Optimal Detector Sensitivity, Number, and Location of Alarm Detectors Against Unknown Instantaneous Air Pollutant Sources

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ABSTRACT

The problem of determining the optimal detector sensitivity, number and location of alarm detectors against instantaneous sources was examined in terms of the conventional Gaussian dispersion model for flat terrain. Analytical expressions for the effective detecting area per detector as a function of detector sensitivity were derived for a simple case. Numerical results were provided for general cases. The optimal number and location of alarm detectors were then inferred from the effective detecting area per detector.

1. Introduction

The problem of air monitoring survey design for continuous sources has been investigated by Noll and Miller (1977). For each air sampler, a representative sampling area was defined in terms of a given probability of measuring the maximum ground concentration. A conventional Gaussian dispersion model for continuous sources was the basis for their analysis.

The present note deals with a related but different problem, namely, the determination of optimal detector sensitivity, number and location of real time pollutant alarm detectors against unknown instantaneous sources such as a sudden release of a hazardous gas at an unknown point. The simple Gaussian dispersion model for instantaneous sources over flat terrain was used as the analytical tool since the purpose here was not model building. The relationship between effective detecting area (EDA) per detector and detector sensitivity was examined and plotted for various cases. The optimal number of detectors was considered to be the ratio of potential monitoring area over EDA. The problem of optimal placement of detectors was discussed in terms of the effective downwind detecting distance.

2. Analysis

The geometry, number, and exact location of instantaneous pollutant sources were assumed to be unknown; neither was a prevailing wind direction assumed. As a conservative approach, a very weak point source was used for all computations. The rationale was that a line source, an area source, or multiple sources would generally result in higher concentrations than a single weak source and therefore were well within the detecting capabilities designed for a single weak point source.

The simple Gaussian dispersion model for an instantaneous point source is

$$\chi = \frac{Q}{(2\Pi)^{3/2}\sigma_x\sigma_y\sigma_z}\exp\left(-\frac{(x-ut)^2}{2\sigma_x{}^2}\right)\exp\left(-\frac{y^2}{2\sigma_y{}^2}\right) \times\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z{}^2}\right)+\exp\left(-\frac{(z+H)^2}{2\sigma_z{}^2}\right)\right], \quad (1)$$

where the source is located at (0, 0, H) and

- χ pollutant concentration at time t and location (x, y, z)
- Q total amount of material released
- σ_x, σ_y, σ_z dispersion coefficients
- u average wind speed.

The dispersion coefficients for an instantaneous source can be expressed as a power function of downwind distance (Slade, 1968):

$$\sigma_x = \sigma_z = ax^b \quad (2)$$

$$\sigma_z = cx^d, \quad (3)$$

where the constants a, b, c and d are functions of meteorological parameters.

Let us first examine a simple but very likely case, namely, ground source and ground detector ($H = z = 0$). Equation (1) reduces to

$$x = \frac{2Q}{(2\Pi)^{3/2}\sigma_x\sigma_y\sigma_z} \times\exp\left(-\frac{(x-ut)^2}{2\sigma_x{}^2}\right)\exp\left(-\frac{y^2}{2\sigma_y{}^2}\right). \quad (4)$$

The maximum concentration occurs at the time $t = x/u$. By setting $x = ut$ and χ equal to a given detector sensitivity s, Eq. (4) represents a time-dependent contour. Each point on the contour will reach the concentration s at $t = x/u$ as the expanding instantaneous cloud moves along the x-axis. The area within this contour may be interpreted as the EDA associated with a detector because the alarm would trigger if the contour centroid falls within this area. The EDA can be easily computed. With $x = ut$ and $\chi = s$, we solve for y, the minor axis of the elliptical EDA:

$$y = \sqrt{2}ax^b\left[\ln \frac{B}{x^{2b+d}}\right]^{1/2}, \qquad (5)$$

where

$$B = \frac{2Q}{(2\Pi)^{3/2}a^2cs}.$$

Since $y = 0$ at $x = 0$ and $x = B^{1/2b+d}$,

$$\mathrm{EDA}(s) = 2\int_0^{B^{1/2b+d}} y dx$$

$$= 2\sqrt{2}a\int_0^{B^{1/2b+d}} x^b\left[\ln \frac{B}{x^{2b+d}}\right]^{1/2} dx. \qquad (6)$$

