatorial zone.

1. Introduction

Understanding the error budget for the retrieval of month-long 5° × 5° box averaged rain rates for the upcoming Tropical Rainfall Measuring Mission (TRMM; Simpson et al. 1990) is clearly a high priority for prelaunch research. One of the largest terms in the error budget is the sampling error variance due to the temporal gappiness associated with low earth orbiting satellites. The main sensor for estimating rain rates over the oceans is the TRMM Microwave Imager (TMI), which is a dual-polarization, multichannel microwave radiometer. The individual channels have different footprint sizes, but when combined they might be thought of as having a nominal 25-km resolution. This means that features in the rain-rate field smaller than this cannot be resolved. When such an instrument is used for the retrieval of rain rates over the ocean, there is inevitably the so-called beam-filling error (BFE).

The BFE comes about because the radiometer measures a field-of-view (FOV) area average of the apparent microwave emission temperature, while the observer desires an estimate of the FOV area-average rain rate. The problem is that the formula relating the point value of the microwave temperature and the point value of the rain rate is nonlinear (approximately a rising saturating exponential for low rain rates). Hence, straightforward insertion of the measured FOV microwave temperature into the formula does not lead to the FOV average rain rate because of the heterogeneity of rain rates within the FOV. In fact, the individual retrieval is not unique—infinitely many FOV average rain rates can be responsible for a single microwave temperature reading. In this paper we have simplified the problem of the BFE to that of an ideal single-channel instrument (we think of the 19.6-GHz channel, but that is not necessary). While use of data from more than one channel can improve accuracy, it can never remove the ambiguity associated with the BFE. This simplification to a single channel allows us to apply simple analytical techniques, bringing out the essential features of the problem.

The BFE has two components, a bias or offset part and a random part with ensemble mean zero. Chiu et al. (1990) used GATE [GARP (Global Atmospheric Research Program) Atlantic Tropical Experiment] rain-rate data to show that these are each large: the bias is of the order of 40% of the actual rain rate, and the standard deviation of the random part is of about the same size for an individual retrieval. Theoretical studies with a variety of random rain-rate fields have shown essentially the same results (Ha and North 1995). Short and North (1990) showed that the BFE was responsible for most of the error in retrieving rain rates from ESMR-5 using data taken from the *Nimbus-5* satellite in coincidence with the GATE experiment in 1974.

Corresponding author address: Dr. Ilya Polyak, Climate Systems Research Program, College of Geosciences and Maritime Studies, Texas A&M University, College Station, TX 77843-3150.
E-mail: polyak@csrp.tamu.edu

The philosophy in compiling month-long averages over grid boxes has been to correct for the bias based upon climatology and assume the random part would under accumulation essentially cancel to a negligible residual. A back-of-the-envelope calculation of the cancelation effect consists of noting that over the month there will be about 12 000 readings for an equatorial 5° × 5° grid box. In this estimate, we took a nominal footprint size of 25 km (about 400 contiguous FOVs cover a grid box) and assume 30 flush (flush means intersection of satellite swath on an overpass and grid box = "grid box") visits (or more realistically 60 half-flush visits) to the grid box during the month. Of the 12 000 FOVs about 10% will be raining (at least in GATE-like areas). This gives 1200 raining FOVs. Under accumulation of the data in the box over a month, cancelation of the random error leads to a reduction in aggregated error at the end of the month of order $\sigma/\sqrt{1200} \approx 1.15\%$ of the mean rain rate, where we have estimated $\sigma \approx 40\% \times$ (average rain rate). This is considered to be a tolerable level of error considering the many other difficulties associated with the measurements. It should be noted, however, that the percentage error rises as one leaves the heavy rain-rate zones near the equator.

The question we pose in this paper is what if the errors from one FOV to another are correlated? The total number of independent random error contributions will be effectively less than 1200. We examine the correlations of random BFE between neighboring FOVs and ask how the effective number of independent FOVs is reduced by this correlation. There are many possible approaches to the problem. We have chosen a conceptually simple one. The steps in our procedure are to first find the spatial correlation of the BFE of adjacent FOVs. Having estimated this correlation for the GATE data, we can estimate the effective number of independent samples across a GATE scene. This part is subtle since the raining areas are not distributed randomly (independently) across a scene. Actually there is considerable clustering of the rainy areas.

