A new simulation technique for spaceborne Doppler radar observations that was developed specifically for inhomogeneous targets is presented. Cloud inhomogeneity affects Doppler observations in two ways. First, line-of-sight velocities within the instantaneous field of view are unequally weighted. As the large forward motion of a spaceborne radar contributes to these line-of-sight velocities this causes biases in observed Doppler speeds. Second, receiver voltages now have time-varying stochastic properties, increasing the inaccuracy of Doppler observations. The new technique predicts larger inaccuracies of observed Doppler speeds than the traditional random signal simulations based on the inverse Fourier transform.

The accuracy of Doppler speed observations by a spaceborne 95-GHz radar [as part of the proposed European Space Agency (ESA)/Japan Aerospace Exploration Agency (JAXA)/National Institute for Information and Communications Technology (NICT) EarthCARE mission] is assessed through simulations for realistic cloud scenes based on observations made by ground-based cloud-profiling radars. Close to lateral cloud boundary biases as large as several meters per second occur. For half of the cloud scenes investigated, the distribution of the in-cloud bias has an rms of 0.5 m s\(^{-1}\), implying that a bias in excess of 0.5 m s\(^{-1}\) will not be uncommon.

An algorithm to correct the bias in observed Doppler observations, based on the observed gradient of reflectivity along track, is suggested and shown to be effective; that is, the aforementioned rms bias reduces to 0.14 m s\(^{-1}\).

1. Introduction

Clouds cause an important, but not fully understood, contribution to the earth’s radiative budget. Better knowledge of their spatial distribution as well as their water content and particle sizes is expected to improve our understanding of climate change (Slingo 1990; Kristjansson et al. 2000; Potter and Cess 2004).

Because of the success of ground-based radars in determining cloud properties, spaceborne equivalents have been proposed and studied (Meneghini and Kozu 1990; Amayenc et al. 1993). The National Aeronautics and Space Administration (NASA) Jet Propulsion Laboratory (JPL) and the Canadian Space Agency (CSA) recently launched a 95-GHz cloud-profiling radar on CloudSat (Stephens et al. 2002), while the European Space Agency (ESA), the Japan Aerospace Exploration Agency (JAXA) and the National Institute for Information and Communications Technology (NICT) are preparing for a 95-GHz Doppler radar for the EarthCARE mission (Harris and Battrick 2001).

Doppler radars not only observe target backscatter power but also target velocities. This yields information on convective flows in clouds but may also improve the retrieval of cloud particle sizes. Because of the stochastic nature of these observations, both reflectivity and Doppler speed have an intrinsic inaccuracy that is not related to calibration issues or atmospheric attenuation. This inaccuracy represents the maximum accuracy achievable by the radar given specific radar parameters and cloud (target) properties.

Amayenc and Testud (2001) performed an initial feasibility study for EarthCARE’s 95-GHz Doppler radar, where they defined scientific requirements and studied the impact of radar parameters like dish size and pulse repetition interval \(\tau\) on Doppler speed accuracy and sensitivity. Kobayashi et al. (2002), Ohsaki (2003), and Kobayashi et al. (2003) searched for optimal pulse repetition times for different pulse patterns. From the
standpoint of radar parameters, the usefulness of a spaceborne 95-GHz Doppler radar was thus proven, and at present a 2.5-m dish \((\theta_0 = 0.09^\circ)\) and a contiguous pulse-pair operation with a variable PRF = 6400 to PRF = 7200 (depending on orbital position) is considered. In their analyses, all authors assumed homogeneous clouds.

Because of the high orbital velocity of a spaceborne platform (7.6 km s\(^{-1}\) for EarthCARE), spaceborne Doppler observations are not as accurate as ground-based observations. First, there will be a larger variation of line-of-sight velocities within the instantaneous field of view (IFOV), increasing the standard deviation of Doppler speed observations (measurements for subsequent pulses have a smaller correlation due to the high platform velocity). Second, when the cloud is inhomogeneous, the line-of-sight contribution by platform velocity may not be balanced across the IFOV. This will cause biases in observed Doppler speed and may also further increase standard deviations. This second effect has previously not been considered in any EarthCARE-related study.

This effect, also called nonuniform beam filling, was, however, considered by Tanelli et al. (2002) in the context of a spaceborne 13.6-GHz precipitation Doppler radar. Tanelli et al. found biases in Doppler speed as large as \(\pm 40\text{ m s}^{-1}\). Moreover, observed Doppler speeds exhibited increased standard deviations. Because of differences in the properties of both the radar and the targets (precipitation is generally less homogeneous than clouds), it is not clear how these results translate to the EarthCARE radar.

In addition, the issue of how to simulate radar signals for inhomogeneous clouds is not fully resolved. Time series of radar receiver voltages may be simulated through individual hydrometeor simulation (Ohsaki 2003) or through the inverse FFT of an assumed Doppler spectrum (Sirmans and Bumgarner 1975; Zrnić 1975). The first technique is versatile and flexible but also time consuming. The latter technique is fairly fast and has been used extensively (e.g., Rajopadyaya et al. 1993; Schafer et al. 2002; Kobayashi et al. 2003; Lucas et al. 2004). One drawback is its implicit assumption of homogeneity, apparent from, for example, the stationary stochastic properties of the fluctuations in a generated time series. Tanelli et al. (2002) argue that, for the radar they consider, this is a valid approximation. Although the argument is sound, no accuracy estimate of this approximation could be provided.

Tanelli et al. (2002) considered a radar with an IFOV of 2260 m and an integrated track of 75–300 m (subsatellite point to subsatellite point). In this case, it makes sense to assume stationarity; for each pulse, the radar essentially observes the same scene. In contrast, EarthCARE’s radar will have an IFOV of 691 m and an integrated track of 1000 m. This radar will see different parts of the same cloud for the duration of a single observation. Clearly the assumption of stationarity is no longer valid.

