Errors Incurred in Sampling a Cyclostationary Field

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(Manuscript received 25 August 1998, in final form 19 July 1999)

ABSTRACT

Low earth-orbiting satellites such as the Tropical Rainfall Measuring Mission (TRMM) estimate month-long averages of precipitation (or other fields). A difficulty is that such a satellite sensor returns to the same spot on the planet at discrete intervals, about 11 or 12 h apart. This discrete sampling leads to a sampling error that is the one of the largest components of the error budget. Previous studies have examined this type of error for stationary random fields, but this paper examines the possibility that the field has a diurnally varying standard deviation, a property likely to occur in precipitation fields. This is a special case of the more general cyclostationary field.

In this paper the authors investigate the mean square error (mse) for the monthly averaging case derived from the satellites whose revisiting intervals are 12 h (sun synchronous) and off 12 h (11.75 h). In addition, the authors take the diurnal cycle of the standard deviation to be a constant plus a single sinusoid, either diurnal or semidiurnal.

The authors have derived an mse formula consisting of three parts: the errors from the stationary background, the cyclostationary part, and a cross-term between them. The separate parts of the mse allow the authors to assess the contribution of the cyclostationary error to the total mse.

The results indicate that the cyclostationary errors due to the diurnal variation appear small for both a 12-h and an off-12-h (11.75 h) revisiting satellite. In addition, the cyclostationary error amounts are similar to each other. For the semidiurnally varying field, the cyclostationary errors increase rapidly as the magnitude of the variance cycle increases for both the 12-h and off-12-h revisting satellites. However, the off-12-h sampling shows the cyclostationary error to be less than that of the exact 12-h sampling.

Furthermore, the authors have evaluated the cyclostationary error as a function of the phase of the satellite visit as it is shifted from the phase of the diurnal cycles (the sun-synchronous case or the start of the month for the off-12-h case). It is found that the cyclostationary error observed from the off-12-h satellite is much less sensitive to the phase shift than the cyclostationary error from the exact 12-h satellite.

1. Introduction

Low earth-orbiting satellites pass over a given location on the earth periodically, with the period depending on latitude and the orbital parameters. One often wants to estimate a time average (nominally 30 days) of some field quantity such as precipitation. The problem is that the discrete sequence of visits might miss important variations of the field during the averaging interval. This could, in principle, lead to large errors of omission that we refer to as the ‘‘sampling error.’’ For planning exercises, the meteorological field might be approximated by a stationary random field with a given space–time spectral density. The random errors committed by such a periodic sampling design have been studied by North and Nakamoto (1989, hereafter NN). A formula for the mean square random sampling error [mean square error (mse)] is

$$\text{mse} = \iiint |H(f, \boldsymbol{\nu})|^2 S(f, \boldsymbol{\nu})\, d\boldsymbol{\nu}\, df, \qquad (1)$$

where f is frequency, $\boldsymbol{\nu}$ is wavenumber, S is spectral density, H is a complex function that depends only on the sampling design, and the integrals run from $-\infty$ to $+\infty$. This formula is very useful because the integrated factors the dependence of the design from the intrinsic second-moment statistics of the random field.

Complicating matters is the diurnal cycle of most fields of interest. If the diurnal cycle consists of a deterministic fundamental, with all of its harmonics embedded in a space–time stationary random field, then the NN formalism applies and the sampling errors can be assessed. The deterministic components enter into the spectral density merely as delta functions. On the other hand, many naturally occurring fluctuations exhibit a periodic ensemble mean, and the second-moment statistics might be periodic as well. For example, the

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standard deviation of the fluctuations might have a diurnal cycle. The general case is a cyclostationary process (see, e.g., Kim and North 1996; Kim and North 1997) for the zero-mean random field $T(t)$ with period P defined by

$$\langle T(t+P)T(t'+P)\rangle = \langle T(t)T(t')\rangle \equiv K(t, t'). \quad (2)$$

The most general case has the space–time-lagged second moments (autocorrelations) dependent on phase, but in this paper we consider only the most simple case in which only the standard deviation depends on the phase of the diurnal cycle.

We have in mind the assessment of errors in experiments such as the Tropical Rainfall Measuring Mission (TRMM) (Simpson et al. 1988) in which a low-altitude, low-inclination satellite passes over $5° \times 5°$ averaging boxes about 60 times each month. The tropical precipitation field is known to have a diurnal cycle (Augustine 1984; Randall et al. 1991) that peaks usually in the midmorning hours. The diurnal cycle of variance is unknown at this time. Absent data, we postulate that the standard deviation is of the form

$$\sigma(t) = 1 + s\cos(2\pi k f_0 t), \qquad k = 1, 2, \ldots, \quad (3)$$

such that we can treat Fourier series harmonics separately ($k = 1, 2, \ldots$). Note that the amplitude of the standard deviation cycle s is expressed as a ratio of the mean. We intend to study the s dependence of the mse ϵ^2 compared to the stationary ($s = 0$) case, ϵ_o^2. The object of the study is to get an understanding of the size of the additional error due to the cyclostationary standard deviation. In what follows, we omit the space dependence of the random field so that the field being sampled is a simple one-dimensional time series.

