Effects of Rain Gauge Temporal Resolution on the Specification of a *Z–R* Relationship

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ABSTRACT

The weather radar is an efficient alternative for measuring spatially varying rainfall covering a large area at a high temporal resolution. This paper studies the impact of rainfall gauge temporal resolution on optimal relationships between radar reflectivity (Z) and rainfall rate (R). Four datasets of radar reflectivity and corresponding rain gauge rainfall data from Sydney and Brisbane, Australia, and one dataset from Bangkok, Thailand, were used in the analysis. Climatological Z–R relationships were calibrated using rainfall aggregated over 1–24 h to investigate the evidence of temporal scaling in the Z–R calibrated parameters. This analysis points to an increase in the multiplicative term (the A parameter) of the Z–R relationship as temporal resolutions become finer. This pattern is repeated in all the datasets analyzed. Thereafter, a simple scaling hypothesis was proposed to develop transformations that could scale the A parameter in the Z–R relation across a range of temporal resolutions. This scaling relationship was found to be suitable, with the scaling exponent attaining values close to 0.055 across all the datasets analyzed. The proposed relationship has a significant role in radar rainfall estimation studies, especially in regions where subdaily gauge rainfall measurements are not readily available to ascertain optimal Z–R parameters.

1. Introduction

The weather radar is a widely used basis for measuring rainfall at fine spatial and temporal resolutions (Collinge and Kirby 1987; Sun et al. 2000; Uijlenhoet 2001; Vieux 2003). Nevertheless, the weather radar does not measure rainfall directly; rather, it infers the rainfall based on the power of electromagnetic waves backscattered by raindrops in the atmosphere and intercepted by the radar. This backscattered power is represented as the radar reflectivity (Z) and is related to a rainfall rate (R) through a power-law relationship $Z = AR^b$ (referred to as the Z–R relationship). This relationship requires the specification of parameters A and b, which are functions of both radar and rainfall characteristics, as discussed in Collier (1996).

Various forms of Z–R relations have been suggested in the literature (Marshall and Palmer 1948; Joss and Waldvogel 1970; Battan 1973). However, these relationships cannot be directly used in any region (Mapiam and Sriwongsitanon 2008). This is because the A and b parameters of the Z–R relationship vary depending on many factors, including their dependence on the rainfall drop size distribution (DSD), which varies in both space and time. We can estimate Z and R directly from the DSD measured using a disdrometer. These Z and R values can then be used to construct a Z–R relationship (Atlas 1964; Battan 1973; Krajewski and Smith 2002; Russo et al. 2005). However, disdrometers are relatively expensive and complicated equipments to operate; hence, it is uncommon for more than one (or even any) of them to be operated in conjunction with a weather radar. The use of DSD to ascertain a Z–R relationship is not possible at all locations. In places where accurate measurements

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of the DSD are not possible, reflectivity data measured by the radar and the rainfall recorded in rain gauges within the radar coverage are generally used.

The approach followed to specify the Z–R relationship at a location involves collating ground rainfall data at the finest temporal resolution possible, accumulating the reflectivity to the same resolution and ascertaining the parameters using a suitable optimization rationale. This relationship is generally used at a resolution finer than the ground-measured rainfall, under the assumption that the relationship is independent of the temporal resolution that it is developed at.

Although past studies have calibrated the Z–R relationship at a range of temporal rainfall resolutions such as hourly, daily, weekly, monthly, seasonal, or even longer (Hitchfeld and Bordan 1954; Smith et al. 1975; Wilson and Brandes 1979; Klazura 1981; Steiner et al. 1995), little has been done to investigate the sensitivity of the relationship to the temporal resolution that is used. Although the assumption that a Z–R relationship developed using hourly rainfall data is not different from application at finer resolutions (such as 6 min) may be appropriate, can the same assumption be made if the relationship is developed using only coarse daily observations instead? This is the main question we investigate in this paper, proposing a rationale for scaling the Z–R relationship developed at one temporal resolution to another, thereby providing an option for specifying the Z–R relationship in locations where subdaily rainfall measurements on a dense rainfall network are not available.

There are two objectives in this study. The first objective is to study the effects of using rain gauge data of different temporal resolutions for calibrating climatological Z–R relationships. Different climatological Z–R relationships were estimated using rainfall aggregated over 1–24 h. Radar reflectivity data from the Kurnell, Mount Stapylton, and Pasicharoen radars located in Sydney and Brisbane, Australia, and Bangkok, Thailand, respectively, as well as corresponding rain gauge data in the three cities, were used in the analysis. The second objective in this study is to propose a generic scaling rule that can be used to estimate radar rainfall at fine temporal resolution for the cases in which only daily rain gauge rainfall data are available for use in the Z–R calibration. A simple scaling hypothesis was applied to construct a scaling transformation equation to ascertain the A parameters at finer temporal resolutions.