2. Definitions

Consider measurements of the rain rate by a single channel microwave radiometer, the FOV of which covers a square composed of 7 × 7 GATE tiles (these are each 4 km × 4 km) and $r(i, j)$ is a value of instantaneous rain rate observed for the tile (i, j). We have taken our nominal FOV to have this 28 km × 28 km size. In fact, we recall that TRMM will make use of several microwave channels, but we are using only a one-channel analog here.

Next we label the tiles within a FOV from 1 to 7 × 7 = 49, and let R_k be the rain rate corresponding to the tile number k. Then the area-average rain rate $[R]_0$ is defined as

$$[R]_0 = \frac{1}{49} \sum_{k=1}^{49} R_k, \qquad (1)$$

and the BFE is

$$\delta_{\mathrm{BF}} = R([T]_0) - [R]_0 = \text{calculated} - \text{true}, \qquad (2)$$

where $R(T)$ is the formula for the point-by-point relationship between microwave temperature and rain rate. A first-order formula for the BFE can be derived (Chiu et al. 1990; Ha and North 1995):

$$\delta_{\mathrm{BF}} = [(R_k - [R]_0)^2]_0 \left(\frac{T''(R)}{2T'(R)} \right)_{R=R_0}, \qquad (3)$$

where the derivatives are to be evaluated at a typical rain rate R_0. For a saturating exponential formula $[T(R) = A - Be^{-CR}$; A, B, C constants depending on microwave frequency], the factor involving derivatives is simply a constant $(-C/2)$ independent of R_0. We see that the BFE to first order is just proportional to the variability (sample variance) of the rain rate within the FOV (Chiu et al. 1990; Ha and North 1995):

$$\delta_{\mathrm{BF}} = -\frac{C}{2}\frac{1}{49} \sum_{k=1}^{49} (R_k - [R]_0)^2$$
$$= -\frac{C}{2} [(R_k - [R]_0)^2]_0. \qquad (4)$$

From one FOV to another in a scene and through time, δ_{BF} is a random variable. Its ensemble mean (when raining) is $\langle \delta_{\mathrm{BF}} \rangle$, which is the bias. The random part by definition has zero mean but may exhibit significant correlation from one FOV to another. This last is our next line of investigation.

3. Statistical dependence of the BFE

First, let us ask if there should be a correlation of the BFE between neighboring FOVs. Since the BFE is approximately linearly related to the variability within, we are asking whether variability is correlated with itself from one place to another. In effect, is the graininess of rain at finescale persistent over distances of order 25 km? It is possible to answer this question theoretically and we present such an argument in the appendix of this paper for the case of normal statistics of the rain rate when raining. The answer is that if there is correlation in the rain field, there will be correlation in the variability field.

Next we turn to the real problem using the GATE1 data. We have calculated $\delta_{\mathrm{BF}}(i, j)$; $i = 1, \cdots, 10$; $j = 1, \cdots, 10$ for each FOV in the 10 × 10 array covering the GATE1 scene. These form 100 time series (1716 terms each for the GATE1 phase). Having these time series one can estimate the spatial correlations for each pair of adjacent $[\delta_{\mathrm{BF}}(i, j), \delta_{\mathrm{BF}}(i', j')]$ FOVs.

One additional parameter has to be established here. This parameter is the minimum number of the simul-

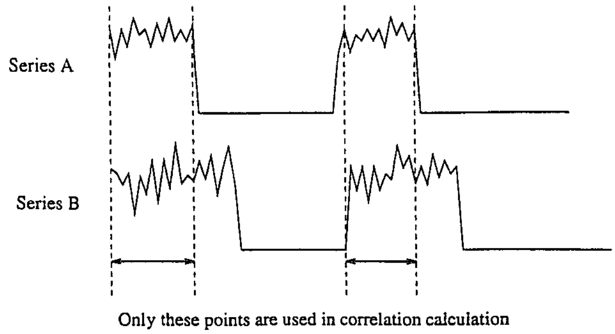

FIG. 1. Scheme of estimation of the correlation coefficient of the adjacent FOVs.