2. Separation of sampling error

Consider a cyclostationary random field $\psi(t)$, which can be described by (Parzen and Pagano 1979)

$$\psi(t) = P(t)\sigma(t), \quad (4)$$

where $P(t)$ is a background oscillating function that can be taken to be a stationary time series and where $\sigma(t)$ is a periodic standard deviation function representing the variation of the field as defined in (3).

Let Ψ be the true average in a time interval T:

$$\Psi = \frac{1}{T}\int_{-T/2}^{T/2} \psi(t)\,dt. \quad (5)$$

Suppose we seek to find the estimate derived from a periodically orbiting satellite whose sampling property is characterized by its discreteness in time. Employing a delta function, the estimate may be designated as

$$\hat{\Psi} = \frac{1}{N}\int_{-T/2}^{T/2} \sum_{n=1}^{N} \delta\left(t - \left(n - \frac{1}{2}\right)\Delta t\right)\psi(t)\,dt, \quad (6)$$

where, respectively, T, Δt, and N denote an averaging period, a sampling interval, and the total number of snapshots taken by the overpassing satellite in the averaging period.

Using a kernel function for this sampling design

$$K(t) = \Delta t \sum_{n=1}^{N} \delta\left(t - \left(n - \frac{1}{2}\right)\Delta t\right), \quad (7)$$

we can obtain the expression for the estimate

$$\hat{\Psi} = \frac{1}{T}\int K(t)\psi(t)\,dt. \quad (8)$$

Now we can form the mse

$$\epsilon^2 = \langle(\Psi - \hat{\Psi})^2\rangle, \quad (9)$$

where the angle bracket $\langle\ \rangle$ means an ensemble averaging operator. Then the mse will be

$$\epsilon^2 = \frac{1}{T^2}\int\int \langle\psi(t)\psi(t')\rangle[1 - K(t)][1 - K(t')]\,dt\,dt'. \quad (10)$$

Most of what follows can be expressed using the classical Fourier transform of a function $g(t)$ defined by

$$\tilde{g}(f) = \int_{-\infty}^{+\infty} g(t)e^{2\pi i f t}\,dt \quad (11)$$

and the inverse Fourier transform

$$g(t) = \int_{-\infty}^{+\infty} \tilde{g}(f)e^{-2\pi i f t}\,df. \quad (12)$$

The integrand in (10), which is the time-lagged covariance, can be represented by

$$\langle\psi(t)\psi(t')\rangle = \sigma(t)\langle P(t)P(t')\rangle\sigma(t') = \sigma(t)\sigma(t')\sigma_p^2 \int_{-\infty}^{+\infty} S(f)e^{-2\pi i f\tau}\,df, \quad (13)$$

where $\tau = t - t'$, and σ_p^2 and $S(f)$ are the variance and the frequency spectral density of $P(t)$, respectively. The spectral density can be represented by the Fourier transform of the autocorrelation function

$$S(f) = \int \frac{\langle P(t)P(t')\rangle}{\sigma_p^2} e^{2\pi i f\tau}\,d\tau. \quad (14)$$

The spectral density has been obtained under the assumption that $P(t)$ is stationary.

Substituting (13) into (10) gives

$$\epsilon^2 = \frac{\sigma_p^2}{T^2} \int_{-\infty}^{+\infty} S(f) \iint e^{-2\pi i f (t-t')} \sigma(t)[1 - K(t)]\sigma(t')[1 - K(t')] \, dt \, dt' \, df. \tag{15}$$

In terms of the Fourier representation, (15) can be rewritten by

$$\epsilon^2 = \sigma_p^2 \int S(f)|H_o(f) + H_c(f)|^2 \, df, \tag{16}$$

where $H_o(t) = (1/T)[1 - K(t)]$ and $H_c(t) = (s/T)\cos(2\pi f_o t)[1 - K(t)]$ for $k = 1$. The functions of frequency $H_o(f)$ and $H_c(f)$ can be referred to as the stationary and cyclostationary filter functions, respectively, acting on the spectral density. The filters may be designed in the same way for any sampling designs.

We may therefore separate the total error into the individual errors contributed from the different processes

$$\begin{aligned}\epsilon^2 &= \sigma_p^2 \int S(f)|H_o(f)|^2 \, df + \sigma_p^2 \int S(f)|H_c(f)|^2 \, df \\ &\quad + 2\sigma_p^2 \int S(f)\mathrm{Re}(H_o(f)H_c^*(f)) \, df \\ &\equiv \epsilon_o^2 + \epsilon_c^2 + \epsilon_{\mathrm{cross}}.\end{aligned} \tag{17}$$

The first term (ϵ_o^2) denotes the the random sampling error from the stationary background time series of the random field, excluding diurnal variation. The modification of the mse due to the diurnal cycle can be anticipated by the last two terms. Here we can note that the second term (ϵ_c^2) always increases the sampling error. The last term ($\epsilon_{\mathrm{cross}}$), which results from the interaction between the stationary background and the diurnal cycle, however, may act as a bidirectional contributor, because it is not necessarily positive. Formula (17) indicates that over all frequencies each contribution to the estimation error can be computed separately with the same spectral density of a stationary background. In the following sections, these contributions will be discussed, in terms of the design filters, for a periodically visiting satellite over a diurnally varying field. For a comparison, the specific case of the semidiurnal cycle ($k = 2$) will be discussed as well. It is assumed in this section that the sampling begins from the peak of our standard deviation cycles: that is, the sampling is in phase with the cycles. The sampling cases that are out of phase will be presented in section 3.