The rest of the paper is as follows: the next section outlines the logic adopted to ascertain the Z–R relationship across a range of rainfall temporal resolutions. Section 3 presents the five datasets used to test the proposed logic and presents the results obtained. Section 4 presents the rationale adopted to develop a simple scaling relationship for the A parameter of the Z–R relationship as a function of the temporal resolution of gauge rainfall. In this section, the scaling logic is applied to the rainfall datasets used and a validation of the proposed logic using alternate rainfall attributes is presented. Conclusions from the study are presented in section 5.

2. Calibration of the climatological *Z–R* relationship

The Z–R conversion error is an important source of error in radar rainfall estimates. The following empirical power-law relationship is used to estimate radar rainfall using measured reflectivity (Battan 1973; Rinehart 1991; Doviak and Zrnic 1992; Collier 1996):

$$Z = AR^b, \tag{1}$$

where A and b are the radar parameters to be estimated and depend on the DSDs that have been sampled; assuming that the terminal velocity of the raindrops is a function of their diameter and that they are falling at terminal velocity through still air (Chumchean et al. 2008), Z is radar reflectivity (mm^6 m^{-3}) and R is the rainfall rate (mm h^{-1}). Although the parameter A is observed to change significantly from one region to another, depending on the nature of the rainfall events that occur, many researchers have suggested that the exponent b does not change as much (Seed et al. 1996; Steiner et al. 1999; Seed et al. 2002; Chumchean et al. 2003). Typical values of the multiplicative term A may range from 100 to 500 (Battan 1973), whereas the exponent b varies from 1 to 3 (Smith and Krajewski 1993), with typical values between 1.2 and 1.8 (Battan 1973; Ulbrich 1983). Doelling et al. (1998), Steiner and Smith (2000), and Hagen and Yuter (2003) investigated an appropriate value of the b parameter using several years of reflectivity data measured by a disdrometer. They found that a value of 1.5 was suitable to represent the b parameter in the Z–R relation. Seed et al. (2002) illustrate that the root-mean-square error (RMSE) of radar rainfall estimates are quite insensitive to the value of b over the range (b = 1.6, 1.5, and 1.4). Based on the above arguments and the result of a climatological calibration using the datasets analyzed in this study, the b parameter of the Z–R relation was fixed at 1.6, whereas the A parameter was ascertained using the procedure described next.

To study the effects of using rain gauge data of different temporal resolutions on Z–R relationships, different Z–R relationships were estimated using rainfall aggregated over 1–24 h. The logic used was as follows:

1) Convert instantaneous radar reflectivity into an initial radar rainfall intensity using the relationship $Z = 200R^{1.6}$ (Marshall and Palmer 1948). Note our

earlier comment on the relative insensitivity of results to changes in the b exponent as our rationale for keeping it fixed equal to 1.6 in our study.

2) Accumulate the initial instantaneous radar rainfall into 1–24-h rainfall resolutions using the accumulation algorithm proposed by Fabry et al. (1994). In this method, the rainfall field is assumed to move at constant velocity and to vary linearly in intensity during the sampling interval. The storm velocity was first computed for each time interval and then used to simulate a 1-min sampling rate by advecting the field observed at the start of the interval toward the field observed at the end of the interval. Represent the accumulated rainfall now via a variable A parameter, denoted A_a, where the subscript denotes the time resolution the rainfall is aggregated over. The resulting radar rainfall (which is a function of A_a) is denoted as $R_{i,t,a}$ where the subscripts denote the gauge, time-step, and temporal resolution, respectively.
3) Ascertain the rain gauge rainfall at station i for time-step t for temporal resolution a (denoted $G_{i,t,a}$) by accumulating it from 1 h to the other durations (1–24 h) considered.
4) Estimate the optimal value of A_a for each time resolution a considered by minimizing the mean absolute error (MAE) between the gauge and radar rainfall estimates. The mean absolute error is expressed as

$$\mathrm{MAE}_a = \frac{1}{N_{t,a}N_G}\sum_{t=1}^{N_{t,a}}\sum_{i=1}^{N_G}|R_{i,t,a} - G_{i,t,a}|, \qquad (2)$$

where $R_{i,t,a}$ is the radar rainfall accumulation at the pixel corresponding to the ith rain gauge for hour t for a temporal resolution a, $G_{i,t,a}$ is the corresponding gauge rainfall for hour t, N_G is the number of rain gauges, and $N_{t,a}$ is the number of time periods for each time resolution a.

It should be pointed out that this study used the mean square error (MSE) and the mean absolute error as two separate error criteria. Although the results from both criteria were similar, the MSE exhibited greater instability for results for larger time resolutions than the MAE, possibly because of fewer gauge–radar pairs being available and the tendency of the measure to magnify the larger differences. As a result, the results presented in later sections are based on the MAE error criterion alone.

