Real-Time Radar Rainfall Estimation. Part I: Algorithm Formulation

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ABSTRACT

A multicomponent radar-based algorithm for real-time precipitation estimation is developed. The algorithm emphasizes the combined use of weather radar observations and in situ rain gauge rainfall measurements. The temporal and spatial scales of interest are hourly to storm-total accumulations for areas of 4 km^2 to approximately 16 km^2. The processing steps include beam–height-effect correction, vertical integration, convective–stratiform classification, conversion from radar observables to rainfall rate, range-effect correction, and transformation of the estimated rainfall rates from polar coordinates to a Cartesian grid. Additionally, the algorithm applies advection correction to the gridded rainfall rates to minimize the temporal sampling effect and, subsequently, aggregates the corrected rainfall rates to 1-hourly, 3-hourly, and storm-total accumulations. The system applies different parameter values for convective and stratiform regimes. The calibration of the system is formulated as a global optimization problem, which is solved using the Gauss–Newton adaptive stochastic method. The algorithm is cast in a recursive formulation with parameters adjusted in real time. Evaluation of the system is based on an extensive dataset from the Melbourne, Florida, WSR-88D radar site.

1. Introduction

A main objective of the National Weather Service's Next Generation Weather Radar (NEXRAD) program is to provide, in real time, accurate quantitative precipitation estimates from a network of weather radars (Heiss et al. 1990; Klazura and Imy 1993). These estimates primarily support flash flood forecasting at Weather Forecast Offices, river-stage forecasts at River Forecast Centers, and numerical weather prediction models at the National Centers for Environmental Prediction (Hudlow 1988). For this purpose, a radar rainfall estimation algorithm, named Precipitation Processing Subsystem (PPS), has been designed to provide rainfall fields at different spatial (1° $\times$ 2 km up to 4 km $\times$ 4 km grid boxes) and temporal scales (1-hourly, 3-hourly, and storm-total accumulations) of aggregation from reflectivity data (Hudlow et al. 1991; Seo et al. 1995; Fulton et al. 1998).

Recent studies by Seo et al. (1995), Smith et al. (1996), Anagnostou and Krajewski (1999), and Anagnostou et al. (1998) show that PPS has not reached its potential in providing accurate rainfall estimates. First, PPS operates with a default set of parameters that are not optimal and do not account for the regional and seasonal rainfall regime differences. Second, there is mean-field systematic underestimation with respect to rain gauge rainfall. Third, there are radar calibration problems that create significant rainfall estimation differences in radar-to-radar comparisons. Finally, Smith et al. (1996) and Seo et al. (1995) show significant range-dependent biases. For example, they show in the cold season (October–March), up to 100% overestimation (underestimation) for radar ranges between 50 and 80 km (>100 km). They also show up to 150% underestimation for close ranges (<50 km), which is attributed to the hybrid-scan construction strategy of the PPS.

This research, inspired by the PPS design, focuses on issues related to the improvement of the accuracy of real-time quantitative precipitation estimation by radar. We attempt to answer several fundamental questions. Is improvement in real-time radar rainfall estimation possible? If so, how significant can this be? How does the algorithm performance depend on radar range? What are the most appropriate space–timescales of aggregation for the algorithm parameter selection? To answer these questions, several existing and herein proposed methodologies are formulated into an integrated precipitation processing system for real-time radar rainfall estimation. The new system includes hybrid scan construction from multiple-tilt radar observations, real-time rainfall regime classification, reflectivity-to-rainfall rate

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(Z–R) conversion, mean-field, and range-dependent systematic error correction, advection correction of the final gridded rainfall products, and recursive parameter estimation.

Special attention is paid to the uncertainty of the Z–R conversion, which can be potentially reduced by applying different Z–R relationships based on real-time convective–stratiform classification. Rain classification also offers potential for range-effect correction according to storm morphology. Investigations of rainfall classification effects were carried out by Steiner et al. (1995), Rosenfeld et al. (1995), and Ciach et al. (1997), among others. These studies, which concentrated on daily-to-monthly timescales of aggregation, showed negligible improvement in radar rainfall accuracy by applying rainfall classification. However, we believe that this was a temporal resolution problem. In this research, classification is investigated at a finer timescale (hourly accumulations).