This integral can be analytically carried out to give:

$$\mathrm{EDA}(s) = \frac{\sqrt{2\Pi}a\left(\frac{2Q}{(2\Pi)^{3/2}a^2cs}\right)^{b+1/2b+d}}{(b+1)\left(\frac{b+1}{2b+d}\right)^{1/2}}. \qquad (7)$$

By using Slade's values for constants a, b, c and d, we obtain a simple numerical relationship between EDA and s for three atmospheric stability categories (omitting complicated units associated with constants):

$$\mathrm{EDA} = 30.1\ (Q/s)^{0.79} \text{ for stable atmosphere} \qquad (8)$$

$$= 5.7\ (Q/s)^{0.75} \text{ for neutral atmosphere} \qquad (9)$$

$$= 1.4\ (Q/s)^{0.76} \text{ for unstable atmosphere,} \qquad (10)$$

where EDA is in m^2, Q in mg and s in mg m^{-3}.

Based on practical experience, we judiciously assign 1 Kg (10^6 mg) to Q as the lower limit of a weak point source. The relationship between EDA and s has been plotted in Fig. 1. For a conservative prediction, the curve for unstable atmosphere should be used because it gives the smallest EDA.

Next, we shall examine the more general cases of elevated sources and elevated detectors. The foregoing analysis still applies except that the integral for EDA is more complex and can not be analytically carried out. We have determined the EDA by numerical summation for elevated sources at 5 m, 25 m and 50 m and elevated detectors at a height of 1 m and 10 m respectively. The results are presented in Fig. 2(a) and (b) for unstable atmosphere. Note that the influence of elevation is not significant when the detector sensitivity is less than 0.1 mg m^{-3} for Q = 1 Kg and the EDA is around 1 km^2. As s approaches 1 mg m^{-3}, the EDA begins to shrink. This indicates that for elevated sources and detectors up to 50 m the detector sensitivity is required to be less than 0.1 mg m^{-3} to ensure protection against unknown sources.

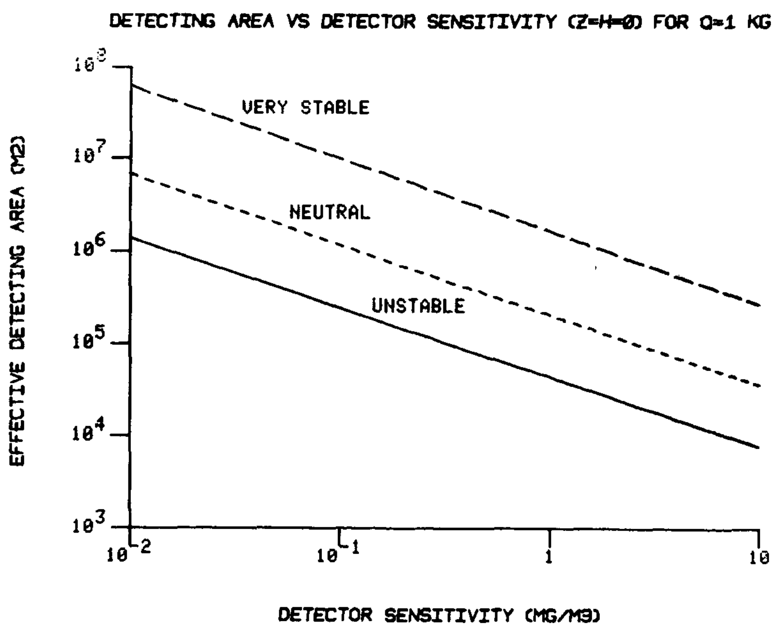

FIG. 1. Effective detecting area versus detector sensitivity for various atmospheric stabilities.