The goal of this paper is twofold: to develop a Doppler radar simulation procedure capable of generating nonstationary time series of radar voltages and to study the effect of cloud inhomogeneity on Doppler observations, in particular for the Doppler radar proposed for EarthCARE (i.e., \(\theta_0 = 0.09^\circ\) and a pulse repetition interval \(\tau = 150\text{ \mu s}\), representative of the above-mentioned interval of PRFs). In section 2, a new simulation technique, akin to random signal simulation but capable of dealing with inhomogeneity, is developed. In section 3, Doppler speed observations are simulated for a variety of clouds, and the associated errors are analyzed. A corrective algorithm is presented as well. A summary of this paper is given in section 4. Unless specified otherwise, all variables will be in mksa units.

2. Aspects of Doppler radar observations

a. Geometry of observation

Before developing the Doppler signal simulation procedure, the geometry of observations and several useful concepts need to be defined.

Consider a spaceborne platform. A short stretch of its orbit may be considered as a straight line above a flat earth (see Fig. 1). Without loss of generality, the platform is assumed to move in the \(y = 0\) plane, with orbital velocity \(\mathbf{u} = (u_x, 0, u_z)\). The component \(u_z\) \((|u_z| \ll |u_x|)\) results from an orbit not parallel to the earth’s local geoid. At \(t = 0\), the platform has an altitude \(h\) over the surface. On board the platform there is a Doppler radar
with circular antenna whose center beam is parallel to \( \mathbf{n} = (n_x, n_y, n_z) \). At the surface the path of the beam center is a straight line approximately parallel to the path of the subsatellite point, and the shape and size of its footprint are approximately constant. The reader may verify both statements by considering that \( |u_i| \ll |u_i| \).

The cloud scene below the radar is specified by volume distributions of radar backscatter \( \eta(\mathbf{r}) \), systematic speed \( \mathbf{v}(\mathbf{r}) \), and turbulent motions specified through a Maxwell distribution with spread \( \sigma_v(\mathbf{r}) \). The exact definition of \( \mathbf{v} \) includes a backscatter weighted average, but in this paper we assume that all cloud drops within a small volume have the same size and move with the same systematic speed \( \mathbf{v} \). Additional broadening of the particle velocity spectrum due to different particle sizes is included by replacing \( \sigma_v \) with \( \sqrt{\sigma_{v_0}^2 + \sigma_{PSD}^2} \) (Kobayashi et al. 2002). Here \( \sigma_{PSD} \) represents broadening due to the particle size distribution.

In Fig. 1, the vector \( \mathbf{m} \) is the line-of-sight vector from the radar to an atmospheric volume.

\[
\mathbf{w} = (\mathbf{v} - \mathbf{u}) \cdot \frac{\mathbf{m}}{|\mathbf{m}|^2},
\]

where negative \( w \) indicates motion toward the radar.

b. From receiver voltage to reflectivity and Doppler speed

The actual measurements of a radar are time series of receiver voltages \( V_k \) \((k = 1, \ldots, N)\), from which total backscatter power and Doppler speed may be estimated using autocovariance processing for contiguous pulses (Doviak and Zrnić 1993; Bringi and Chandrasekar 2001):

\[
P = \frac{1}{N} \sum_{k=1}^N V_k^2,
\]

\[
u_D = \frac{\lambda}{4\pi} \text{arg} \left[ \frac{1}{N-1} \sum_{k=1}^{N-1} V_k V_{k+1}^* \right],
\]

where \( N \) is the number of measurements, \( \tau \) the time between measurements (i.e., the pulse repetition time), and \( \lambda \) the operating wavelength of the radar. The receiver measurements are complex voltages (*) denotes the complex conjugate) measured for the same range gate.

c. Stochastic properties of receiver voltages

Receiver voltages are stochastic time series due to constant turbulent reshuffling of the scatterers. For each radar pulse, the sums of the backscattered waves add up differently at the receiver. The (complex) receiver voltages will have a Gaussian distribution around zero (\( \langle V \rangle = 0 \), where the brackets denote an ensemble average). Consequently, the voltages’ stochastic properties are completely determined through their covariances. For a radar probing the same volume with subsequent pulses, the covariances are given by (Doviak and Zrnić 1993; Bringi and Chandrasekar 2001)

\[
\langle V(0)V^*(t) \rangle = \frac{g^2 \lambda^2 P_e}{(4\pi)^2} \int f^2(\theta, \phi) |W(\mathbf{r})|^2 \eta(\mathbf{r}) e^{-2\text{Si}^2(\mu(\theta, \phi)/\lambda)} e^{4\pi i \nu \tau(\theta, \phi) / \lambda} dV,
\]

where \( g \) is the radar power gain, \( P_e \) the emitted power, \( f(\theta, \phi) \) the normalized one-way power gain of the radiation pattern, and \( l \) the one-way attenuation loss due to scattering and absorption (\( W \) is the range weighting function, which is only mentioned here for completeness but is not relevant to the present discussion). A spherical coordinate system centered at the radar with the radar central beam at \( \theta = 0^\circ \) is used in Eq. (4). Note that the covariance decreases as \( t \) increases. For \( t = 0 \), Eq. (4) becomes the standard deviation of the instantaneous voltage, and for \( t = \tau \) it describes the correlation between neighboring pulses.

Note that expected power and Doppler speed may be derived from these covariances, hence the formalisms in Eqs. (2) and (3). For \( \tau \lambda \ll 1 \text{ s}^{-1} \) and a nonmoving platform, this yields the familiar expression where Doppler speed is the average of \( \nu(x) \) weighted by \( \eta(x) \) and the two-way gain pattern \( f^2(\theta, \phi) \).

The remainder of this paper this expression (not explicitly given) will serve as a reference truth when assessing the accuracy of Doppler observations.

d. A model for Doppler radar observations over inhomogeneous scenes

Equation (4) was derived for a radar observing the same cloud volume for a time period \( t \). In the case of a ground-based cloud-profiling radar, this is certainly a good approximation. However, it is not obvious that the same equation can be used for spaceborne obser-
vations, when both the backscatter $\eta$ and line-of-sight velocities $w$ change from pulse to pulse.