Another important aspect of this study is the calibration of the algorithm. The parameters are estimated in real time, which is a completely different approach than the conventional data-fitting exercises done in the past (Battan 1973). The problem is solved by optimizing a selected rainfall product quality criterion (objective function). This procedure estimates the system's parameters, simultaneously accounting for their possible interactions. It is designed to handle difficulties such as the high dimensionality and nonlinear structure of the system, feedback and system memory, and the high cost of computing the objective function. These features make the use of standard optimization techniques infeasible. We propose the use of a recursive stochastic optimization approach (Ljung and Soderstrom 1981), combined with highly efficient data preprocessing (Anagnostou and Krajewski 1999).

The algorithm also includes a mean-field bias model for real-time adjustment of the systematic rainfall estimation error. Mean-field bias is modeled as a state–space stochastic–dynamic system (Anagnostou et al. 1998). State estimates are obtained on an hourly basis from corresponding rain gauge and radar rainfall accumulations. Bias estimations and 1-h-ahead predictions are based on Kalman filter equations (Gelb 1974), which optimally combine observations and model estimates, taking into account uncertainties associated with both. This bias adjustment scheme, although conceptually similar to the PPS (both schemes use Kalman filter equations), employs a different method for evaluating the Kalman filter's model and observation error variances. Data-based Monte Carlo simulation experiments, performed by Anagnostou et al. (1998), showed that the proposed bias prediction algorithm was unbiased and outperformed the PPS bias scheme at different scales (1-hourly, 3-hourly, and 6-hourly accumulations) and seasons [warm (May–October) and cold (November–April)].

Other important features of our algorithm are the following. First, a beam–height-effect correction is applied at the level of raw volume scan reflectivity data. Second, an advection correction is used to estimate rainfall accumulations. Correcting for beam–height effect, in real time, is particularly important for events with strong vertical reflectivity gradients (i.e., bright band). Our algorithm uses a mean-field vertical reflectivity profile-based technique, proposed by Andrieu and Creutin (1995), for reducing the beam–height effect due to brightband and snow region sampling. The advection correction procedure uses velocity fields that are calculated based on a modified cross-correlation method (Bellon et al. 1991).

We present development of the algorithm and its testing as a two-paper series. This paper is organized as follows. Detailed description of the algorithm's components is provided in section 2. In section 3, we discuss the parameter estimation procedure. We summarize the proposed algorithm in section 4. The paper includes two appendixes. Appendix A describes the advection correction scheme, and appendix B presents the Gauss–Newton adaptive stochastic optimization method. The results of the algorithm implementation and performance demonstration are described in the companion paper (Anagnostou and Krajewski 1999).

2. Algorithm description

a. Algorithm summary

The purpose of the proposed algorithm is to produce real-time 1-hourly, 3-hourly, and storm-total rainfall accumulation maps based on volume scan radar reflectivity measurements. The algorithm satisfies the following general features. First, it provides unbiased hourly rainfall estimates. Second, it accepts external information, that is, rain gauge observations in real time. Third, it is formulated in a general way, parameterized to adjust to seasonal and geographic rainfall regime variations. Finally, its calibration, performed in real time, forces statistical consistency and convergence for the parameter values.

The algorithm produces rainfall maps in the Hydrologic Rainfall Analysis Project (HRAP) coordinates (Fulton et al. 1998), which cover a range of 200 km and have spatial resolution of approximately 16 km^2. The aforementioned spatiotemporal scale is based on WSR-88D operational product requirements. The structure of the algorithm's processing components is described herein.

The first step in the algorithm is a beam–height correction based on a mean vertical reflectivity profile (VRP) method, activation of which depends on the type of the precipitation system. The second step is construction of a hybrid radar reflectivity observation map by integrating the two lowest sweeps. The hybrid map reflectivities are classified into two types, convective and stratiform, and are converted to rainfall rates by applying the Z–R transformation and range-effect correction. The Z–R transformation and range correction use different parameter values for each precipitation type. Every time a volume scan is collected, the algorithm produces the scan-to-scan rainfall accumulations. These accumulations are interpolated from polar coordinates to a 2 km $\times$ 2 km Cartesian grid, corrected for advection, accumulated to form hourly rainfall, and subsequently projected to the HRAP grid plane. The HRAP gridded hourly rainfall accumulations are corrected for mean-field systematic errors. Every so often, when an adequate number of rain gauge observations are available, the algorithm updates its parameter values using an adaptive global optimization scheme. Three-hourly and storm-total accumulations are constructed from the hourly maps.