a. Contribution of stationary background

The contribution of the stationary design filter $|H_o(f)|^2$ to ϵ_o^2 has been discussed previously in NN. This section is a brief review of NN. The filter for a satellite

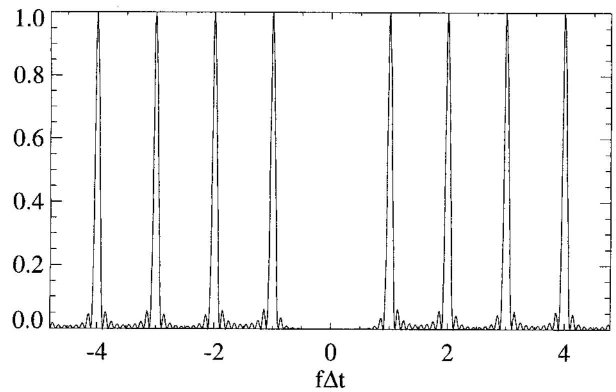

FIG. 1. The stationary design filter $|H_o(f)|^2$ given by (18) for $N = 10$. The front tooth of the Dirac comb is eliminated by the high-pass filter $(1 - G)^2$. A narrower width of the peaks will be seen as N increases. However, the oscillations in between do not depend on N but on the sampling interval Δt.

collecting samples with the revisit frequency $1/\Delta t$ is given by

$$|H_o(f)|^2 = \frac{G(\pi f T)^2}{G(\pi f \Delta t)^2}[1 - G(\pi f \Delta t)]^2, \tag{18}$$

where $G(\pi x)^2 \equiv [\sin(\pi x)/\pi x]^2$, which is called the "Bartlett filter" (Blackman and Tukey 1959), and $T = N\Delta t$. The first factor in $|H_o(f)|^2$ can be interpreted readily in terms of the so-called Dirac comb:

$$\lim_{T\to\infty} \frac{G(\pi f T)^2}{G(\pi f \Delta t)^2} \approx \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{\Delta t}\right). \tag{19}$$

This form is thus seen to be a series of spikes at multiples of twice the Nyquist frequency, equivalently, at the repetition period of the satellite overpasses. However, the front tooth of the comb is eliminated (at the origin), since the second factor serves as a high-pass filter, preserving the original amplitudes at $f = n/\Delta t$ but eliminating the amplitude at the origin. We can note also that the larger N is, the narrower the peaks are, but the oscillation of the wave train does not depend on N (Fig. 1). From the abovenoted considerations, the error attributable to the stationary background is accumulated as a sum over multiples of the satellite visiting frequency. In other words, the error is dependent on the sampling frequency of a measuring design for a given background spectral density. For a red noise spectral density, therefore, the longer the satellite sampling interval, the larger the sampling error. In NN, the mse equation, based upon the stationarity assumption of the rain field, explained that the sampling error increases as the sampling interval is made larger than the time-scale of a field.

b. Cyclostationary effect

The additional contribution from the diurnal cycle is determined by the cyclostationary design filter $|H_c(f)|^2$ whose shape varies as a function of the internal cycle frequency of the random field. Based on a single harmonic contribution, the filter function can be represented by

$$H_c(f) = \frac{s}{T}\int e^{2\pi ift}\cos(2\pi kf_0t)[1 - K(t)]\,dt. \quad (20)$$

Using the trigonometric identity

$$e^{2\pi ift}\cos(2\pi kf_0t) = \frac{1}{2}\{\exp[2\pi i(f + kf_0)t] + \exp[2\pi i(f - kf_0)t]\} \quad (21)$$

and making rearrangements yields

$$\begin{aligned}|H_c(f)|^2 = \frac{s^2}{4}\Bigg[&G(\pi(f + kf_0)T)^2 + G(\pi(f - kf_0)T)^2 + 2G(\pi(f + kf_0)T)G(\pi(f - kf_0)T) - 2\frac{G(\pi(f + kf_0)T)^2}{G(\pi(f + kf_0)\Delta t)}\\ &- 2\frac{G(\pi(f - kf_0)T)G(\pi(f + kf_0)T)}{G(\pi(f - kf_0)\Delta t)} - 2\frac{G(\pi(f - kf_0)T)^2}{G(\pi(f - kf_0)\Delta t)} - 2\frac{G(\pi(f + kf_0)T)G(\pi(f - kf_0)T)}{G(\pi(f + kf_0)\Delta t)}\\ &+ \frac{G(\pi(f + kf_0)T)^2}{G(\pi(f + kf_0)\Delta t)^2} + \frac{G(\pi(f - kf_0)T)^2}{G(\pi(f - kf_0)\Delta t)^2} + 2\frac{G(\pi(f + kf_0T)G(\pi(f - kf_0)T)}{G(\pi(f + kf_0)\Delta t)G(\pi(f - kf_0)\Delta t)}\Bigg]. \quad (22)\end{aligned}$$