TABLE 2. Number of observation pairs used to estimate correlations in Table 1.

	j								
i	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10
1	15	39	45	61	58	57	78	147	207
2	20	49	36	75	40	95	93	157	164
3	13	54	71	77	58	110	107	154	148
4	0	39	63	88	113	147	127	145	167
5	42	59	89	81	171	165	138	161	183
6	43	99	129	123	183	170	127	162	194
7	40	97	160	184	181	141	131	157	222
8	51	103	158	192	173	153	144	165	224
9	37	61	136	189	178	154	106	117	40
10	0	47	112	134	108	93	49	0	0

taneously nonzero terms (see Fig. 1) of each pair of the adjacent series for which the spatial correlation coefficient is estimated. This number was taken to be equal to 10.

If i is the row number and j is the column number (of the 10×10 array) then, fixing the i, one can find 9 correlation coefficients between the adjacent time series $j = 1$ and $j = 2$; $j = 2$ and $j = 3$; $\cdots$; $j = 9$ and $j = 10$. Repeating this procedure for each i, one obtains 90 estimates of the correlation coefficient that characterize the statistical dependence of the BFE along an east–west line.

The results in Table 1 give the correlation coefficient estimates when the threshold value (for the FOV area-averaged rain rate) is taken to be 1 mm h^{-1}. The estimates vary significantly [as well as the number of the corresponding nonzero pairs of FOVs (Table 2) used for their estimation].

The t-statistic values (Devore 1994) given in Table 3 show that there are 41, about 45%, statistically significant (at 95% level) and $43 = 84 - 41$ statistically insignificant estimates in this case. The number of the statistically significant estimates is too large not to reject the hypothesis about a zero value of the spatial correlation coefficient analyzed. [Our anonymous reviewer showed that, under some assumptions, the likelihood to get 41 (or more) successes out of 83 by chance is less than 10^{-4}.]

Analogous estimates (which we do not provide here) for each pair of the adjacent FOVs along the north–south line and for other threshold values (2 and 3 mm h^{-1}) lead to approximately the same conclusion. We repeat the calculation for different thresholds because it may be that the microwave retrieval algorithms in actual practice may need some flexibility in the choice of threshold.

The summary results (Table 4) obtained by averaging the correlation coefficients over the entire number of cases (90), over the nonzero estimates and over the statistically significant estimates, show that the number of the statistically significant estimates varies from about 29% to 48%; the number of the statistically insignificant estimates varies from about 48% to 51%; and the number of cases for which the results could not be obtained are from 3% to 20% (for the interval of the GATE1 observations).

The roughly equal number of the statistically significant and insignificant estimates is, possibly, explained

TABLE 1. Spatial correlations (threshold is 1 mm h^{-1}, west–east direction).

	j								
i	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10
1	−0.03	0.56	0.12	0.13	0.44	−0.01	−0.06	0.14	0.36
2	0.31	−0.04	0.03	0.34	0.44	0.04	0.21	0.14	0.05
3	0.09	0.56	0.09	0.19	0.47	0.17	0.03	0.15	0.00
4	0.00	−0.12	0.01	0.15	0.18	0.25	0.11	0.18	0.26
5	−0.06	0.15	0.36	−0.11	0.21	0.54	0.13	0.08	0.07
6	0.64	0.03	0.34	0.05	0.22	0.19	0.47	0.15	0.38
7	0.56	−0.03	−0.01	0.21	0.09	−0.03	0.20	0.29	0.23
8	0.50	0.21	0.52	0.36	0.03	0.46	0.38	0.12	0.14
9	0.28	0.00	0.32	0.02	0.19	0.80	0.67	0.18	0.22
10	0.00	0.12	0.03	0.37	0.35	0.28	0.18	0.00	0.00

TABLE 3. The t-statistic values corresponding to the correlations in Table 1.

	j								
i	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10
1	−0.1	4.1	0.8	1.0	3.6	−0.1	−0.6	1.7	5.5
2	1.4	−0.3	0.2	3.1	3.0	0.4	2.0	1.8	0.6
3	0.3	4.9	0.7	1.7	4.0	1.7	0.3	1.9	0.1
4	0.0	−0.8	0.1	1.5	2.0	3.1	1.3	2.2	3.5
5	−0.4	1.2	3.6	−1.0	2.8	8.1	1.5	1.0	0.9
6	5.3	0.3	4.1	0.6	3.0	2.6	6.0	1.9	5.7
7	4.2	−0.3	−0.1	2.9	1.2	−0.3	2.4	3.8	3.5
8	4.0	2.1	7.6	5.4	0.3	6.4	4.9	1.5	2.0
9	1.7	0.0	3.9	0.3	2.6	16.3	9.3	2.0	1.4
10	0.0	0.8	0.4	4.6	3.8	2.8	1.3	0.0	0.0

TABLE 4. Summary results: mean values of the correlations obtained by averaging over different number of cases.