b. *Vertical reflectivity profile effect correction*

A vertical reflectivity gradient is created when the phase or intensity of precipitation changes with height. If this gradient is strong, radar measurements, which are taken at high elevations (1–4 km) in the atmosphere, are not representative of surface rainfall. Interaction between droplets, updrafts and downdrafts, evaporation and accretion of drops under the cloud base, and mixed-phase precipitation (bright band) are the physical causes that can create strong gradients in the vertical reflectivity profile. Studies related to this problem have been reported by Zawadzki (1984), Austin (1987), Joss and Waldvogel (1990), and Kitchen and Jackson (1993), among others.

The beam–height effect due to VRP is particularly important in radar measurements of stratiform precipitation systems. The VRP of these systems results in radar underestimation or overestimation of rainfall depending on the range from the radar (e.g., Joss and Waldvogel 1990; Kitchen and Jackson 1993). The main factors for this beam–height effect are 1) strong reflectivity enhancement at ranges in which the beam intercepts the melting layer (bright band), 2) reflectivity reduction when the beam samples the snow region, and 3) nondetection at far ranges in which the beam overshoots the cloud tops. The influence of bright band is significant, resulting in overestimation up to a factor of 10, at which the zero isotherm is at low altitudes relative to the radar (see Smith et al. 1996). Partial detection and nondetection are important in cases of low-top precipitating clouds (e.g., widespread precipitation, warm rain processes, and orographic enhancement processes) and in cases in which the beam is at a far range from the radar (i.e., $>$150 km).

This algorithm is particularly effective in correcting beam–height effects due to strong VRP gradients. Such gradients can originate from strong reflectivity enhancements due to bright band in midlatitude stratiform clouds and from range degradation of reflectivity due to partial beamfilling in widespread precipitation systems (Seo et al. 1996). The procedure uses mean VRP values determined in real time (Andrieu and Creutin 1995). The attractive features of this procedure include the use of 1) the Kalman filter inverse method to deduce mean VRP from observed ratio curves (the ratio of reflectivities from two sweeps at discrete ranges from the radar) and 2) the radar beam geometry in the construction of the beam–height correction function.

A limitation of this procedure is the assumption of spatial uniformity of VRP. As a result, application to precipitation systems with spatially varying reflectivity profiles (i.e., squall lines, tropical convective storms, and midlatitude convective systems, etc.) would introduce additional uncertainty to the radar rainfall estimates. Therefore, the VRP correction method is deactivated in these cases. Instead, a simpler procedure, which uses a climatologically based range-effect correction formula, is applied to the rainfall rates at the 1° $\times$ 1 km scale. The procedure is discussed in section 2e.

On the other hand, widespread stratiform precipitation systems, whose dominant vertical structure (bright-band) is less spatially variable, should be better predicted by this technique. A simulation study by Borga et al. (1997) has shown that, for these systems, mean VRP-based beam–height corrections can significantly [$\sim$40% root-mean-square error (rmse) reduction] reduce the error due to bright band. Little improvement (less than 10% rmse reduction) was shown, though, at distances beyond the range in which the 0° isotherm intercepts the radar beam and in which the 0° isotherm is at high altitudes ($>$3 km).

c. *Hybrid reflectivity scan construction*

After the raw volume scan reflectivity data have been corrected for the beam–height effect, the algorithm produces a hybrid reflectivity map that consists of a weighted averaging of reflectivities from the two lowest sweeps. The following formulas are used to calculate the weights applied to each sweep:

$$W_0 = \begin{cases} S/S_x, & S \le S_x, \\ 1, & S > S_x, \end{cases} \tag{1}$$

and

$$W_1 = \begin{cases} 1 - S/S_x, & S \le S_x, \\ 0, & S > S_x, \end{cases} \tag{2}$$

where W_0 and W_1 are the first and second sweep's reflectivity weights, respectively; S is the distance (km) of the radar cell from the radar; and S_x (km) is the range beyond which only the first sweep's reflectivities are used in the hybrid scan construction. The range S_x is defined as a parameter.