3. Application

a. Radar and rain gauge data

Radar reflectivity data from the Kurnell, Mount Stapylton, and Pasicharoen radars located in Sydney, Brisbane, and Bangkok, respectively, and corresponding rain gauges data representing large networks in the three cities were used for the analyses presented in this paper. Three datasets of the 1.5-km constant altitude plan position indicator (CAPPI) reflectivity data at the Kurnell radar and hourly rain gauge data obtained from a network of 227 rain gauges during November 2000–April 2001, August–December 2006, and January–May 2007 were used for the Z–R development for the Sydney area. The Mount Stapylton radar data used represented the November 2006–March 2007 period, with corresponding rain gauge data obtained from a network of 202 rain gauges. For the Z–R calibration in Bangkok area, one dataset of the 0.5° plan position indicator (PPI) reflectivity data at Pasicharoen radar and 15-min rain gauge data obtained from a network of 61 rain gauges during June 2005–October 2006 were used.

The Kurnell radar is a C-band Doppler radar that transmits the radiation with the wavelength of 5.3 cm and produces a beamwidth of 0.94°, the Mount Stapylton radar is an S-band Doppler radar that transmits the radiation with the wavelength of 10.7 cm and produces a beamwidth of 1°, and the Pasicharoen radar is a C-band minimax Doppler radar that transmits the radiation with the wavelength of 5.4 cm and produces a beamwidth of 0.90°. The radar reflectivity data achieved from the Sydney and Brisbane stations are in a Cartesian grid with 256 km × 256 km extent with 1 km^2 spatial resolution and 10-min temporal resolution, whereas the reflectivity data achieved from Bangkok station are in a Cartesian grid with 240 km × 240 km extent with 1 km^2 spatial resolution and 10-min temporal resolution.

Rain gauge rainfall data used in this study were obtained from the networks of 227, 202, and 61 continuous tipping-bucket gauge stations located within 100 km from the Kurnell, Mount Stapylton, and Pasicharoen radars, as illustrated in Fig. 1. The rain gauges used for Sydney and Brisbane are owned and operated by the Australian Bureau of Meteorology and the Sydney Water Corporation, whereas the rain gauges used for Bangkok are owned and operated by the Bangkok Metropolitan Administration (BMA). The tipping-bucket gauge can systematically underrecord the true rainfall accumulation during the hour by the volume of water required to initially wet the funnel plus the volume of water stored in the tipping bucket at the end of hour. However, the amount of rainfall required to wet the funnel of the gauge before it starts to drain into the tipping bucket is very small and considered to be insignificant in this study. The rain gauges used in this study have tipping-bucket sizes of 1.0 and 0.5 mm. Because tipping-bucket rain gauges record the time of the tips, they are subject to significant high quantization