As an example, each term in (22) will be examined in the limit of large N and with a fixed sampling interval of 12 h. For a field with a diurnal cycle ($k = 1$, exactly half of the sampling frequency), the first two terms are simply the Bartlett filter functions centered at the diurnal frequency. The third term is negligible since

$$\lim_{T\to\infty}\frac{\sin(\pi(f + f_0)T)}{\pi(f + f_0)}\frac{\sin(\pi(f - f_0)T)}{\pi(f - f_0)} = \delta(f + f_0)\delta(f - f_0). \quad (23)$$

For the fourth through seventh terms, we can approximate that

$$\lim_{N\to\infty}\frac{\sin^2(N\pi(f + f_0)\Delta t)}{N(\pi(f + f_0)\Delta t)\sin(\pi(f + f_0)\Delta t)} \approx \frac{1}{\Delta t}\delta(f + f_0). \quad (24)$$

Thus, the fourth and fifth terms approach the Dirac delta function at the diurnal frequency, and the other peaks are cut off drastically. The fourth and fifth terms cancel each other out. A similar cancellation occurs with the sixth and seventh terms at the diurnal frequency. The last three terms become Dirac combs, as shown previously in (19). The sum of the eighth and ninth terms are compensated by the last term. Finally, we can arrive at the formula for the cyclostationary (diurnal) design filter in the case of 12-h sampling:

$$|H_c(f)|^2 = \frac{s^2}{4}[G(\pi(f + f_0)T)^2 + G(\pi(f - f_0)T)^2]. \quad (25)$$

The shape of the diurnal design filter $|H_c(f)|^2$ for $s = 1$ is illustrated in Fig. 2a. It is simply the sum of the two Bartlett filters centered at the diurnal frequency ($\pm f_0$) and multiplied by the coefficient $s^2/4$. For sufficiently large N, the filter has two meaningful pulses at the diurnal frequency. The error from the diurnal cycle is determined by the spectral density of the background only at the diurnal frequency. Thus, if a significant diurnal cycle is involved, the diurnal characteristic of the field might not be a negligible contributor to the total error for a given red background spectral density, even for a sun-synchronous satellite.

For the semidiurnally varying field ($k = 2$), however, the existence of a semidiurnal cycle no longer allows such term-by-term cancellations, since each term turns out to have the same sign of amplitude. Hence, the terms that canceled each other in the diurnal case are added constructively to enhance their magnitudes, which results in the exaggeration of the semidiurnal, bias for the sun-synchronous satellite. The corresponding filter for the field having a semidiurnal cycle is depicted in Fig. 2b. We find that peaks exist at the multiples of the semidiurnal frequency, including zero frequency. From this property, we may expect that 12-h sampling of the field quantity undergoing semidiurnal variation can cause a larger cyclostationary bias than the 12-h sampling from the diurnally varying field.

c. Cross-term contribution

The cross-term contribution to the total error, representing the cross interaction between the stationarity of

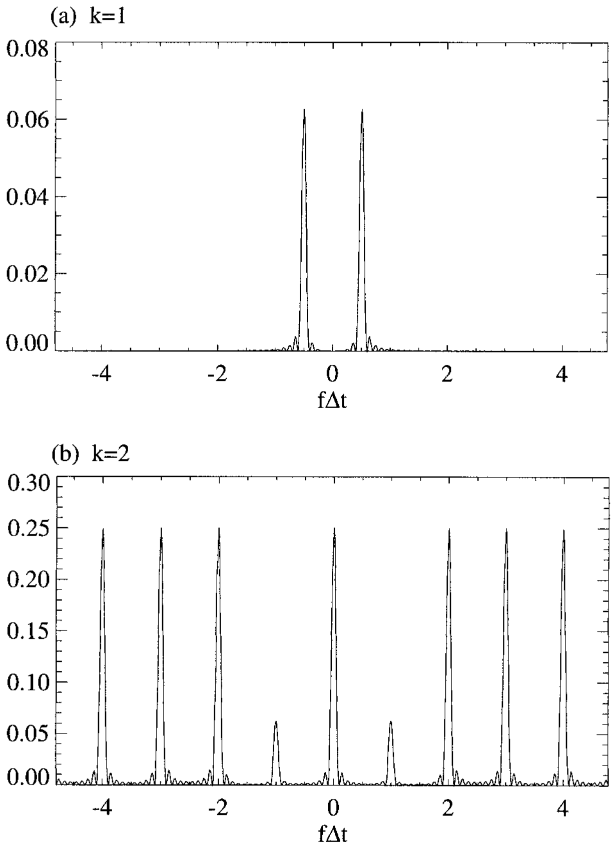

FIG. 2. The cyclostationary design filter $|H_c(f)|^2$ represented by (22) for the 12-h sampling, and $s = 1$. (a) The filter for the diurnal cycle has only two pulses at the diurnal frequency, resulting from the sum of the two Bartlett filters. (b) The semidiurnal filter consists of the peaks at the multiples of the semidiurnal frequency.