Vertical reflectivity integration has the following two justifications. First, it is expected to reduce the high temporal and spatial variability of the radar data. Second, it includes the algorithm information relevant to surface rainfall accumulation (Green and Clark 1972; Ciach et al. 1997). The weighted averaging is performed to provide a smooth transition between the two lower sweeps. Note that the presented formulation is not applicable to radar operations in mountainous regions in which portions of the lower beam may be embedded in the terrain. In such situations, different schemes, which account for the whole volume scan information (Joss and Lee 1995), may be more appropriate.

d. Classification of reflectivity measurements

Many studies in the past (e.g., Atlas and Chmela 1957; Zawadzki 1984; Austin 1987; Smith and Krajewski 1993) have shown significant variability in the relationship between radar reflectivity and surface rainfall rate. This variability sources from differences in the microphysical properties and vertical structure of raining clouds associated with different precipitation systems. These systems can be distinguished based on geographic location (tropical, maritime, continental, orographic, and warm rain processes, etc.), season (warm, cold), and precipitation type (convective, stratiform).

Classification of reflectivity measurements associated with the aforementioned precipitation systems, prior to their conversion to rainfall rate, has strong physical and observational justification. Houze (1993), using cloud models and radar observations, presents significant differences in the microphysical properties and vertical structure of raining clouds associated with convective and stratiform regimes. Also, Tokay and Short (1996) show significant differences in the multiplier coefficient of the Z–R relationship associated with convective and stratiform rain types. Anagnostou and Kummerow (1997) show that convective and stratiform clouds have a different effect on the spatial variability of the 85-GHz microwave brightness temperature observed by satellites. It is expected that uncertainty in the Z–R conversion can potentially be reduced by applying different Z–R relationships for different precipitation systems, seasons, and types.

A modified form of the approach presented by Steiner et al. (1995) is used to classify instantaneous reflectivity patterns to convective and stratiform origin. The original approach uses the instantaneous horizontal radar reflectivity patterns, and the reflectivity itself, as predictors of the echo type. This algorithm has been evaluated objectively based on radar reflectivity fields taken from a shipborne radar during the Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (Kucera et al. 1996). Modification of this technique consists of applying the classification results at the instantaneous volume scan reflectivities. First, the reflectivities in polar coordinates are interpolated to a 2 km $\times$ 2 km Cartesian grid. Second, the algorithm by Steiner et al. (1995) is used to classify the gridded reflectivity values. Third, the gridded classification map is converted to the radar's polar coordinate system. Based on this coordinate transformation, each radar cell's reflectivity value is assigned a convective–stratiform index.

Reports from other radar rainfall studies regarding this classification scheme did not show any significant benefit to radar rainfall estimation (Ciach et al. 1997; Steiner and Houze 1996; Yuter and Houze 1996). However, these studies concerned a very specific precipitation system (i.e., tropical oceanic convection) and much higher temporal scales (i.e., daily to monthly accumulations). The question of how well this technique can apply to hourly scales of aggregation in continental mesoscale systems is still open for research.

e. Conversion of reflectivities to rainfall rates

The reflectivities of the hybrid reflectivity map, which exceed a specified minimum reflectivity threshold ($Z_{\min}$), are converted to rainfall rates through the following nonlinear operation:

$$R_p(s, t, \theta, r) = \left[\frac{Z_p(s, t, \theta, r)}{A}\right]^{(1/B)}, \quad \text{for } Z_p > Z_{\min}, \qquad (3)$$

where Z_p and R_p are the corresponding integrated reflectivity (mm^6 m^{-3}) and rainfall rate (mm h^{-1}) values at hour s, scan t, radar cell azimuth θ, and range r (km); A and B are the Z–R relationship parameters, which correspond to the conventional Z–R relationship form.

Note that reflectivity values greater than a maximum reflectivity threshold ($Z_{\max}$) are assigned the maximum reflectivity value. This procedure is similar to PPS and is considered a quick and inexpensive way to reduce the hail contamination effect (Austin 1987).