Threshold	Over all estimates		Over nonzero estimates		Over statistically significant estimates	
	N of cases	ρ	N of cases	ρ	N of cases	ρ
Along rows (W–E direction)						
1	90	0.20	84	0.21	41	0.36
2	90	0.20	81	0.22	36	0.42
3	90	0.21	72	0.27	26	0.51
Along columns (N–S direction)						
1	90	0.20	87	0.21	43	0.35
2	90	0.23	85	0.24	39	0.44
3	90	0.22	81	0.24	36	0.47

by the predominate direction of wind for the GATE1 time interval (see Fig. 2). When this direction is along the adjacent FOV areas (from one to another), the correlation is maximized; when it is in normal with the adjacent FOV areas, the correlation is minimized.

We think that these results confirm the theoretical conclusion about spatial statistical dependence of the BFE for the adjacent FOVs. The most reliable, statistically significant estimates show that the correlation coefficient is in the interval of 0.35–0.5.

These results characterize only the natural statistical dependence of the BFE. In the case of satellite observations, some additional component, conditioned by the possible features of the particular measurement device used can slightly increase the correlations.

4. Equivalent number of statistically independent FOVs

Next consider the real situation as represented in the GATE data. We will show results for several different spatial correlation values between neighboring FOVs. The bottom line is that because rainy areas tend to be clustered, there is considerable reduction in the effective number of independent raining FOVs. Depending on threshold and correlation coefficient, one can see that the number is reduced by a factor of roughly 1/3.

We assume that the BFE field is approximately isotropic and its spatial correlations can be approximated by the exponential correlation function

$$\rho(ij;\, i'j') = \rho^{d_{ij;i'j'}}, \tag{5}$$

where $\rho(ij;\, i'j')$ is the spatial correlation of the (i, j) and (i', j') FOVs, $d_{ij;i'j'}$ is the distance between FOVs (i, j) and (i', j').

The average BFE over a scene is

$$E_{\text{scene}} = \frac{\sigma^2}{N_{\mathcal{K}}} \sum_{(i,j)\in\mathcal{K}} \delta_{\text{BF}}(i, j), \tag{6}$$

where summations must be conducted over the FOVs with $[R]_0 \geqslant$ threshold, $\mathcal{K}$ is the set of the FOVs for which $[R]_0 \geqslant$ threshold, and $N_{\mathcal{K}}$ is the number of members of the set $\mathcal{K}$.

The ensemble average of E_{scene} is just the bias referred to earlier. We are interested in the variance of E_{scene}, whose square root gives us an estimate of the spread of random errors for a typical scene. If the terms in the sum for E_{scene} were statistically independent, we would find the variance to be $\sigma^2/N_{\mathcal{K}}$. When the spatial dependence is taken into account we must include cross terms involving correlation; we find

$$\frac{\sigma^2}{M} = \frac{\sigma^2}{N_{\mathcal{K}}} \sum_{(i,j)\in\mathcal{K}} \sum_{(i',j')\in\mathcal{K}} \rho(ij;\, i'j'), \tag{7}$$

where M is the equivalent number of the FOVs with statistically independent BFEs. This quantity M is the desired result of our calculations. Clearly, if $\rho(ij;\, i'j')$ vanishes for $i \neq i'$ and $j \neq j'$, then $M = N_{\mathcal{K}}$. (It should be noted that the data effects the results by determining which FOVs are above the threshold.)

Before passing to the actual GATE clusters, consider a case where there are only 9 rainy FOVs and these are in a 3×3 array. We used the above procedure to estimate M. Depending on where the correlation coefficient lies in the interval (0.35–0.50) we can show that the effective number of independent variates is roughly 2.5–3.0 (instead of 9 in the case of no correlation between neighbors). This extreme situation shows the upper bound on the reduction of independent variates in a scene. Next we use actual GATE data in the computation.