Before one can resolve this issue, two components of Eq. (4) require further specification: the one-way gain of the radiation pattern $f(\theta, \phi)$ and the line-of-sight velocity $w$. Significant reflections only occur in a layer very close to the surface compared to the altitude ($h = 440$ km) of the satellite. Thus the gain pattern may be considered the same at every atmospheric height. If the observation of a ground track starts at $t = 0$, this gain pattern at spatial coordinate $r = (x, y, z)$ can be expressed as

$$f^2(x, y, t) = \exp \left( -\frac{\ln^2(\frac{x-u_xt-n_h y + x y-n_z h}{h^2 \tan^2\theta_0/2})}{\frac{2}{\ln^2}} \right),$$

(5)

where $\vec{n} = n/|n|$, and $\theta_0$ is the conventional one-way 3-dB beamwidth, and the radar is pointing close to nadir, $|n_x|, |n_y| \ll |n_z|$ (see Fig. 1).

Similarly, the line-of-sight velocity $w$ in Eq. (4) can be expressed through Eq. (1) as

$$w(x, y, z, t) = u_z(x - u_z t - n_z h, y - n_y h, z)$$

$$= \left( n_x + \frac{x - u_x t}{h} \right) u_x - u_z,$$

(6)

where a fairly narrow beam ($\theta_0 < 1^\circ$) was assumed to ensure that for relevant cases $\vec{n} \cdot \vec{u} \approx 1$.

In the case of a spaceborne radar, decorrelation of radar signals occurs for three reasons. First, there is the variation of line-of-sight velocities within an IFOV. With ground-based radars this is mostly due to turbulent atmospheric motions, but for spaceborne radars the systematic variation of line-of-sight velocities over the IFOV [the term $-x u_z/h$ in Eq. (6)] often dominates. Second, there is variation in the contribution of the same atmospheric volume to the receiver voltage as the antenna gain pattern $f^2(x, y, t)$ moves over the atmosphere. Finally, there is the variation in line-of-sight speeds of the same atmospheric volume as it is viewed from different angles by the moving radar [the term $u_z t/h$ in Eq. (6)].

For these three processes, time scales $T$ may be derived that allow us to determine whether any is dominant. The first time scale is related to the spread $\sigma_w$ of line-of-sight velocities within the IFOV. The smaller this spread, the better two consecutive measurements will correlate. Kobayashi et al. (2002) argue that $\sigma_w = \sigma_u^2 + \sigma_{psd}^2 + \sigma_{pl}^2$. Here $\sigma_{pl}$ is due to the systematic variation of line-of-sight velocities in the IFOV for a moving platform [the term $-x u_z/h$ in Eq. (6)] and is easily the largest contributor. Using the expressions in Kobayashi et al. (2002), we can estimate that $\sigma_w^2 \approx 8.2 - 24.2$ m$^{-1}$, depending on whether the vertical wind shear reinforces or mitigates the variation of line-of-sight velocities across the IFOV. The associated time scale is given by

$$T_1 = \sqrt{\frac{\lambda^2}{8 \pi^2 \sigma_w^2}}.$$  

(7)

The second time scale is related to the change in antenna gain for the same atmospheric volume as the radar moves over it. From Eq. (5), one can estimate an $\epsilon$-folding time scale by considering the change in gain as $t$ varies from 0 to $T_2$:

$$T_2 = \begin{cases} 
\frac{h \tan \theta_0/2}{u_x \sqrt{2 \ln 2}}, & \text{for beam center,} \\
\frac{h \tan \theta_0/2}{u_x \sqrt{2 \ln 2}} \left( \sqrt{2 \ln 2} + \sqrt{1 + 2 \ln 2} \right), & \text{for beam edge (3 dB).}
\end{cases}$$

(8)

Note that this time scale only has meaning in the case of inhomogeneous clouds. When the atmosphere is homogeneous, translation invariance implies that this time scale effectively becomes infinite.

Finally, there is the time scale on which the line-of-sight velocity of the same volume element changes as the radar moves over it. Here a time scale can be derived from the period with which the last factor in Eq. (4) changes:

$$T_3 = \min \left[ \frac{\sqrt{\lambda h}}{2u_x^2}, \sqrt{\frac{\lambda}{2u_x}} \frac{\partial v_x}{\partial x} \right].$$

(9)

From the above discussion it may seem that $T_1$ and $T_3$ should have a similar form (they both result from changing line-of-sight velocities). But note that Eq. (4) contains an integration over $x$, while $t$ is constant. Again, the time scale $T_3$ only has meaning in the case of
inhomogeneous clouds. It is effectively infinite for homogeneous clouds because of translation invariance.

Figure 2 shows the effect of decreasing $\theta_0$ on these time scales, using EarthCARE parameters. It may be concluded that usually $T_1$ is significantly smaller than the other two time scales. Consequently, the variation of line-of-sight velocities within the IFOV drives the decorrelation of receiver voltages. Hence, Eq. (4) is adequate for describing the correlations between receiver voltages when relative motion between radar and scene exists, certainly for the presently considered antenna ($\theta_0 = 0.09^\circ$).

**e. Simulating receiver voltages**

Once a cloud scene has been specified through $\eta(\mathbf{r})$, $\mathbf{v}(\mathbf{r})$, and $\sigma_r(\mathbf{r})$, covariances between neighboring pulses can be calculated from Eq. (4). It then remains to develop a technique to generate time series of correlated receiver voltages, which will from now on be written as the complex sum of two independent real components $V = I + jQ$ (in-phase and quadrature-phase components). Equation (4) readily yields the covariances for the phase components, when one realizes that they are due to the very same physical process,

\[
\left\langle I(0)I(t) \right\rangle = \left\langle Q(0)Q(t) \right\rangle = \frac{1}{2} \Re(V(0)V^*(t)),
\]

\[
\left\langle I(0)Q(t) \right\rangle = -\left\langle Q(0)I(t) \right\rangle = -\frac{1}{2} \Im(V(0)V^*(t)).
\]

Furthermore, it is convenient to represent a time series of voltage components by a vector, for example, $\mathbf{I}$, whose elements are the measurements at different times. The covariances between the elements of this vector $\mathbf{I}$ can be represented by a matrix $(\mathbf{I} \otimes \mathbf{I})$, whose elements are given by Eq. (4). Here $\otimes$ is the dyadic product. When the matrix $(\mathbf{I} \otimes \mathbf{I})$ is diagonal, a time series of in-phase components is uncorrelated and easily generated with standard random number generators.