In cases of mesoscale convective systems and tropical storms, a climatologically based range-effect correction procedure is applied. (In these types of storms, beam–height correction based on the mean VRP-based method is not applied.) The range-effect procedure uses different parameterization for convective and stratiform rain types. The range-effect-correction equations for these types are given below:

$$R_p(s, t, \theta, r) = \begin{cases} b_c\left(1 + \dfrac{rA_{\mathrm{rc}}}{S_0}\right)R_p(s, t, \theta, r), & \text{for convective rain,} \\ \left(1 + \dfrac{rA_{\mathrm{rs}}}{S_0}\right)R_p(s, t, \theta, r), & \text{for stratiform rain,} \end{cases} \qquad (4)$$

where b_c is a parameter that differentiates the convective and stratiform multipliers of the Z–R relationship. Parameters A_{rc} and A_{rs} control the range correction applied to the rainfall estimates of the convective and stratiform types, respectively. Both classification (b_c) and range-correction (A_{rc}, A_{rs}) parameters are applied uniformly across the radar domain. In Eq. (4), S_0 equals 200 km, which is the algorithm's effective radar range for rainfall estimation.

f. Rainfall products generation

The instantaneous rainfall-rate maps are used to compute the scan-to-scan and hourly accumulation maps. First, the instantaneous maps in polar coordinates are interpolated to a large domain (400 km $\times$ 400 km) Cartesian grid with a 2 km $\times$ 2 km pixel resolution. The spatial interpolation is performed by dividing the radar sampling volume into many elementary subdivisions along the azimuth and then taking the average rainfall rates for those that project into the Cartesian pixels. This approach is fully parameterized to handle any combination of radar characteristics and Cartesian field specifications. The transformation is performed with the use of lookup tables to accelerate the conversion.

The next step is to create the scan-to-scan accumulation map from two consecutive instantaneous maps. A typical way (i.e., PPS) of doing so is by averaging the corresponding values of the two maps. However, recent studies (e.g., Bellon et al. 1991; Liu and Krajewski 1996) have shown that in cases of fast-moving storms the temporal sampling effect of the radar observations can lead to significant errors in the estimated rainfall accumulation maps. Our algorithm uses a temporal interpolation scheme (advection correction procedure) that accounts for the shift of rainfall patterns between consecutive radar rainfall maps. This advection correction procedure converts the two consecutive radar rainfall maps into a scan-to-scan accumulation. Details about the advection correction procedure are provided in appendix A. Note that advection correction is not applied when the time between the consecutive scans is greater than 30 min. This time interval is selected following previous studies, suggesting that the error tolerance of statistical extrapolation techniques is significant at larger time lags (Fabry et al. 1994). When the sampling interval exceeds 30 min, the algorithm follows a procedure, similar to the PPS, that flags the excess time period, centered midway between the two scan times, as a missing period. The incremental period accumulations, 15 min before and after the missing period, are computed using the corresponding single-scan rain-rate maps.

Subsequently, the hourly accumulation rainfall maps are computed by summing the scan-to-scan accumulations from the incremental periods falling within the hour. Note that if the missing periods within an hour are more than a certain threshold (10 min), then no accumulation is computed for this hour. From the hourly accumulations, the algorithm computes the 3-hourly and storm-total accumulations.

Finally, the accumulations in Cartesian coordinates are projected to the HRAP grid plane [otherwise known as 1/40th Limited Fine Mesh (LFM) grid], which is used for the entire National Weather Service River Forecasting System operational system within the Office of Hydrology of the National Weather Service. The LFM grid is a limited polar-stereographic projection and covers essentially the coterminous (i.e., lower 48) United States, plus some surrounding territory.

g. Mean-field systematic error adjustment

The mean-field systematic error (bias) adjustment is applied to the final (i.e., HRAP grid) hourly rainfall products. The decision of applying range correction (4), prior to mean-field bias adjustment, is supported by recent studies based on NEXRAD radar rainfall products (Seo et al. 1995). Mean-field bias adjustment is applied by scaling the rainfall accumulations by a constant factor (B_s):

$$A(s, j) = A_c(s, j)B_s, \tag{5}$$

where A_c and A (mm h^{-1}) are the hourly rainfall accumulations at hour s and HRAP pixel j before and after the bias adjustment, respectively. The bias adjustment factor B_s is estimated based on an hourly mean-field bias model described herein.