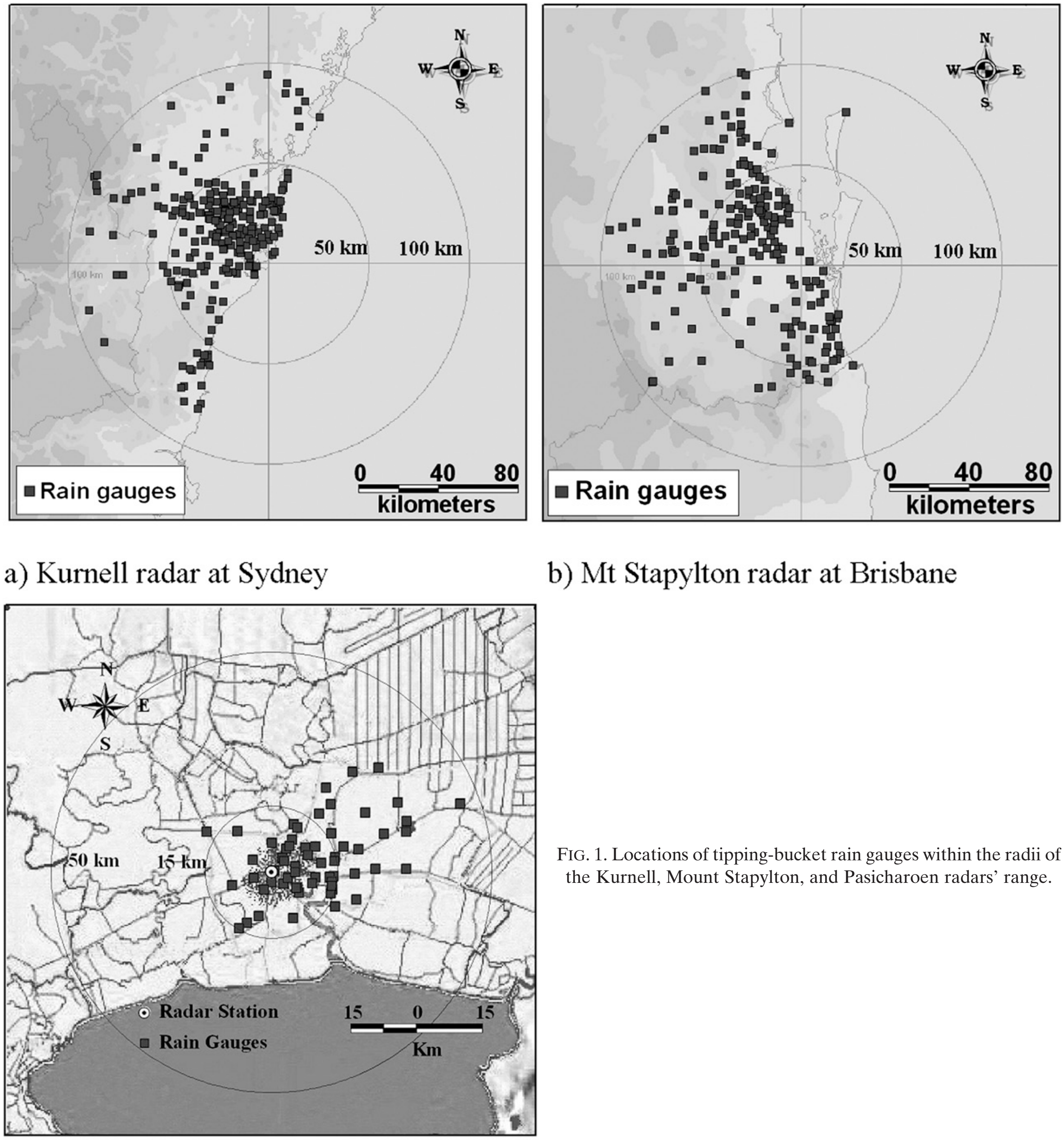

FIG. 1. Locations of tipping-bucket rain gauges within the radii of the Kurnell, Mount Stapylton, and Pasicharoen radars' range.

error at low rainfall intensity (Chumchean et al. 2003, 2004, 2006a,b). Therefore, only the rainfall amounts that are greater than the volume of that gauge's tipping bucket were used in this analysis. It should be noted that quality control of these data has been performed by considering rainfall data from adjacent gauges and the plots of time series. If unusual rainfall data were found, these data were excluded from the analysis.

To avoid the effects of the bright band and different observation altitudes in radar reflectivity, the CAPPI reflectivity data of the Kurnell and Mount Stapylton radars at the altitude below the climatological freezing levels of Sydney and Brisbane, respectively, and the PPI reflectivity data of the Pasicharoen radar at the lowest elevation angle within the radar range that gives the height of radar beams below the freezing level of Bangkok were used in the analyses.

The climatological freezing levels for Sydney, Brisbane, and Bangkok are about 2.5, 3.0, and 4.7 km, respectively (Chumchean et al. 2003, 2004). In this study, the 1.5-km CAPPI reflectivity data of both radars in Australia, the 0.5° PPI reflectivity data of the Bangkok radar, and only the reflectivity and rain gauge data that lie within 100 km of the radars were used. Also note that the height of the base scan beam center at this range is about 1.8 km above the ground, which is also below the freezing levels for the three cities and can be considered to not be overly different from the 1.5-km CAPPI height. Therefore, we consider that the reflectivity data used in this study are free from the effects of the bright band and different observation altitudes.

To avoid the effects of noise and hail in the measured radar reflectivity, the reflectivity values less than 15 dBZ were assumed to represent a reflectivity of 0 mm^6 m^{-3} and the reflectivity values greater than 53 dBZ were assumed to be 53 dBZ. Additionally, the errors resulting from the effects of ground clutter were also removed from the reflectivity data by finding the clutter locations from the map and discarding the radar measurements in these areas.

b. *Climatological Z–R relations for variable temporal resolutions*

The five datasets considered in this study were used to develop Z–R relationships by following the calibration procedure outlined in section 2. Numbers of radar–gauge pairs used for Z–R calibration for each temporal resolution in each dataset are also presented in Table 1. As mentioned earlier, the results presented here correspond to the use of the mean absolute error as the objective function to be minimized. The A parameters of the Z–R relationships derived by using different temporal rainfall resolutions for the five datasets are illustrated in the Fig. 2. Note that the datasets Sydney 1, Sydney 2, and Sydney 3 represent data for the periods November 2000–April 2001, August–December 2006, and January–May 2007, respectively. The results in Fig. 2 illustrate clearly that the highest value of parameter A is obtained when the temporal resolution is 1 h, with the parameter value decreasing as the aggregation period increases. The difference between the calibrated A parameters for Sydney, Brisbane, and Bangkok are large, whereas there are only small differences among the Sydney 1–3 datasets. Given the differences in the dominant storm types that could be expected between the three cities, this is to be expected.

From the above results, it can be seen that the multiplicative term A of the Z–R relationship varies as a function of the temporal resolution of the rainfall used in the calibration. Derivation of the A parameter using coarse (e.g., daily) rainfall and subsequently applying it to estimate fine-resolution (e.g., hourly or subhourly) rainfall can lead to significant biases in the resulting estimates. For situations where only daily rain gauge rainfall data are available for use in the Z–R calibration, a transformation function for converting the A parameters to other resolutions is needed.