To generate correlated voltage components, we modify a technique used for a single correlated time series (Peebles 1980; Fishman 1996) and write

\[
\mathbf{I} = \mathbf{A} \cdot \zeta,
\]

\[
\mathbf{Q} = \mathbf{B} \cdot \zeta + \mathbf{C} \cdot \xi.
\]
where the constant matrices $A$, $B$, and $C$ are yet to be determined, and $\zeta$ and $\xi$ will be called seed vectors. A seed vector $\zeta$ or $\xi$ has uncorrelated elements that are normally distributed with $\langle \zeta_k \rangle = 0$ and $\langle \xi_k \rangle = 1$. Standard random number generators suffice to generate these seed vectors. Substituting Eqs. (12) and (13) in $I$ and $Q$, we derive

$$A = (I \otimes I)^{1/2},$$

$$B^\dagger = A^{-1} \cdot (I \otimes Q),$$

$$C = ((I \otimes I) - B \cdot B^\dagger)^{1/2}. \tag{16}$$

In the derivation, the following identities were used ($A$, $B$, and $C$ arbitrary matrices, and $^\dagger$ signifying the transpose):

$$\langle (A \cdot \zeta) \otimes (B \cdot \zeta) \rangle = A \cdot B^\dagger, \tag{17}$$

$$\langle (A \cdot \zeta) \otimes (C \cdot \zeta) \rangle = 0. \tag{18}$$

Note that if we use for example, $\langle Q \otimes Q \rangle$ and $\langle Q \otimes I \rangle$, the same solution is found. An explicit solution (many will exist) of Eqs. (14) and (16) may be calculated using eigenvalue or Cholesky decomposition (see Golub and van Loan 1996).

The $I$ and $Q$ generated with $A$, $B$, and $C$ through random $\zeta$ and $\xi$ will have all the required stochastic properties described in the previous subsections.

Summarizing our technique for simulating Doppler signals over inhomogeneous scenes, we proceed as follows. First, we prescribe the scene itself through backscatter $\eta(r)$, systematic speed $v(r)$, and turbulent motions $w(r)$. Second, we calculate the correlations between $N$ subsequent receiver voltages using Eq. (4). Here it is assumed that for any given pulse, the three pulses just preceding it and following it are measuring the same stationary process (see the discussion at the end of section 2d). Beyond these six pulses, covariances are effectively zero anyway. Third, we compute the matrices $A$, $B$, and $C$ from Eqs. (14)–(16). Fourth, we generate time series of receiver voltages through Eqs. (12) and (13). These voltages result from atmospheric reflections only. To introduce receiver noise, independent fluctuations of appropriate magnitude and with a Gaussian distribution should be added to both $I$ and $Q$. Finally, we generate Doppler radar observations from Eqs. (2) and (3). Each instance of two seed vectors $\zeta$ and $\xi$ yields one simulated observation of power $P$ and Doppler speed $v_D$. 

Fig. 3. Relative increase in the standard deviation of observed Doppler for inhomogeneous clouds, when predicted with the new technique instead of the Sirmans and Bumgarner (1975) technique. The scene used is described in detail in section 3a.
f. Comparison with the Sirmans and Bumgarner technique

For homogeneous clouds, the new technique should give identical results to the old technique by Sirmans and Bumgarner (1975). Simulations were performed for a homogeneous cloud with downward velocities (away from the radar) of either 1 or 5 m s\(^{-1}\). For the radar parameters, a range of values around typical EarthCARE values was chosen: \(\tau \in [60, 200] \mu\text{s}\) and \(\theta_0 = 0.045^\circ, 0.09^\circ,\) or \(0.18^\circ\). A single observation was made up of 1000 measurements (pulses).

Ten thousand \((10^4)\) observations of power and Doppler speed were generated and their mean and standard deviation computed. Typically, the mean of the power and Doppler velocity samples agree within 0.2\% for both techniques. Similarly, the standard deviation of power and Doppler speed samples agree within 2\%. For the majority of cases, means of power and Doppler speed agree within the 95\% confidence interval for a \(10^4\) sample size. Here we excluded those cases where severe decorrelation or aliasing caused the distribution of observed speeds to become strongly non-Gaussian.

In the next section, simulated observations over a discontinuous inhomogeneous scene (12-dB change in reflectivity) will be shown that cause significant biases in observed Doppler speeds. Here the same scenes are used to compare the new and old simulation techniques for inhomogeneous clouds. The Sirmans and Bumgarner (1975) technique is now adapted according to Tanelli et al. (2002), who averaged the expected Doppler spectra (weighted by reflectivity) for many small volumes in an inhomogeneous target. Now statistically significant deviations exist for the means of power and Doppler speed. However, the actual deviations are still small (a few percent or a few centimeters per second). The standard deviations of power and Doppler speed are more strongly affected. Maximum difference in the standard deviation of power or Doppler speed depends on opening angle \(\theta_0\), but mostly on how inhomogeneous the observed scene is. Here the scene consists of two clouds, one with backscatter \(\eta_+\), the other with \(\eta_-\). When \(\eta_+ / \eta_- = 4\), the relative difference is maximally 15\% \((\theta_0 = 0.09^\circ)\) and 20\% \((\theta_0 = 0.045^\circ)\). When \(\eta_+ / \eta_- = 16\), the relative difference is maximally 50\% \((\theta_0 = 0.09^\circ)\) and 70\% \((\theta_0 = 0.045^\circ)\). Both power and Doppler

![Figure 4](image-url)
speed are similarly affected. In all cases the new technique predicts higher standard deviations (see also Fig. 3).

It would appear safe to conclude that both techniques yield identical results for homogeneous scenes, but for inhomogeneous scenes the new technique predicts larger uncertainties in observed power and Doppler speed, as expected.

3. Simulated observations for inhomogeneous clouds

a. Discontinuous variation of \( \eta \)

To better understand the effect inhomogeneity has on Doppler radar observations, first some highly abstract cloud scenes will be considered. Observations at a single altitude will be simulated, so the cloud profile is ignored. Also, receiver noise will be excluded for now. As a model for a lateral cloud boundary, consider a cloud with constant speed \( v = 1 \text{ m s}^{-1} \) (downward, away from the radar) and piecewise constant backscatter:

\[
\eta(x) = \begin{cases} 
\eta_- & x < 0, \\
\eta_+ > \eta_- & x \geq 0.
\end{cases}
\]

Considering the extreme nature of this inhomogeneity, this scenario likely represents a worst-case scenario.