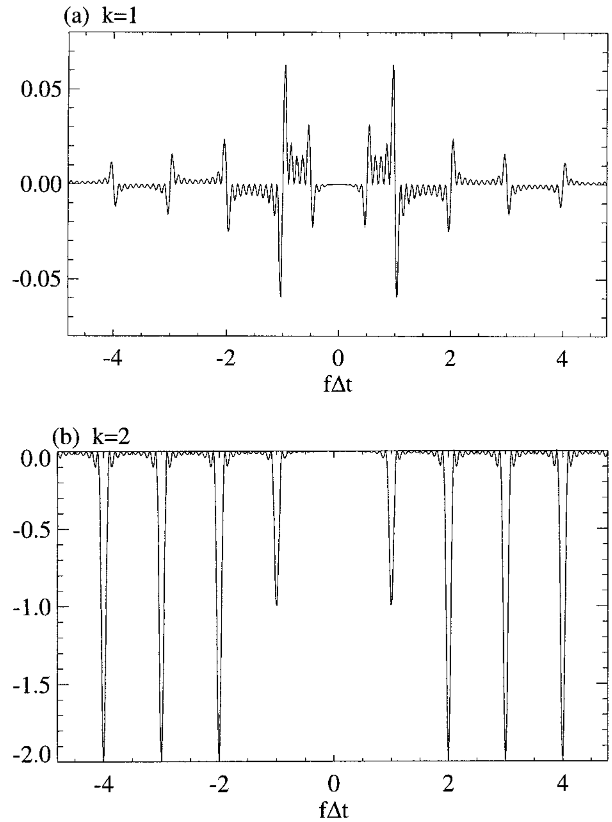

FIG. 3. Plot of the cross filters for the (a) diurnal and (b) semidiurnal cases when $N = 10$.

the background and the diurnal variability of the variance in the field, can be described by the term

$$2\mathrm{Re}(H_o^*(f)H_c(f)) = s\left[G(\pi fT) - \frac{G(\pi fT)}{G\pi f\Delta t}\right] \times \left[G(\pi(f + kf_0)T) + G(\pi(f - kf_0)T) - \frac{G(\pi(f + kf_0)T)}{G\pi(f + kf_0)\Delta t} - \frac{G(\pi(f - kf_0)T)}{G\pi(f - kf_0)\Delta t}\right]. \quad (26)$$

The cross filters for the diurnally and semidiurnally varying cases are plotted in Fig. 3 for a 12-h revisiting satellite. In the upper panel, representing the diurnal case, the small peaks, enough to be negligible in the monthly averaging ($N = 60$), are found at the diurnal frequency and its multiples. Meanwhile, the cross filter for the semidiurnal cycle in the lower panel has peaks with negative amplitudes at multiples of the semidiurnal frequency, except for the zero frequency. This implies that the increased error due to the semidiurnal cycle tends to be reduced to some extent because of the negative contribution of the cross term.

3. Examples with a background red noise

As shown in section 2, the diurnal effect for a satellite taking snapshots at 12-h intervals (e.g., a sun-synchronous satellite) is essentially restricted to the first diurnal frequency, and the semidiurnal effect seems to be concentrated at the semidiurnal frequency, as well as its multiples. From this characteristic of the filter functions, the bias due to a semidiurnal cycle exceeds the diurnal bias. We will examine the cyclostationary bias in the estimation of monthly averages due to the different aspects of diurnal and semidiurnal effects in combination with a postulated background for a sun-synchronous satellite and another satellite whose sampling interval is slightly less than that of a sun-synchronous satellite: for example, the TRMM satellite ($\Delta t = 11.75$ h). The spectral density used in this example is generated by an autoregressive process of order 1. The corresponding timescale of the field is 12 h. The case without phase shift between samplings and diurnal cycles is reviewed first, and the discussion will be extended to the case of various phase shifts.

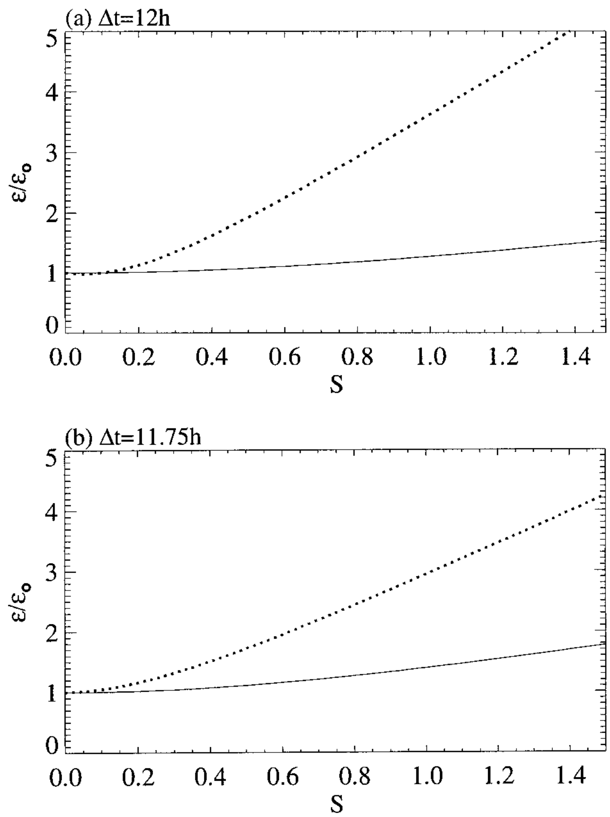

FIG. 4. The ratio (ϵ/ϵ_o) of the total error to the error of the stationary process as the function of s in monthly averaging estimation from (a) the satellite whose revisiting interval is exactly 12 h (e.g., a sun-synchronous satellite) and from (b) an off-12-h (11.75 h) revisiting satellite (e.g., the TRMM satellite), when the beginning of the sampling is in phase with the diurnal harmonics. That is, the satellite visits begin from the maximum variance of the field. The solid line denotes the ratio calculated when the field has a diurnal cycle, and the dotted line indicates the semidiurnal case.