The bias governing equation is

$$\beta_s = \beta_{s-1} + W_s, \tag{6}$$

where β_s is the $\log_e$–bias, and $W_s \sim N(0, Q)$ is the model error that is a normally distributed random variable with zero mean and variance Q. The model error W_s accounts for mean-field bias fluctuations, which are due mainly to temporal variations in the hydrometeors' size distribution and the transmitted beam power. The bias and the data are linked by the observation equation

$$Y_s = \beta_s + M_s, \tag{7}$$

where Y_s is the natural logarithm of the observed bias at hour s:

$$Y_s = \log_e\left[\frac{\sum_{j=1}^{N_g} R_g(s, j)}{\sum_{j=1}^{N_g} A_c(s, j)}\right], \tag{8}$$

where R_g is the rain gauge hourly rainfall accumulation, j is the rain gauge index, and N_g is the total number of rain gauges under the radar umbrella. It should be noted that bias is computed only from the common radar–rain gauge pairs. Therefore, one should expect significant bias observation error due to limited sampling of the rain gauge network. Anagnostou et al. (1998) have shown that for a typical rain gauge network density (i.e., 30–40 gauges within a 200-km radius from the radar) the bias observation sampling error variance can be up to 300% of the mean bias, depending on the season and precipitation type. In this study, the observation error M_s is defined as a normally distributed variable with zero mean and variance R_M.

TABLE 1. The parameters of the real-time radar rainfall estimation algorithm.

Name	Description
S_x	Range parameter in Eqs. (3) and (4)
$Z_{\min}$	Lower reflectivity threshold
$Z_{\max}$	Upper reflectivity threshold
A	The Z–R relationship multiplier (5)
B	The Z–R relationship exponent (5)
b_c	Ratio of convective–stratiform Z–R multipliers
A_{rc}	Convective-range-correction parameter (6)
A_{rs}	Stratiform-range-correction parameter (7)
Q	Variance of the bias model error (9)
R_M	Variance of the bias observation error (10)

The $\log_e$-bias prediction and update are based on Kalman filter equations (Gelb 1974). The Kalman filter requires specification of two unknown parameters: the model error variance Q and the observation error variance R_M. These parameters are adjusted recursively so that the Kalman filter's innovation sequence becomes uncorrelated. Innovations are the differences between predicted and observed biases and are uncorrelated if the Kalman filter works optimally (Mehra 1972; Anagnostou et al. 1998). If the initial estimates of Q and R_M lead to a correlated innovation sequence, a number of steps are followed, based on a procedure by Mehra (1972), to obtain better estimates of Q and R_M. This procedure has been used by Anagnostou et al. (1998) in their comparison of radar-rainfall mean-field bias estimation methods. The mean of the bias adjustment factor $\overline{B}_s$ and its estimation variance Σ_B are computed from the predicted log-bias mean β_s and variance V_s using the following transformations:

$$\overline{B}_s = \exp(\hat{\beta}_s + 0.5V_s) \tag{9}$$

and

$$\Sigma_B = \overline{B}_s^2(\exp V_s - 1). \tag{10}$$

With the analysis of the mean-field systematic error correction procedure, we complete the description of the algorithm-processing stages. The algorithm's parameter estimation procedure is described next.

3. Parameter estimation procedure

The proposed system has several parameters that control the different processing components. The parameters of the proposed algorithm are listed in Table 1. Proper implementation of the algorithm to a given radar-rainfall regime requires determination of its optimal parameter values.

The estimation of the algorithm's parameters is formulated as a global optimization problem. The optimization criterion is a minimization of the root-mean-square (rms) difference between radar (HRAP grid average) and rain gauge hourly rainfall accumulations:

$$\mathrm{rms} = \left(\frac{1}{N_s N_g}\sum_{s=1}^{N_s}\sum_{j=1}^{N_g}[(R_g(s,j) - A(s,j)^2]\right)^{0.5}, \tag{11}$$

where all variables have been previously defined. The rms is assessed after the mean radar–rain gauge bias has been adjusted. This is performed by excluding parameter A of the Z–R relationship from the optimization and by tuning its value to remove the mean bias. As mentioned earlier, the objective function is applied on the final radar rainfall products. This approach is conceptually similar to the radar rainfall optimization approach proposed by Ciach et al. (1997). Use of a global optimization procedure has two main justifications. First, global procedures provide an integral assessment of the algorithm's performance. Second, because of the highly complex and nonlinear processes involved in radar rainfall measurement and estimation, separate optimization of selected processing steps does not ensure optimal performance of the algorithm at the final stage.