For different IFOVs (\( \theta_0 = 0.045^\circ \) and \( 0.09^\circ \)) and PRIs (\( \tau = 100 \) and \( 150 \mu s \)), observations starting at different locations \( x_s \) along track were simulated (sample size: \( 10^6 \)). Each observation consists of a 1-km along-track integration and comprises either \( 1308 \) (\( \tau = 100 \)) or \( 872 \) (\( \tau = 150 \)) voltage measurements (or pulses). As the satellite moves toward ever larger \( x \), it moves toward the more reflective cloud.

Figure 4 shows the mean Doppler speed \( \bar{D} \) at different locations along track. Far away from the lateral cloud boundary (\( x_s = -1400 \text{ m} \) or \( x_s = 400 \text{ m}, \) where \( x_s \) is the starting location of the ground track), observed Doppler speeds will be unbiased, but close to the boundary large biases exist. The deviations depend on the relative brightness of the two cloud halves (\( \eta_- / \eta_+ \)) and on the radar IFOV (\( \theta_0 \)) but may reach several meters per second for the proposed EarthCARE radar. Also, aliasing of Doppler speeds may occur (near \( x_s = -1100 \text{ m} \)). That is, many line-of-sight velocities within the IFOV are larger than the Nyquist velocity. As a result, observed Doppler speeds experience aliasing where positive speeds fold onto negative speeds and vice versa. In these instances the histogram of observed Doppler speeds is no longer Gaussian in shape but has two peaks at the negative and positive Nyquist velocity.

![Fig. 5. Maximum and minimum values of bias and standard deviation of Doppler speeds for a scene with harmonically varying \( \eta(x) \), as a function of wavelength \( L \). Based on simulations made for standard EarthCARE parameters.](image-url)
and the bias (shift in mean Doppler speed) may actually become positive.

The EarthCARE radar’s IFOV yields a circular footprint with a 700-m diameter at the surface, which smears out any gradients encountered along track. Consequently, the above results do not change drastically when continuous models for \(\eta(x)\) are used. When \(\eta(x)\) is assumed to vary linearly within a transition region (of width \(\Delta x = 100, 200, 400, \) or \(800\) m), the changes in the biases in the mean Doppler speed are small (\(0.02\) m s\(^{-1}\) for \(\Delta x \leq 200\) m; \(0.06\) m s\(^{-1}\) for \(\Delta x \leq 400\) m; and \(0.22\) m s\(^{-1}\) for \(\Delta x \leq 800\) m). Simulations were performed for proposed EarthCARE radar parameters, scenes with \(\eta(x)\) and \(u(x)\), \(\eta(x)\) or \(u(x)\) will be discussed. For each scenario (a particular choice of \(L\) and \(x_s\)), \(10^4\) observations were simulated and used to determine the mean and standard deviation of observed power and Doppler speed. In all cases, both \(P\) and \(v_D\) are distributed normally. Aliasing of Doppler speeds is not an issue for these scenarios. Because the observed scene is periodic in \(x\), the mean observed power and Doppler speed are likewise periodic in \(x\).

For standard EarthCARE parameters, Fig. 5 shows the total variation in mean and standard deviation of \(v_D\) for varying \(L\). Since the integrated FOV is 1000 m, only small biases in \(v_D\) occur for \(L = 500\) and 1000 m, while large biases are found for \(L = 700\) and 2000 m. The standard deviation of \(v_D\) behaves similarly.

Just like in the previous subsection, biases increase with increasing gradients in \(\eta(x)\) and show a strong correlation with \(-\partial P(x)/\partial x\) (not shown). On the other hand, standard deviations correlate with \(\partial^2 P(x)/\partial x^2\) (not shown). For a scenario centered on a local minimum in

b. Harmonic variation of either \(\eta\) or \(u\)

Next, as an idealized model of up- and downdrafts or gravity-wave-induced motions, clouds with a harmonic variation in either backscatter or particle speed will be considered:

\[
\eta(x) = \eta_0 + \Delta \eta \sin(2\pi x/L), \quad \text{or} \quad u(x) = \Delta u \sin(2\pi x/L),
\]

where \(L\) takes on the values \(L = 400, 500, 600, \ldots, 1500, \) and 2000 m. Simulations were made for the same radar parameters as in the previous subsection, starting the observation at equidistant phases along the oscillation in either \(\eta(x)\) or \(u(x)\).

First, results for a scene with \(\Delta \eta/\eta_0 = 0.5\) and \(\Delta u = 0\) will be discussed. For each scenario (a particular choice of \(L\) and \(x_s\)), \(10^4\) observations were simulated and used to determine the mean and standard deviation of observed power and Doppler speed. In all cases, both \(P\) and \(v_D\) are distributed normally. Aliasing of Doppler speeds is not an issue for these scenarios. Because the observed scene is periodic in \(x\), the mean observed power and Doppler speed are likewise periodic in \(x\).

For standard EarthCARE parameters, Fig. 5 shows the total variation in mean and standard deviation of \(v_D\) for varying \(L\). Since the integrated FOV is 1000 m, only small biases in \(v_D\) occur for \(L = 500\) and 1000 m, while large biases are found for \(L = 700\) and 2000 m. The standard deviation of \(v_D\) behaves similarly.
\( \eta(x) \), line-of-sight velocities near the FOV’s forward and backward edges will be favored, increasing the standard deviation in observed Doppler speeds. For a scenario centered on a local maximum in \( \eta(x) \), line-of-sight velocities near the FOV’s center will be favored, decreasing the standard deviation in observed Doppler speeds.

For scenes with \( L \leq 1100 \) m, the maximum variation along track in averaged \( P(x) \) is less than its standard deviation. That is, based on observations of reflectivity, one cannot conclude the scene is inhomogeneous. There is nevertheless a strong effect on Doppler speed (see also Fig. 6, filled black circles).