For the case of sampling in phase with the variation cycle, the ratio (ϵ/ϵ_o) of the total estimation error for monthly averages to the error from the stationary process (representing the contribution of the cyclostationary bias due to the diurnal or semidiurnal cycle to the estimation error) is presented as a function of s in Fig. 4a for a sun-synchronous satellite, and in Fig. 4b for the TRMM (off-12-h revisiting) satellite. Figure 4a shows that the ratio remains fairly close to unity up to about $s = 0.3$, when the sun-synchronous satellite collects the data over the field having diurnal cycle, and about $s = 0.13$ over the semidiurnally dependent variance in the field. This indicates that the cyclostationary (diurnal or semidiurnal) effect turns out to be unimportant for both cases when the amplitudes of a diurnal or a semidiurnal cycle are about 30% or 13% of the mean, respectively. Above these values of s, the ratio for the diurnal case gradually rises, and for the semidiurnal case it increases more rapidly. Therefore, much larger cyclostationary effect seems to be involved in the sampling error when the satellite passes over a semidiurnally varying field in the revisiting interval of 12 h.

The cyclostationary effect for the TRMM satellite passing over tropical regions with a revisit interval of 11.75 h (11 h 45 min) can be seen in Fig. 4b. The TRMM satellite may have a slightly larger sampling error, due to its diurnal variation cycle, than the sun-scynchronous satellite has. Its interpretation is that the term-by-term cancellation in the cyclostationary (diurnal) design filter, discussed in section 2b, is not performed as much as the case of exact 12-h revisits, since the synchrony of the spikes needed for cancellation is offset slightly. This enhances the contribution associated with the cyclostationarity. However, the bias incurred from the semidiurnal cycle has become smaller when sampled by the TRMM satellite. The reason for the larger bias (semidiurnal) in a sun-synchronous satellite is the coincidence of the sampling frequency and the semidiurnal frequency of the field, which tends to increase the power of the cyclostationary filter at the semidiurnal frequency, resulting in the enhancement of the bias.

At this point, we would like to see how much the cyclostationary bias is altered when the beginning of the sampling is out of phase with the diurnal cycles. The sampling problem may be resolved by introducing the phase shift θ into the cyclostationary standard deviation formula (3) as follows:

$$\sigma(t) = 1 + s\cos(2\pi k f_0 t + \theta), \qquad k = 1, 2, \ldots, \tag{27}$$

and then the mse formula may be reestablished readily.

Figures 5 and 6 show the ratio (ϵ/ϵ_o) as the function of s and θ for the sampling cases from a sun-synchronous satellite and from the TRMM satellite, respectively. In both figures, the upper panel shows the ratio obtained with the diurnal cycle, and the lower panel shows the ratio obtained with the semidiurnal cycle. In Fig. 5, when a sun-synchronous satellite collects samples from the random field, the cyclostationary bias is likely to depend on phase shift for both the diurnal and semidiurnal cycles. The cyclostationary bias from the diurnal cycle appears to be maximized by 6- or 18-h out-of-phase sampling, and by 0- or 12-h shifted-phase sampling, with the semidiurnal cycle producing the largest cyclostationary bias. The phase dependence of the cyclostationary bias for a sun-synchronous satellite can be viewed by the schematic illustration in Fig. 7. In the upper panel, the samplings taken every 12 h, which are shifted by 0 or 12 h from the phase of the diurnal cycle (empty arrows), are associated with the ridges and troughs of the variation cycle. It seems that the interaction between the cyclostationary filter and the minimum amplitudes at the troughs in the cycle lets the cyclostationary bias be small during half of the visits. Shifting the sampling time, however, by 6 or 18 h from