The main feature that differentiates our procedure from the approach proposed by Ciach et al. (1997) is that parameters are adjusted adaptively by optimizing the objective function (11) in real time. Because of the high zero intermittence and variability of rainfall (Seed and Austin 1990; Crane 1990) at the hourly timescale, only conditional radar–rain gauge pairs are included in the rms calculations. Solution of the real-time calibration problem is based on an adaptive global optimization procedure.

The adaptive optimization procedure is the stochastic Gauss–Newton method (Ljung and Soderstrom 1981). The choice of a stochastic optimization method stems from the fact that optimal estimation algorithms are difficult to implement in this situation because 1) the statistical characteristics of the model parameters are not well defined, 2) the error processes are likely to be non-Gaussian, and 3) the model structure is highly nonlinear for accurate approximation by linearization. In these cases, stochastic approximation algorithms, although not optimal in a strict sense of optimizing the true error characteristics, yield recursive model parameter estimates with well-defined convergence properties. The Gauss–Newton method is described in appendix B.

4. Closing remarks

This work presents a multicomponent radar rainfall estimation algorithm suitable for real-time applications. This algorithm provides a potential framework for operational rainfall estimation from the nationwide WSR-88D radar network. New parameterizations for hybrid reflectivity scan construction, reflectivity to rainfall rate conversion, and range-effect correction are introduced. In addition, the algorithm includes procedures for beam–height-effect correction, rainfall classification, advection correction, and recursive parameter estimation. The new parameterizations and procedures distinguish our approach from the operational PPS and other real-time radar algorithms.

The parameter estimation procedure is based on the stochastic Gauss–Newton optimization method. The rms of the radar–rain gauge conditional hourly rainfall difference is used as the objective function. Estimation of the objective function at the final rainfall products was chosen since it provides an overall assessment of the algorithm's performance. This calibration procedure is completely different from the conventional Z–R fitting problem. It is based on storm morphology (i.e., it has a physical basis), and the parameters are estimated in real time, accounting for possible interactions and feedbacks.

We point out that the choice of the minimization of the rms radar–rain gauge difference as the optimization criterion is arbitrary. The proposed framework remains valid even if a different criterion is used. The criterion should be selected based on the mission requirements for the system operators and users of the rainfall products. We decided to use the rms, as we believe it is a good overall criterion that serves well both the practical needs of flood forecasting and the other applications of the products, such as water resources system management and climate monitoring.

The global optimization approach provides the basis for assessing the significance of the different components of the algorithm in the context of rainfall estimation accuracy. This is presented in Anagnostou and Krajewski (1999), based on an extensive dataset of two months of volume scan reflectivity and rain gauge data from the Melbourne, Florida, WSR-88D radar site. Other issues investigated in the study by Anagnostou and Krajewski (1999) are 1) the convergence properties of the algorithm's adaptive calibration procedure, 2) the sensitivity of the system with respect to changes in the parameter values, and 3) the relative rms reduction with respect to the operational PPS for the newly proposed algorithm.

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APPENDIX A

Advection Correction

The advection correction scheme used in the algorithm consists of the following steps. First, the rainfall maps' Cartesian domain (200 × 200 pixels) is divided into N subdomains of equal size: in this study the subdomain size was 20 × 20 pixels. Second, for each subdomain the average advection vector $\Delta u_o(j)$ between two subsequent rainfall maps is determined based on the cross-correlation method (Bellon et al. 1991). The advection vector (unitless) is in terms of the number of radar pixels: j represents the subdomain's index. The average advection vector is the spatial shift between two subsequent rainfall maps that maximizes the cross correlation between the two maps' rainfall values of the corresponding subdomain:

$$\Delta u_o(j) = \max[0, \rho_j(\Delta u)] \tag{A1}$$

$$\rho_j(\Delta u) = \frac{1}{\sigma_{1j}\sigma_{2j}}\left[\sum_n R_{1j}(u_n - \Delta u)R_{2j}(u_n) - \mu_{1j}\mu_{2j}\right], \tag{A2}$$

where ρ is the cross correlation; $R_1(u)$ and $R_2(u)$ are consecutive radar rainfall rates at pixel u; μ_{1j}, μ_{2j} and σ_{1j}, σ_{2j} (mm h^{-1}) are the mean values and standard deviations of the jth subdomain's rainfall; Δu is the distance (in terms of the number of pixels) between two points in the domain; and the summation is over all of the u_n pixels of the jth subdomain.