A comparison for different choices of radar parameters is shown in Fig. 6. For the standard EarthCARE parameters, inhomogeneity can cause biases as large as 0.8 m s\(^{-1}\) and increases in standard deviations of as much as 100%. Note that that these effects are uncorrelated. Choosing shorter PRIs (\( \tau \)) decreases the susceptibility of the standard deviation of \( v_0 \) to inhomogeneity. Choosing smaller IFOVs (\( \theta_0 \)) also reduces the bias.

Lastly, results for a scene with \( \Delta \eta/\eta_0 = 0 \) and \( \Delta v = 1 \) m s\(^{-1}\) are briefly discussed. Again, the mean observed Doppler speed is periodic in \( x \) (mean observed power is constant), and aliasing of Doppler speeds is inconsequential. The effect of inhomogeneity on Doppler speeds is apparent from a figure very similar to Fig. 5 (not reproduced here). Figure 7 summarizes the results for different choices of radar parameters. Obviously, not only inhomogeneity in \( \eta(x) \) causes errors in observed Doppler speed. Inhomogeneity in \( v(x) \) has a similar effect, although it may be less important in real clouds, as \( \eta \) usually varies more strongly.

c. Realistic variations in \( \eta(x) \) and \( v(x) \)

To conclude this study, realistic cloud scenes derived from ground-based observations of reflectivity and Doppler speed will be considered. Six datasets will be used in this section, observed by either the NICT 95-GHz SPIDER Doppler radar at Kokubunji, Japan (Horie et al. 2000), or the Royal Netherlands Meteorological Institute (KNMI) 35-GHz radar at Cabauw, Netherlands (http://www.knmi.nl/samenw/cloudnet/documents/35GHz.html). In addition to reflectivity and Doppler speed, the Cabauw dataset also contains observations of the Doppler width, \( \sigma_v^2 + \sigma_{PSD}^2 \approx 0.2^2 \) m s\(^{-1}\). For the SPIDER datasets, a typical value of \( \sigma_v^2 + \sigma_{PSD}^2 = l^2 \) m s\(^{-1}\) was used (Kobayashi et al. 2002).

The Cabauw data are provided as 16.5-s averages,
while the SPIDER data were averaged over 10 s to obtain reliable Doppler speeds. Four datasets show thick, extended ice clouds (with different amounts of inhomogeneity), and two others show a more broken cloud field (see Figs. 8 and 9). All clouds show downward speeds of $0 \rightarrow 1 \text{ ms}^{-1}$.

For the SPIDER data, radiosonde data [from World Meteorological Organization (WMO) station 47646 at Tateno, Japan] were used to interpret time series of reflectivity and Doppler speed as spatial tracks of backscatter $\eta(x)$ and particle speed $v(x)$. For the Cabauw data, European Centre for Medium-Range Forecasts (ECMWF) wind analysis data were used for the same purpose. The aforementioned time averaging and the prevalent horizontal wind speeds lead to tracks with a horizontal resolution of 240–450 m (SPIDER) or 155–560 m (Cabauw). Linear interpolation was used to obtain $\eta(x)$ and $v(x)$ at shorter sampling intervals. Both the time averaging and the linear interpolation suppress the true variation of cloud properties and, consequently, the present analysis can only be seen as providing minimum errors in Doppler speed.

Spaceborne observations were simulated for standard EarthCARE radar parameters ($\theta_0 = 0.09^\circ$, $\tau = 150 \mu s$), including correct range-weighting (pulse length: 1000 m) and realistic receiver noise levels ($-20 \text{ dBZ}$). Observed tracks of 1 km were simulated every 250 m (exception: 500 m for Cabauw on 26 February 2002). The gate separation of the simulated profiles was larger than 400 m, so that no correlations between altitudes existed. All Doppler statistics are based on samples of $10^4$ observations (per 1-km track) (exception: $10^3$ for Cabauw on 26 February 2002).

As in the previous sections, biases in mean observed Doppler speed and standard deviations of observed Doppler speeds were analyzed. Figure 10 shows standard deviations of observed Doppler speeds as a function of target reflectivity for each dataset. The solid line is the standard deviation estimated from traditional analysis [i.e., under the assumption of homogeneity (Sirmans and Bumgarner 1975)]. It is flat for high signal-to-noise ratios but increases with decreasing reflectivity ($Z \leq -15 \text{ dBZ}$) because of receiver noise. Because of cloud inhomogeneity, actual standard deviations cluster around the traditional estimate, although the distribution is skewed and favors values in excess of the traditional value. In addition, some scenes (open circles) cause such large shifts in line-of-sight velocities...
that some Doppler observations are aliased. As a consequence, distributions of observed Doppler speeds are, strictly speaking, no longer Gaussian. Typically less than 1% of the scenes suffer from this. The exception to this is the broken cloud scene of 10 September 2003 (SPIDER), where some 10% of the scenes suffer from quite strong aliasing. For the scenes with minimal aliasing the strongly increased (w.r.t. the traditional value) standard deviation reflects a strong broadening of the distribution of simulated Doppler speeds. This usually occurs for scenes with a local minimum in $\eta(x)$ near the center of the integrated FOV.

In the following, only observations with $Z > -20$ dBZ and minimal aliasing will be considered. The reflectivity threshold is due to the accuracy requirement of EarthCARE (which stipulates a minimum reflectivity). Scenes that cause aliasing are not considered, as the non-Gaussian distribution of Doppler speeds makes the analysis more complicated. A summary of the datasets and the relevant simulated observations is shown in Table 1. It contains the total number of 1-km scenes available, the spatial resolution (which varies with altitude because of variations in wind speed), and the rms and maximum value of biases in Doppler speed.

The biases in mean simulated Doppler speed are uncorrelated with the standard deviations, as in the previous section. For each dataset the biases themselves are normally distributed around zero, with an rms as given in Table 1. For several scenes one-third of the simulated Doppler speeds have biases in excess of 0.5 m s$^{-1}$. Some scenes even cause biases as large as 3.1 m s$^{-1}$. Figure 11 shows all biases as a function of the normalized difference along track in the observed reflectivity, for different values of $\Delta x$, which is the sampling interval of the data. The standard EarthCARE value would be $\Delta x = 1000$ m; note that $P$ is still an average over a 1-km track (irrespective of $\Delta x$). Also shown are the results for the model clouds discussed in the previous sections.