The results of the calculations for different $\rho = 0.1, 0.2, \cdots, 0.9$, are given in Table 5, which shows that the mean number of the observed FOVs containing rain rate above threshold varies from about 12 to 19, depending on the threshold value (the smaller the threshold, the greater the number of corresponding FOVs considered). This means that, when it is rainy somewhere in the GATE scene, only 12%–19% (on average) of FOVs had enough rain to be used in our calculations. Another important point is that the variance of the number of rainy FOVs from scene to scene is

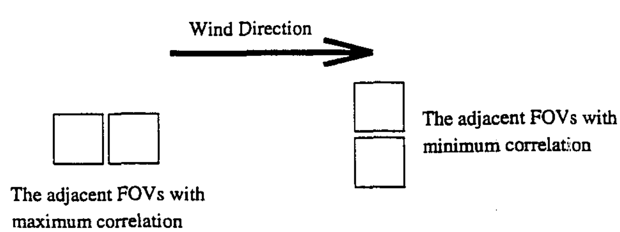

FIG. 2. Dependence of the correlation values on the wind direction.

Table 5. Mean number of the FOVs ($\bar{M}$) with statistically independent BFEs and the mean number ($\bar{K}$) of the observed BFEs [Eq. (2)] (averages over all rainy situations) for different spatial correlation values.

Threshold	Number of fields (situations)	Correlations ρ for lag $d = 1$									Mean number of the observed FOVs, $\bar{K}$
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
1	977	14.8	11.1	8.4	6.4	4.8	3.6	2.7	2.0	1.4	19.4
2	837	11.8	9.1	7.0	5.5	4.2	3.3	2.5	1.9	1.4	15.2
3	731	9.9	7.8	6.1	4.8	3.8	3.0	2.3	1.8	1.3	12.5

very large; when it is raining in a scene, the rain tends to be widespread, leading to an enlargement of the correlation effect we are studying here.

The equivalent number of statistically independent FOVs in a scene, when the correlation coefficient value is between 0.4 and 0.5, varies from about 4 to 6; it is about a factor of 3 less than the total number of rainy FOVs. This result tells us that the degree of clustering in a GATE scene is great.

5. Final comments

We have shown (theoretically and experimentally) that the BFEs of two adjacent FOVs are statistically dependent. The most reliable, statistically significant estimates show that the correlation coefficient between BFEs of adjacent FOVs is about 0.35–0.5. The equivalent number of the FOVs with independent BFEs is about three times less than the observed ones.

In the ideal accumulation of data from FOVs over a month, one might expect a great amount of cancelation of the random part of the measurement error. In the case of the random part of the BFE, there is significant correlation between the error in an FOV and that in its neighbors (when there is rain in each). Hence, the degree of cancelation of the errors is less. In the case of GATE data it was found that the degree of clustering of rainy FOVs is great, leading to an enhancement of the effect. In fact, for GATE-like rain data, the reduction in the number of independent random error terms is reduced by about a factor of 3. This means that in the example of 12 000 readings with about 1200 of them rainy, we have only of the order of 400 independent error terms. This would lead one to think that the contribution from the accumulation of random errors is about a factor of $\sqrt{3}$ larger than previously estimated (few percent). As one moves away from the equator and the associated heavy rain band, the number of rainy FOVs will be much less than the 1200 estimated here. It would be interesting to repeat the procedures invoked here for subtropical conditions.

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APPENDIX

Theoretical Proof of the Dependence of the BFEs

In this appendix we address the problem of whether a random variable like $\delta_{BF}(i, j)$ should be expected to exhibit dependence between evaluations at (i, j) and nearby locations (i', j') when the underlying rain-rate field $r(i, j)$ does exhibit spatially lagged correlation. Roughly speaking, is the "graininess" of the field from place to place correlated?

To proceed we introduce a convenient matrix notation. The BFE (4) is proportional to

$$\frac{1}{49}\sum_{k=1}^{49}(R_k - [R]_0)^2 = \mathbf{X}^{\mathrm{T}}\mathbf{X}, \tag{A1}$$

where T means transposition, and the random vector

$$\mathbf{X} = \frac{1}{N}\{R_k - [R]_0\}_{k=1}^{N\times N}, \tag{A2}$$

where $N \times N$ is the number of GATE tiles in a FOV. The right-hand side of the formula (A1) is a quadratic form conveniently considered in statistics for a random vector $\mathbf{X}$.