4. Simple scaling hypothesis for the multiplicative term *A*

a. *Climatological scaling transformation function*

As a means of explaining the variation of A with temporal resolution in Fig. 2, we hypothesize here that A exhibits simple scaling behavior in time (Mandelbrot 1982; Chumchean et al. 2004). Such a hypothesis has been used by others to describe rainfall extremes at fine temporal resolutions (Menabde et al. 1999), to develop cascade models of rainfall disaggregation in time (Sivakumar and Sharma 2007), and to explain bias in radar rainfall estimates in space (as a function of distance from the radar; Chumchean et at. 2004). The simple scaling hypothesis [as defined by Gupta and Waymire (1990) and used by Chumchean et al. (2004) in a spatial scaling context] can be expressed as

$$A_t \overset{\text{dist}}{=} (t/T)^{-\eta} A_T, \tag{3}$$

where $\overset{\text{dist}}{=}$ represents the equality of the probability distribution for the multiplicative factor A, t/T is a scale factor, t (h) is the temporal resolution at which the rainfall needs to be estimated, T (h) is the reference temporal resolution of the radar rainfall, η is a scaling exponent that needs to be ascertained, and A_T and A_t represent the parameter A at temporal resolutions T and t, respectively. The distributional equality represented in (3) implies that the quantiles and the moments of any order of the calibrated multiplicative term A are scale invariant. Raising both sides of Eq. (3) with different power q, the relationship between the qth moment can be written as

$$\langle A_t^q \rangle = (t/T)^{-\eta q} \langle A_T^q \rangle. \tag{4}$$

The brackets $\langle\rangle$ denote the expected value of the qth-order moment for the multiplicative term A. The estimation of the scaling exponent η that best describes the distributional equality expressed in (3) and (4) now

TABLE 1. Number of radar–gauge pairs used for Z–R calibration at each temporal resolution for (top) Sydney and (bottom) Brisbane and Bangkok datasets.

	Sydney 1			Sydney 2			Sydney 3		
Temporal resolution (h)	0–100 km	0–50 km	50–100 km	0–100 km	0–50 km	50–100 km	0–100 km	0–50 km	50–100 km
1	31 405	22 527	8878	9664	7857	1807	17 294	13 299	3995
2	20 298	14 676	5622	6514	5281	1233	11 891	9151	2740
3	15 649	11 384	4265	5195	4175	1020	9319	7134	2185
4	13 143	9567	3576	4385	3565	820	7815	6003	1812
5	11 165	8119	3046	3862	3119	743	7057	5423	1634
6	9871	7200	2671	3582	2885	697	6290	4806	1484
7	8959	6576	2383	3136	2522	614	5773	4427	1346
8	8622	6305	2317	2930	2310	620	5253	4033	1220
9	7454	5428	2026	2878	2284	594	4903	3752	1151
10	6890	4965	1925	2544	2001	543	4771	3628	1143
11	6414	4612	1802	2300	1811	489	4490	3408	1082
12	6009	4401	1608	2357	1884	473	4061	3072	989
13	5760	4184	1576	2146	1724	422	4116	3119	997
14	5550	4087	1463	2130	1683	447	3825	2881	944
15	5145	3746	1399	1872	1468	404	3701	2757	944
16	5135	3759	1376	1806	1422	384	3476	2634	842
17	4741	3474	1267	1793	1403	390	3516	2644	872
18	4563	3351	1212	1892	1497	395	3189	2388	801
19	3987	2901	1086	1802	1404	398	3289	2486	803
20	3883	2792	1091	1676	1311	365	3007	2238	769
21	3749	2722	1027	1579	1227	352	3196	2402	794
22	3879	2827	1052	1519	1158	361	2952	2181	771
23	3639	2655	984	1548	1197	351	2695	1989	706
24	3424	2479	945	1438	1112	326	2653	1983	670

	Brisbane			Bangkok		
Temporal resolution (h)	0–100 km	0–50 km	50–100 km	0–50 km	0–15 km	15–50 km
1	14 949	8696	6253	868	563	305
2	9849	5747	4102	703	467	236
3	7652	4485	3167	645	418	227
4	6628	3880	2748	567	371	196
5	5908	3489	2419	536	345	191
6	5222	3033	2189	548	354	194
7	4902	2878	2024	503	325	178
8	4514	2636	1878	473	298	175
9	3914	2265	1649	506	336	170
10	3680	2142	1538	471	301	170
11	3864	2259	1605	493	323	170
12	3875	2239	1636	471	306	165
13	3229	1880	1349	458	293	165
14	3308	1926	1382	465	306	159
15	3122	1805	1317	422	278	144
16	3067	1765	1302	414	272	142
17	2892	1718	1174	493	312	181
18	2758	1616	1142	490	315	175
19	2712	1570	1142	554	344	210
20	2477	1420	1057	450	288	162
21	2566	1479	1087	526	326	200
22	2730	1558	1172	510	325	185
23	2555	1477	1078	476	323	153
24	2378	1376	1002	465	316	149