It would appear that, to first order, a linear relation exists between the normalized difference and the bias...
Fig. 10. Standard deviations of simulated Doppler speeds for all scenes of realistic clouds, as a function of target reflectivity. Open circles represent scenes that give rise to aliased Doppler speeds. Solid lines are standard deviations estimated from traditional analysis.
in the Doppler speed. It is also clear that a higher horizontal sampling than \( \Delta x = 1000 \) m yields a better relation (less scatter). Deviations from this line can probably be explained from unresolved structure in \( \eta(x) \) and unaccounted variations in \( \sigma(x) \). For each value of \( \Delta x \), a line was fitted to the September 2002 and 2003 Cabauw data (these scenes have fairly high spatial resolution and a fairly wide range of biases). Any offset in the fitted lines is due to the limited number of data points (one would expect zero offset because of symmetry).

This linear relation between normalized difference and bias can be used to correct biases in observed Doppler speeds. First, the rms of residual biases for each scene is investigated as a function of \( \Delta x \) (see Table 2). This clearly shows the beneficial effects of shorter sampling intervals. Table 3 shows the percentages of simulated observations that fail to meet the Earth-CARE accuracy requirements. This occurs when the absolute total error (bias + random) is larger than 1 m s\(^{-1}\) (for a 1-km track) or 0.2 m s\(^{-1}\) (for a 10-km track). These percentages were calculated using either the traditional analysis (i.e., assuming homogeneous clouds) or the current analysis (inhomogenous clouds) with or without application of the bias correction formula.

The results in Table 3 indicate that inhomogeneity can significantly increase the number of observations that fail the accuracy requirement. The results for the 1-km Cabauw data in particular are telling, as signal-to-noise ratios are very high, and receiver noise does not influence the observations. Further, Doppler speeds averaged over 10 km will more often fail the accuracy requirement than Doppler speeds averaged over 1 km. This is understandable, as the requirement decreases by a factor of 5, while the standard deviation of the observations decreases by \( \sqrt{10} \). Finally, applying the bias correction formula clearly improves the observations, especially (and understandably) when signal-to-noise levels are high. As a matter of fact, errors are typically reduced to error levels derived from the traditional analysis for homogeneous clouds. Since this latter error is entirely due to the stochastic nature

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**Table 1. Observational datasets used in this paper.**

<table>
<thead>
<tr>
<th>Dataset</th>
<th>( N )</th>
<th>Resolution (m)</th>
<th>Rms ( \Delta u_D ) (m s(^{-1}))</th>
<th>Max ( \Delta u_D ) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPIDER (22 Jun 2005)</td>
<td>988</td>
<td>265–450</td>
<td>0.25</td>
<td>1.0</td>
</tr>
<tr>
<td>SPIDER (26 Jun 2005)</td>
<td>1514</td>
<td>240–410</td>
<td>0.17</td>
<td>0.9</td>
</tr>
<tr>
<td>SPIDER (10 Sep 2005)</td>
<td>652</td>
<td>240–300</td>
<td>0.53</td>
<td>3.1</td>
</tr>
<tr>
<td>Cabauw (26 Feb 2002)</td>
<td>942</td>
<td>460–560</td>
<td>0.46</td>
<td>2.3</td>
</tr>
<tr>
<td>Cabauw (21 Sep 2002)</td>
<td>471</td>
<td>235–315</td>
<td>0.45</td>
<td>1.6</td>
</tr>
<tr>
<td>Cabauw (7 Sep 2003)</td>
<td>420</td>
<td>155–200</td>
<td>0.36</td>
<td>2.0</td>
</tr>
</tbody>
</table>

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**Fig. 11. Biases in mean simulated Doppler speeds for all scenes of realistic clouds \((Z \approx -20 \text{ dBZ})\), as a function of normalized difference along track in reflectivity. Open circles represent scenes that give rise to aliased Doppler speeds. Results for the model clouds have been included for comparison (note that they were calculated without noise).**
of radar observations, it also represents a minimum error. Table 4 shows the influence of \(\Delta x\) on the percentage of observations that do not meet the accuracy requirement for 1-km along-track integration. Again, we see the positive effect of shorter sampling distance. For the 10-km along-track integration (fail statistics not shown), the bias correction does make a difference (see also Table 3), but the value of \(\Delta x\) seems not very important.

4. Summary

This paper assesses the accuracy of Doppler radar observations from a spaceborne platform. The error in observed Doppler speeds consists of both a systematic and a random component. The random component is due to the stochastic nature of Doppler radar measurements as radar waves are scattered back by distributed hydrometeors in turbulent motion. The systematic component is due to motion of the radar relative to a reference frame fixed to the earth. Even when the radar’s velocity vector is perpendicular to the imaginary line connecting radar and target, the finite beamwidth may cause systematic errors in Doppler speed, since the radar’s motion still contributes to the line-of-sight velocities in the instantaneous field of view. In case of cloud inhomogeneities, these contributions will not cancel out when integrated over the IFOV.

For the proposed ESA/JAXA/NICT space mission EarthCARE, the systematic errors in Doppler speeds can be quite large, because of the large platform speed (7.6 km \(\text{s}^{-1}\)). Simulated Doppler speeds for a variety of cloud scenes show systematic errors of 0–3 m \(\text{s}^{-1}\), depending on how inhomogeneous the scene is. The largest errors can be expected for lateral cloud boundaries, but within clouds, biases of 0.5 m \(\text{s}^{-1}\) will not be uncommon. In addition, inhomogeneity can both increase or decrease the random error of Doppler speeds, although increases are more likely to occur. Finally, as a result of these changes in the Doppler speed statistics, aliasing of Doppler speeds may occur in the case of cloud inhomogeneity, even for slow-moving hydrometeors.

These errors were determined using a new simulation technique that was developed to deal explicitly with along-track inhomogeneity. The new technique belongs to the class of random signal simulators but is not restricted to stochastically stationary processes. Instead, variation in the stochastic properties of radar measurements can be tracked on a pulse-by-pulse basis. The new technique predicts random Doppler errors larger by 50% than the traditional random signal simulators for very inhomogeneous scenes and EarthCARE radar parameters.