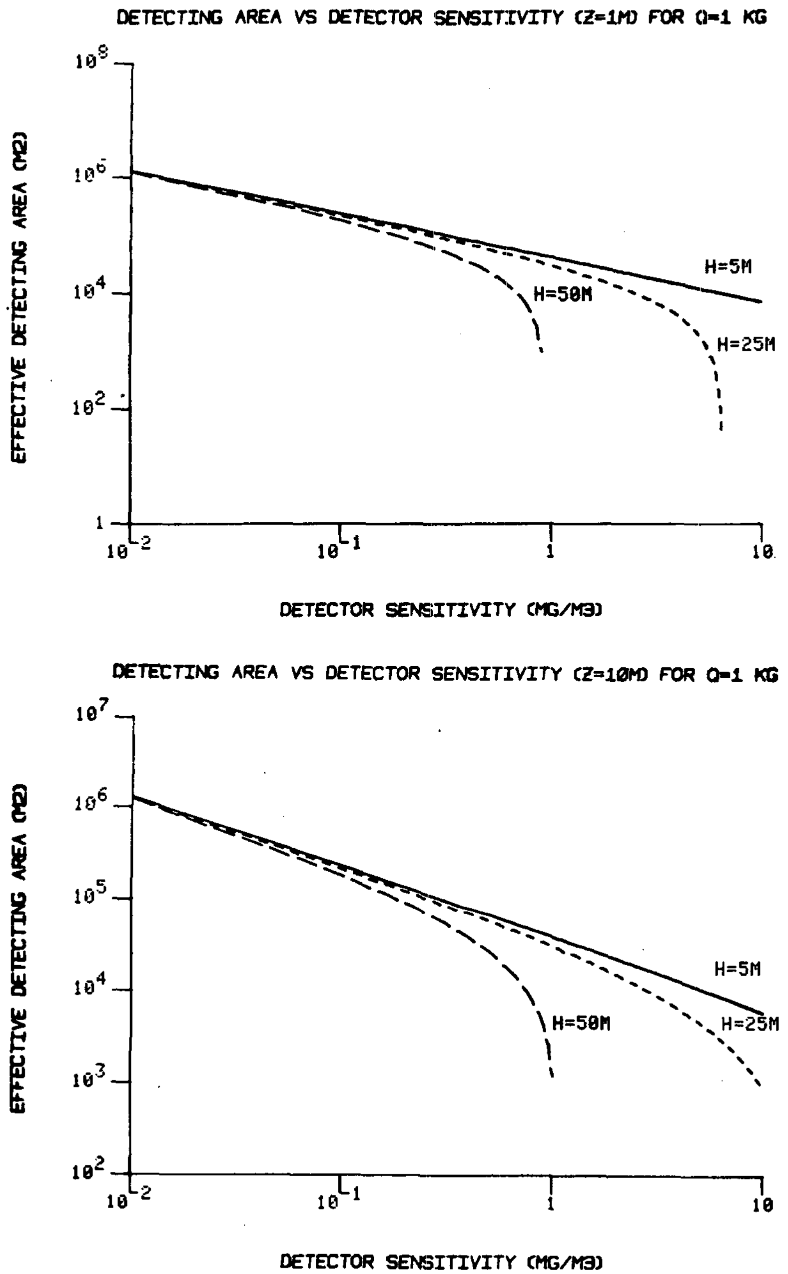

FIG. 2. Effective detecting area versus detector sensitivity for (a) detector at 1 m height (b) detector at 10 m height due to elevated sources.

3. Optimal number and location of detectors

Given a large potential monitoring zone within which potential unknown instantaneous sources are believed to exist at unknown points, the optimal number of detectors required is simply the ratio of potential monitoring zone/EDA, if the potential monitoring zone is completely filled with EDAs.