The mean advection vector Δu_o of each subdomain is subsequently used to determine the subdomain's scan-to-scan accumulation as

$$R_j(u) = \frac{1}{T^2}\sum_{t=1}^{T}\left\{(T - t)R_{1j}\left[u - \frac{t}{T}\Delta u_o(j)\right] + tR_{2j}\left[u + \frac{T - t}{T}\Delta u_o(j)\right]\right\}, \tag{A3}$$

where T (min) is the time period between the two consecutive rainfall fields and t is the time interval (1 min) of the discrete temporal interpolation.

APPENDIX B

Gauss–Newton Stochastic Optimization Method

The most common categories of recursive optimization algorithms are discussed in Ljung and Soderstrom (1981). These categories are 1) modifications of offline optimization methods, 2) nonlinear filtering methods (i.e., extended Kalman filter), 3) stochastic approximation methods, and 4) model reference approaches. These methods are described in detail by Rajaram and Georgakakos (1987). Optimization algorithms involve two main features. The first is the definition of the error function that will represent the discrepancies between model outputs and ''true'' physical values. The second is the specification of an objective function in terms of some statistics of the error function.

In this study we selected a stochastic approximation method to solve the algorithm's optimization problem.

We decided to use a stochastic approach for the following reason. It is not based on strong assumptions and/or approximations, such as the normality of noise processes, the linearity of process dynamics, and/or the well-defined statistical characterization of the parameters, among others. In the following, we present the Gauss–Newton stochastic optimization method. Further details can be found in Ljung and Soderstrom (1981).

The estimation error is defined by

$$\varepsilon(t,\ \theta) = \overline{R}_g(t) - \overline{A}(t,\ \theta), \tag{B1}$$

where $\overline{R}_g$ and $\overline{A}$ represent average (over all rain gauge locations at hour t) rain gauge and radar hourly rainfall (mm h^{-1}), respectively. The following equations, derived from the quadratic objective function of Eq. (11), consist of the Gauss–Newton adaptive optimization method:

$$\mathbf{W}(t) = \mathbf{W}(t-1) + \gamma(t)[\varepsilon(t)\varepsilon^{\mathrm{T}}(t) - \mathrm{W}(t-1)], \tag{B2}$$

$$\mathbf{S}(t) = \psi^{\mathrm{T}}(t)\mathrm{P}(t-1)\psi(t) + \lambda(t)\mathrm{W}(t), \tag{B3}$$

$$\mathbf{L}(t) = \mathbf{P}(t-1)\psi(t)\mathbf{S}^{-1}(t), \tag{B4}$$

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}(t-1) + \mathbf{L}(t)\varepsilon(t), \quad \text{and} \tag{B5}$$

$$\mathbf{P}(t) = [\mathbf{P}(t-1) - \mathbf{L}(t)\mathbf{S}(t)\mathbf{L}^{\mathrm{T}}(t)]/\lambda(t), \tag{B6}$$

where t (unitless) denotes the discrete time increment (1 h); $\boldsymbol{\theta}(t)$ is a vector containing the algorithm's parameters (unitless); $\mathbf{P}(t)$ is a vector containing the parameter estimation error variance (unitless); $\mathbf{W}(t)$ is a weight matrix (mm^2 h^{-2}); $\gamma(t) = 1/t$ (unitless), $\mathbf{L}(t)$ (mm^{-1} h^{-1}), and $\mathbf{S}(t)$ (mm^2 h^{-2}) are intermediate matrices; $\lambda(t)$ is

$$\lambda(t) = \gamma(t-1)\ [1 - \gamma(t)]/\gamma(t); \tag{B7}$$

and $\psi(t)$ (mm h^{-1}) is a vector containing the derivatives of the algorithm with respect to its parameter vector θ, at time step t (h),

$$\psi(t) = \frac{\partial}{\partial\theta}[f(H,\ t,\ \theta)], \tag{B8}$$

where f represents the algorithm's equations and H are the radar and rain gauge data input to the algorithm during the $[t-1,\ t]$ time interval.

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