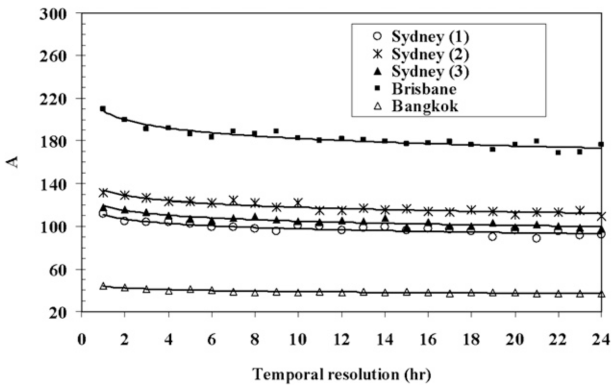

FIG. 2. Coefficient A of the Z–R relationship derived using a fixed exponent $b = 1.6$, for the Kurnell, Mount Stapylton, and Pasicharoen radars as a function of varying temporal rainfall resolutions.

proceeds by fitting the relationship in (4) across a range of moment orders q.

Moment orders q equal to 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, and 6.0 were used to ascertain the optimal value of η for the five rainfall datasets used. Results from this analysis are illustrated in Fig. 3a. An optimal value of η was ascertained as the slope of $\partial \log(A_t^q)/\partial \log(t)$ plotted as a function of moment order q in Fig. 3b. Note that Fig. 3 only represents results for the Sydney 1 data. Similar results were obtained for the other four datasets used.

The values of the scaling exponent for the other four datasets were 0.0548, 0.0566, 0.0574, and 0.0528 for Sydney 2, Sydney 3, Brisbane, and Bangkok, respectively. Consequently, we propose a uniform scaling exponent equal to 0.055. The proposed climatological scaling equation then becomes

$$A_t = (t/24)^{-0.055} A_{24}. \tag{5}$$

It should be noted that the data used in the study comprise of a range of storms from short-lived convective events to more sustained stratiform ones. Although the argument can be made that the mix of these types of events could be responsible for the observed scaling behavior, the fact that similar scaling relations are derived in the three climatologically different locations the study focused on suggests the scaling may be due to other factors. However, future work in this research will investigate the effects of storm types on the temporal scaling behavior of the Z–R relationship for these two rainfall types, using an operational storm classification approach of the type outlined in Chumchean et al. (2008).

b. *Investigation of impact of attenuation on the temporal scaling relationship*

Attenuation for C-band radar is considered to be a severe problem for measurement of high-intensity rainfall (reflectivity >50 dBZ; Hildebrand 1978; Austin 1987). However, the impact of attenuation can be ascertained by studying the relationship between gauge and radar rainfall as a function of distance from the radar (Burrows and Attwood 1949). In the results reported below, we investigate the effects of attenuation on the temporal scaling behavior of the Z–R relationship by considering distance from the radar site as a surrogate for possible attenuation. We assume that radar data within a given range interval have common attenuation effects. According to the spatial distribution of the three rain gauge networks of the three cities, the three datasets for Sydney and one dataset for Brisbane were separated into 0–50-km and 50–100-km range intervals and the dataset for Bangkok was separated into 0–15-km and 15–50-km range intervals because the farthermost gauge of the Bangkok network is located at

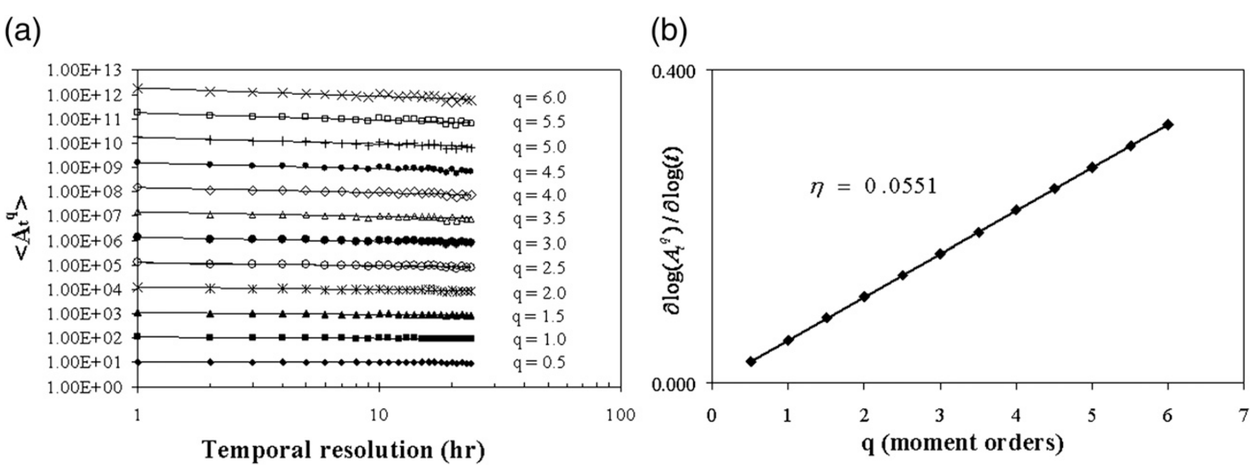

FIG. 3. Verification of scaling hypothesis for Sydney 1 rainfall data representing the period November 2000–April 2001. (a) Scaling of the moments for A parameters. (b) Scaling exponent for A parameters at different moment orders (q).