Since random wind direction is assumed, these detectors should be symmetrically deployed in the potential monitoring zone in rectangular arrays with equal spacing between detectors. The problem of detector location warrants more attention if protection is to be given to a selected area of importance, but devoid of potential sources, such as a population center. It is then more convenient to construct two concentric circles of radii R and r with the smaller circle r enclosing the target area and the annular area between r and R, $\Pi(R^2 - r^2)$, representing the potential monitoring zone. The required detectors

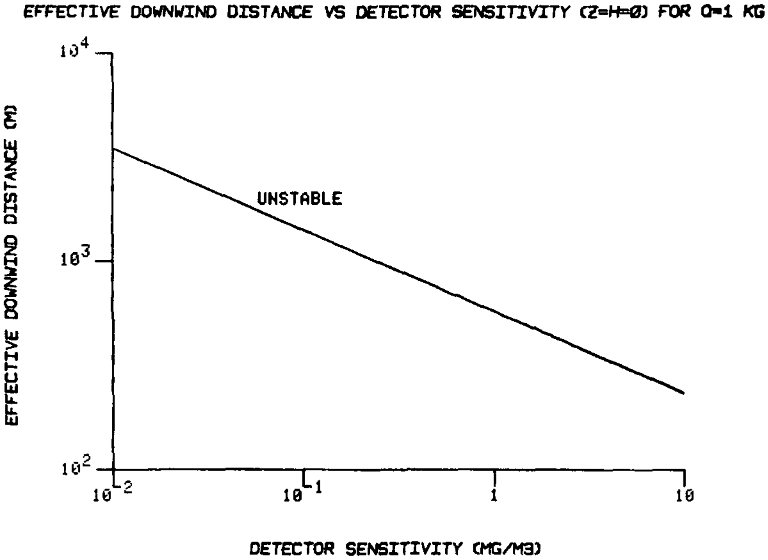

FIG. 3. Effective downwind distance versus detector sensitivity for unstable atmosphere and ground level source and detector.

should be symmetrically deployed as concentric layers in the annular area. It is also necessary to determine the number of concentric layers required in this case. The length of the elliptical EDA is the effective downwind distance defined to be the x value corresponding to an axial concentration s in the Gaussian dispersion model. The number of concentric layers required can be estimated from $(R - r)$/effective downwind distance. Again, as a conservative approach, we shall use the effective downwind distance for unstable atmosphere with $Q = 1$ Kg and $z = H = 0$. The plot of effective downwind distance versus detector sensitivity is given in Fig. 3.

Regardless of numerical prediction, the minimum number of detectors should be four, deployed just outside the smaller circle in the four major wind direction sectors to avoid the possibility that the cloud of hazardous pollutant may cross the target area before reaching a detector if the computed optimal number of detectors turns out to be less than four.

4. Concluding remarks

A simple and logical strategy has been worked out to determine the optimal number and location of alarm detectors against unknown instantaneous sources in terms of the concept of an effective detecting area per detector.

This analysis has also suggested a detector sensitivity requirement of 0.1 mg m^{-3} for alarm protection against unknown sources. Since this value is based on a very weak point source it should be adequate for stronger sources with this built-in safety factor.

For simplicity, a simple Gaussian model was used to illustrate the procedure for obtaining the relationship between detecting sensitivity and the effective detecting area for dispersion of material over flat terrain. When complex terrain is involved, it is sometimes tempting to solve the advection–diffusion differential equation coupled to a three-dimensional wind field with a proper numerical scheme (K-Theory approach), described, for instance, by Hanna *et al.* (1982). However, exact source locations as well as wind speeds and directions at a number of observation stations are required in such an approach and render it unsuitable for the problem considered here. Note that wind speed and direction were not involved in our computations at all. Source locations were, by definition, unknown. The Gaussian model can be used for complex terrain if the dispersion coefficients (σ's) are properly modified; numerous procedures are available in the literature. For instance, Bass *et al.* (1981) have shown how dispersion coefficients can be modified to account for terrain-induced kinematic constraints for a variety of terrain features.

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