As the new simulation technique is very capable in describing Doppler observations of inhomogeneous clouds, the main limitations of the present study are in the limited number and spatial resolution of cloud scenes. The realistic cloud scenes based on observations by ground-based radars have spatial resolutions of 200–

### Table 2. Effect of \(\Delta x\) on rms bias per scene.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Rms (\Delta v_0) (m (\text{s}^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncorrected</td>
</tr>
<tr>
<td>SPIDER (22 Jun 2005)</td>
<td>0.25</td>
</tr>
<tr>
<td>SPIDER (26 Jun 2005)</td>
<td>0.17</td>
</tr>
<tr>
<td>SPIDER (10 Sep 2005)</td>
<td>0.53</td>
</tr>
<tr>
<td>Cabauw (26 Feb 2002)</td>
<td>0.46</td>
</tr>
<tr>
<td>Cabauw (21 Sep 2002)</td>
<td>0.45</td>
</tr>
<tr>
<td>Cabauw (7 Sep 2003)</td>
<td>0.36</td>
</tr>
</tbody>
</table>

### Table 3. Percentage of Doppler observations that fail the accuracy requirement (\(\Delta x = 500\) m).

<table>
<thead>
<tr>
<th>Dataset</th>
<th>1-km track</th>
<th>10-km track</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Homogeneous</td>
<td>Inhomogeneous</td>
</tr>
<tr>
<td>SPIDER (22 Jun 2005)</td>
<td>2.8</td>
<td>5.3</td>
</tr>
<tr>
<td>SPIDER (26 Jun 2005)</td>
<td>4.0</td>
<td>4.9</td>
</tr>
<tr>
<td>SPIDER (10 Sep 2005)</td>
<td>6.0</td>
<td>16.0</td>
</tr>
<tr>
<td>Cabauw (26 Feb 2002)</td>
<td>0.9</td>
<td>9.6</td>
</tr>
<tr>
<td>Cabauw (21 Sep 2002)</td>
<td>1.6</td>
<td>10.0</td>
</tr>
<tr>
<td>Cabauw (7 Sep 2003)</td>
<td>1.0</td>
<td>6.3</td>
</tr>
</tbody>
</table>
600 m because of data-sampling and accuracy constraints. In contrast, the EarthCARE radar’s IFOV is 700 m wide, and the observations will be integrated along track for 1 km. It is well known that cloud liquid water content exhibits variations on many different length scales, down to a few centimeters. The error budgets derived in the present study should therefore be considered minimum error budgets.

Simulations suggest that up to 16% of the EarthCARE Doppler observations will fail the accuracy requirement for 1-km along-track integration (1 m s\(^{-1}\) for targets \(Z > -20 \, \text{dBZ}\)). As the requirement is phrased as the maximum allowable 1σ level of the error distribution, this seems acceptable. On the other hand, the accuracy requirement for 10-km along-track integration (0.2 m s\(^{-1}\) for \(Z > -20 \, \text{dBZ}\)) will not be met for 16%–33% of the observations. Given that these are minimum error budgets, it seems unlikely that the accuracy for 10-km along-track integration can be met as is. Of course, these conclusions are limited by the low number of cloud cases that were studied.

The simulations also suggest that a fairly simple linear relation exists between the systematic error in Doppler speeds and the gradient in observed reflectivity along track. This allows for the correction of systematic errors in Doppler speeds but necessitates that observations of reflectivity are available over shorter tracks than what is currently planned (1000 m). The present study suggests 250 m instead, although it remains to validate the corrective algorithm for many different cloud systems. Still, the fact that the biases in Doppler speed are dominated by the reflectivity gradients suggests that they are quite insensitive to possible variations in the \(Z \sim v_D\) relation. Variations on length scales below the IFOV are smeared out, and the exact details probably do not affect the value of the bias. Small-scale variability (<100 m) is likely to add to the standard deviation but not cause any biases.

Clearly, more work remains to be done. Preferably, additional realistic cloud scenes should be considered to improve upon the statistics of the error budget. These cloud fields should have a high spatial resolution and preferably exhibit variation across track as well (3D clouds).

The present study assumed a radar that pointed exactly nadir while the platform moved parallel to the surface. Since neither of these conditions will be met for a real satellite, additional errors will result. In particular, it may be assumed that aliasing of Doppler speeds will occur rather frequently. This also requires further study.

This work was done in preparation for EarthCARE, which is a combined ESA/JAXA/NICT space mission aimed at synergetic studies of cloud properties.

Acknowledgments. The author wishes to thank Dr. Y. Ohno and Dr. H. Kumagai for interesting discussions. SPIDER observational data were provided by Dr H. Horie. The author acknowledges the Cloudnet project (European Union Contract EVK2-2000-00611) for providing Doppler radar observations, which were produced by KNMI using measurements from the Cabauw Experimental Site for Atmospheric Research (CESAR). This work was supported by the Research Fund B-4 for the Global Environment of the Japanese Ministry of the Environment. The author gratefully acknowledges useful criticism on earlier drafts of this paper by Dr. D. Donovan and Dr. H. Kumagai, and by two unknown reviewers.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Rms (\Delta v_D) (m s(^{-1}))</th>
<th>(\Delta x = 1000 , \text{m} )</th>
<th>(\Delta x = 500 , \text{m} )</th>
<th>(\Delta x = 250 , \text{m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPIDER (22 Jun 2005)</td>
<td>5.3</td>
<td>3.4</td>
<td>3.0</td>
<td>2.9</td>
</tr>
<tr>
<td>SPIDER (26 Jun 2005)</td>
<td>4.9</td>
<td>4.0</td>
<td>3.9</td>
<td>3.8</td>
</tr>
<tr>
<td>SPIDER (10 Sep 2005)</td>
<td>16.0</td>
<td>9.4</td>
<td>6.8</td>
<td>5.7</td>
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<td>Cabauw (26 Feb 2002)</td>
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<td>2.1</td>
<td></td>
</tr>
<tr>
<td>Cabauw (21 Sep 2002)</td>
<td>10.0</td>
<td>4.9</td>
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<td>2.5</td>
</tr>
<tr>
<td>Cabauw (7 Sep 2003)</td>
<td>6.3</td>
<td>2.8</td>
<td>1.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

REFERENCES