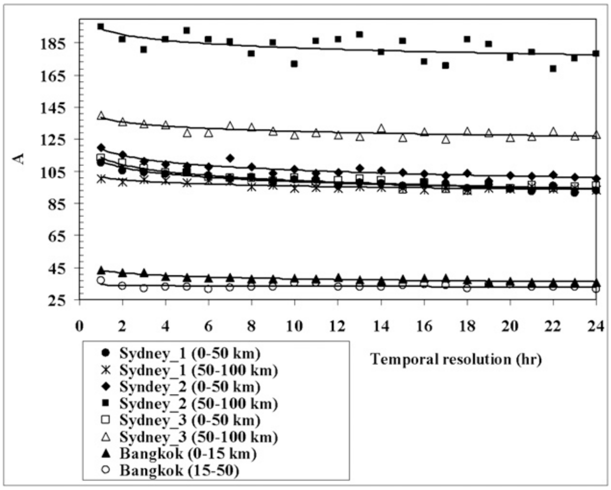

FIG. 4. As in Fig. 2, but at different range intervals.

around 46 km from the radar. To investigate the effects of attenuation on the temporal scaling of the Z–R relationship, the A parameters of each range interval for each radar were calculated; the results are presented in Fig. 4. Differences in the A parameters of each range interval for the same radar might be due to the differences in rainfall characteristic of each range, the differences being partly due to attenuation. Once the A parameters of each range interval were derived, the scaling transformation equations for each dataset were also estimated; the results are presented in Table 2. Note that the Sydney 1, Sydney 2, and Sydney 3 datasets represent data for the periods November 2000–April 2001, August–December 2006, and January–May 2007, respectively.

From the results presented in Table 2, it can be seen that the scaling exponents (η) of the scaling transformation Z–R equations obtained from the data lying within far range intervals of the C-band radar (50–100 km for Kurnell radar and 15–50 for Pasicharoen radar) are lower than using the data lying in the inner range intervals. This is possibly because of attenuation. For a situation where an intense rainfall cell passes between the radar and a rain gauge location, the reflectivity recordings from the pixel above the gauge will be underestimated. The calibration procedure would therefore return higher values of the coefficient A of the Z–R relationship than if there was no signal attenuation. Attenuation is a more severe problem during convective events and there may be a bias toward the prevalence of convective events at shorter durations over longer durations. Attenuation is more likely to produce lower A values at shorter durations than longer durations and consequently a lower value of the scaling exponent, η. This was indeed observed from both of the

TABLE 2. Scaling exponents at different range intervals for five datasets.

City	0–50 km	50–100 km	0–15 km	15–50 km
Sydney 1	0.0529	0.0250	—	—
Sydney 2	0.0513	0.0276	—	—
Sydney 3	0.0578	0.0273	—	—
Brisbane	0.0577	0.0557	—	—
Bangkok	—	—	0.0572	0.0114

C-band radar, with much lower values of η for the outer range band, where attenuation (if not corrected for) is more pronounced. The lower scaling exponent indicates an overestimated scaling exponent because attenuation has not been corrected for the data at far ranges. From this result, it is evident that attenuation has affected the temporal scaling behavior of the Z–R relations of the C-band radars. However, for the S-band radar, the scaling exponents of the two range intervals are not appreciably different. This confirms the temporal scaling hypothesis of the Z–R relationship because the attenuation problem can be neglected for the S-band radar.

c. *Verification of proposed scaling transformation function*

Because the proposed scaling transformation is based on the assumption of distributional equality, it can be expected that the function leads to reasonable results when applied to ascertain specified quantiles of the data. Consequently, the scaling function was verified by using it to ascertain the probability distribution of the maximum intensity of rainfall burst as a function of rainfall duration and compared to the distribution of similar maximum intensity of rainfall burst observed in each rain gauge location. It is to be noted that the term ''rainfall burst'' has been used to represent rainfall bursts of fixed durations. The 24-h A-parameter value was used as a reference, based on which parameters for other temporal aggregation periods were estimated. Additionally, to show an effectiveness of the proposed scaled Z–R transformation equation, the frequency distributions of the maximum radar rainfall obtained from the scaling and 24-h Z–R relationships were also compared with rain gauge data, as presented in Figs. 5–7. It can be noted that the distributional attributes of the estimated maximum radar rainfall are more similar to those of the rain gauge rainfall than the maximum radar rainfall obtained from the 24-h Z–R relationship. This lends further credibility to the assumptions that were used to formulate the proposed scaling transformation function.