Let $\mathbf{X}_1$ and $\mathbf{X}_2$ be two vectors of the type (A2) corresponding to two adjacent FOVs. Then the problem that we are going to consider can be expressed as follows: Are two quadratic forms, $\mathbf{X}_1^{\mathrm{T}}\mathbf{X}_1$ and $\mathbf{X}_2^{\mathrm{T}}\mathbf{X}_2$, statistically dependent?

For normally distributed random vectors $\mathbf{X}_1$ and $\mathbf{X}_2$, the answer can be easily obtained from the theory presented, for example, in Rao (1973). According to this theory two quadratic forms $\mathbf{Z}^{\mathrm{T}}\mathbf{A}_1\mathbf{Z}$ and $\mathbf{Z}^{\mathrm{T}}\mathbf{A}_2\mathbf{Z}$ (where $\mathbf{Z}$ is a normal random vector, $\mathbf{A}_1$ and $\mathbf{A}_2$ are nonrandom matrices, elements of which are numbers) are independent if and only if

$$\mathbf{A}_1\mathbf{\Sigma}\mathbf{A}_2 = 0 \tag{A3}$$

($\mathbf{\Sigma}$ is a covariance matrix of vector $\mathbf{Z}$).

To apply this theory to our case, let us introduce notations for $\mathbf{Z}$, $\mathbf{\Sigma}$, $\mathbf{A}_1$, $\mathbf{A}_2$ as follows:

$$\mathbf{Z} = \begin{pmatrix}\mathbf{X}_1\\ \mathbf{X}_2\end{pmatrix}, \tag{A4}$$

$$\mathbf{\Sigma} = \begin{pmatrix}\mathbf{C}_1 & \mathbf{C}_{12}\\ \mathbf{C}_{12} & \mathbf{C}_2\end{pmatrix}, \tag{A5}$$

$$\mathbf{A}_1 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \tag{A6}$$

and

$$\mathbf{A}_2 = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}, \tag{A7}$$

where $\mathbf{C}_1$ and $\mathbf{C}_2$ are the covariance matrices of the vectors $\mathbf{X}_1$ and $\mathbf{X}_2$; $\mathbf{C}_{12}$ is their cross-covariance matrix; $\mathbf{I}$ is the unit matrix of the $N \times N$ size; $\mathbf{0}$ is the matrix of the $N \times N$ size, all element of which are zeros.

Because in our case

$$\mathbf{A}_1\Sigma\mathbf{A}_2 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{C}_1 & \mathbf{C}_{12} \\ \mathbf{C}_{12} & \mathbf{C}_2 \end{pmatrix} \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{C}_{12} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \tag{A8}$$

the two quadratic forms

$$\mathbf{Z}^{\mathrm{T}}\mathbf{A}_1\mathbf{Z} = \mathbf{X}_1^{\mathrm{T}}\mathbf{X}_1 \tag{A9}$$

and

$$\mathbf{Z}^{\mathrm{T}}\mathbf{A}_2\mathbf{Z} = \mathbf{X}_2^{\mathrm{T}}\mathbf{X}_2 \tag{A10}$$

are independent if two sets of observations, $\mathbf{X}_1$ and $\mathbf{X}_2$, are independent ($\mathbf{C}_{12} = \mathbf{0}$).

Therefore, the two quadratic forms of the vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ of the type (A2) are independent only if the vectors are independent. This result is really obvious and, in spite of the fact that rain-rate observations (when above threshold) are not normally distributed, one can expect that the theory will hold in our case as well.

Consider next the dependence of $\mathbf{X}_1$ and $\mathbf{X}_2$. As it was shown, for example, by Polyak and North (1995) and by Nakamoto et al. (1990) rain-rate observations are highly spatially correlated or $\mathbf{C}_{12} \neq \mathbf{0}$. Spatial correlations can be traced up to 100 km and more. Moreover, for distances of about 35 km, the spatial correlations of temporally averaged rain rate can be approximated by the exponential correlation function $\rho(d) = 0.9^d$ where d is lag in kilometers.

This means that the above quadratic forms (and, therefore, the BFEs of two adjacent FOV) are *statistically dependent.*

The reasoning and outcome presented in this appendix are confirmed by the findings based upon the observations in the GATE data as shown in the text.

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