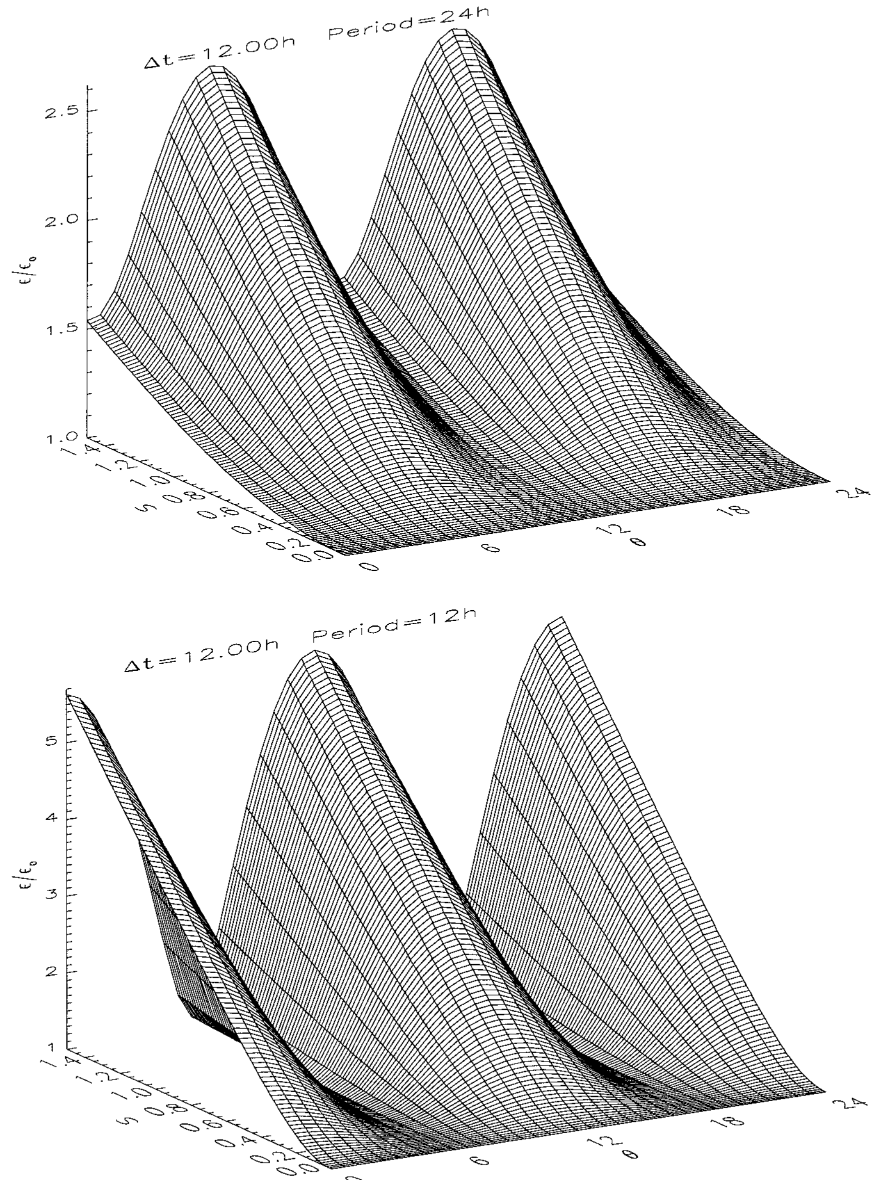

FIG. 5. The ratio (ϵ/ϵ_o) of the total error to the error of the stationary process as the function of s and θ in monthly averaging estimation from a 12-h revisiting satellite. The θ indicates phase shift between the beginning of the sampling and the phase of the diurnal variation cycles. The phase shift is in units of hours. The upper panel is for the diurnal case, and the lower panel for the semidiurnal case.

the peaks of the cycle selects the middle amplitude, and it seems to enlarge the cyclostationary bias on every visit as the amplitude of the cycle increases. Note from the lower panel of Fig. 7 that if the 12-h sampling over the semidiurnally varying field begins 6 or 18 h out of phase, the amplitude of the cycle will be the smallest one, so that it leads to the smallest cyclostationary bias. However, when the sampling starts in phase or 12 h out of phase, the cyclostationary bias approaches the largest one, since the maximum amplitudes of the semidiurnal cycle are involved.

On the other hand, the cyclostationary bias that occurred in off-12-h sampling from the TRMM satellite seems to be less sensitive to the phase shift for both the diurnal and semidiurnal cycles, since all of the amplitudes in the full cycle are considered when we estimate a sufficiently long-term average, such as a monthly estimate (Fig. 6). Note that sampling and averaging over a large number of months would eliminate the phase dependence, since the phase slips in each visit. The error values given here are for an ensemble average of months all having the same phase.

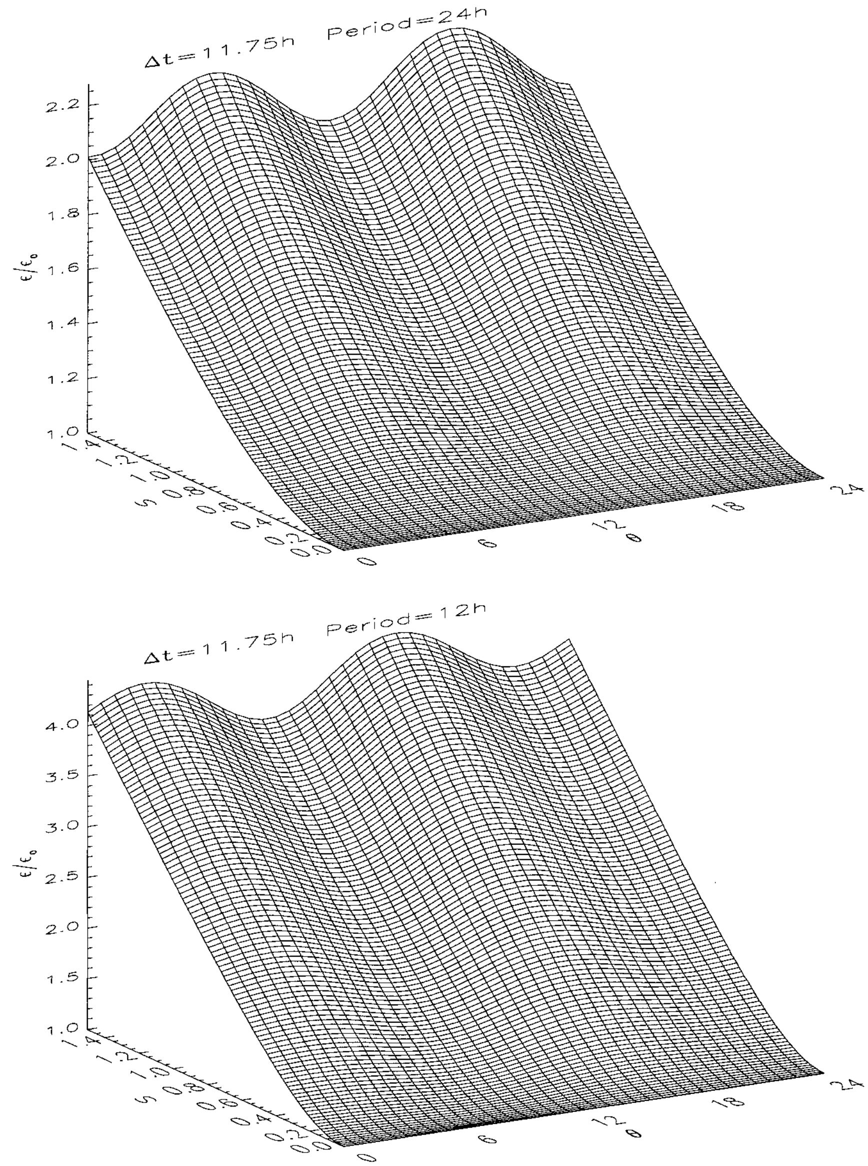

FIG. 6. As in Fig. 5 but for off-12-h sampling from the TRMM satellite.

4. Conclusions

We have investigated the additional sampling error due to a superimposed cyclostationary standard deviation over a red noise stationary background in estimating a time average of a field using periodic sampling. The mse formula proposed in NN has been extended to cover a class of such cyclostationary fields. The extended formula for a cyclostationary random field allows the total error to be decomposed into individual terms that explain the contributions from the stationary background and a single harmonic of the diurnal cycle of the variance separately.

We have found that the cyclostationary variation of the field tends to increase the error depending on the strength of the diurnal or semidiurnal harmonic in the variance for both of 12-h and off-12-h sampling cases. From the case of a semidiurnal varying field, it has been shown that off-12-h sampling appears to have a smaller additional estimation error than that from the 12-h sampling. It may imply that the estimation of a time average the sampling intervals slightly different from the inherent period of a variance cycle may reduce the cyclostationary bias.

Furthermore, we have shown the cyclostationary bias as a function of phase shift between the diurnal overpass phase of the satellite at the beginning of the month with respect to the local hour. A strong dependence of the cyclostationary bias was found for a sun-synchronous satellite, since the extreme amplitudes of the variation cycle can interact strongly with the cyclostationary filter,

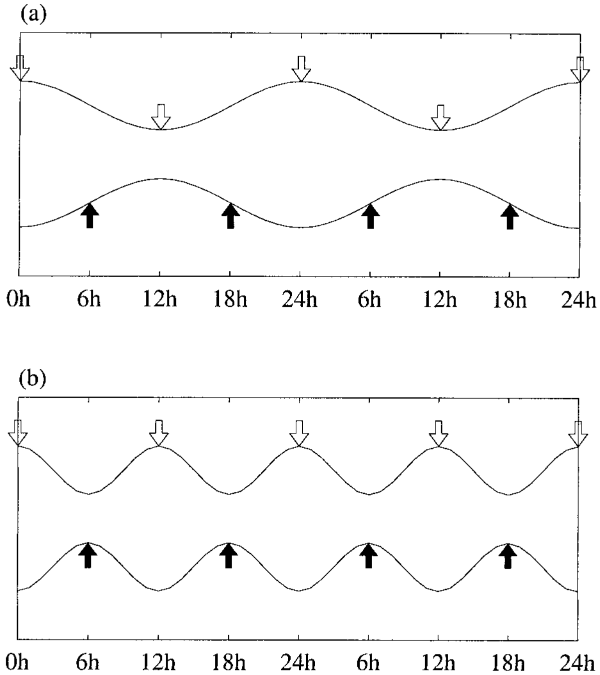

FIG. 7. Schematic illustration of 12-h sampling of the field having (a) diurnal or (b) semidiurnal variation. The samplings in 0 or 12 h out of phase are represented by the empty arrows, and the black arrows indicate the samplings beginning with 6- or 18-h phase difference from the variation cycles.

depending upon relative phase. However, off-12-h sampling from a TRMM-like satellite shows a weaker dependence on the phase shift. The dependence is weaker because the satellite revisit times drift through the local hour of day over the course of a few weeks.

The mse formula used here covers the case of multiple harmonics of the diurnal cycle, so that the sampling error over a random field having more complicated variations can be resolved as long as the variations are known.

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