The mean square errors for the maximum intensity of rainfall bursts for six time resolutions (1, 2, 4, 6, 12,

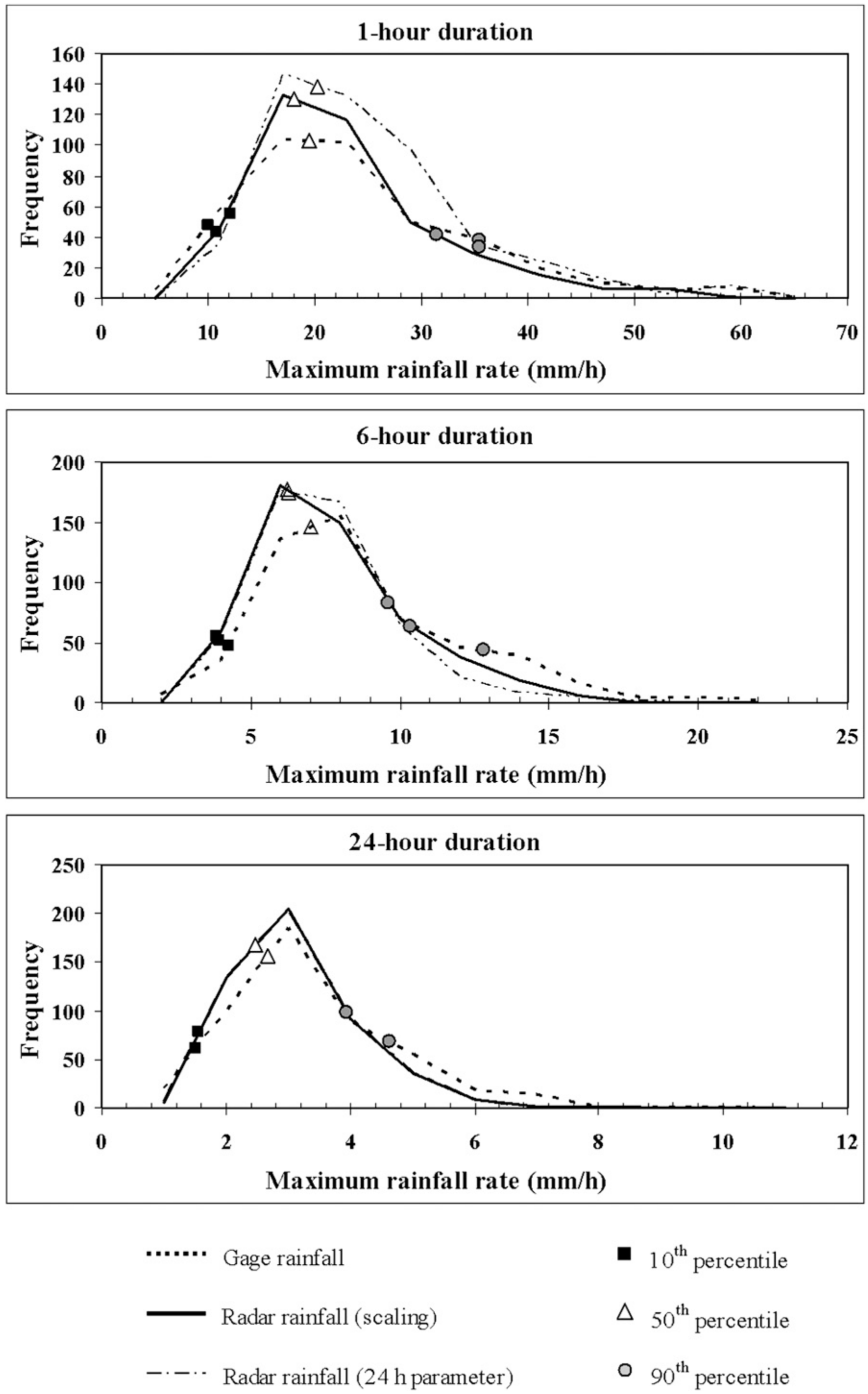

FIG. 5. Frequency of maximum gauge rainfall and the scaling transformation–based estimated radar rainfall for 1-, 6-, and 24-h durations for the combined Sydney datasets.

and 24 h) for the combined Sydney data, the Brisbane data, and the Bangkok data are presented in Table 3. In addition to this, the mean square error for the full rainfall data, excluding zero radar rainfall, was also calculated. The ''optimal'' case in the table refers to the estimation of the optimal A coefficient based on mean absolute error as described in section 2. For contrast, values of the error that would be expected were the 24-h A parameter to be used are also given. Percentages of errors in radar rainfall estimates based on two different A parameters that were derived from the temporal scaling transformation equation and the

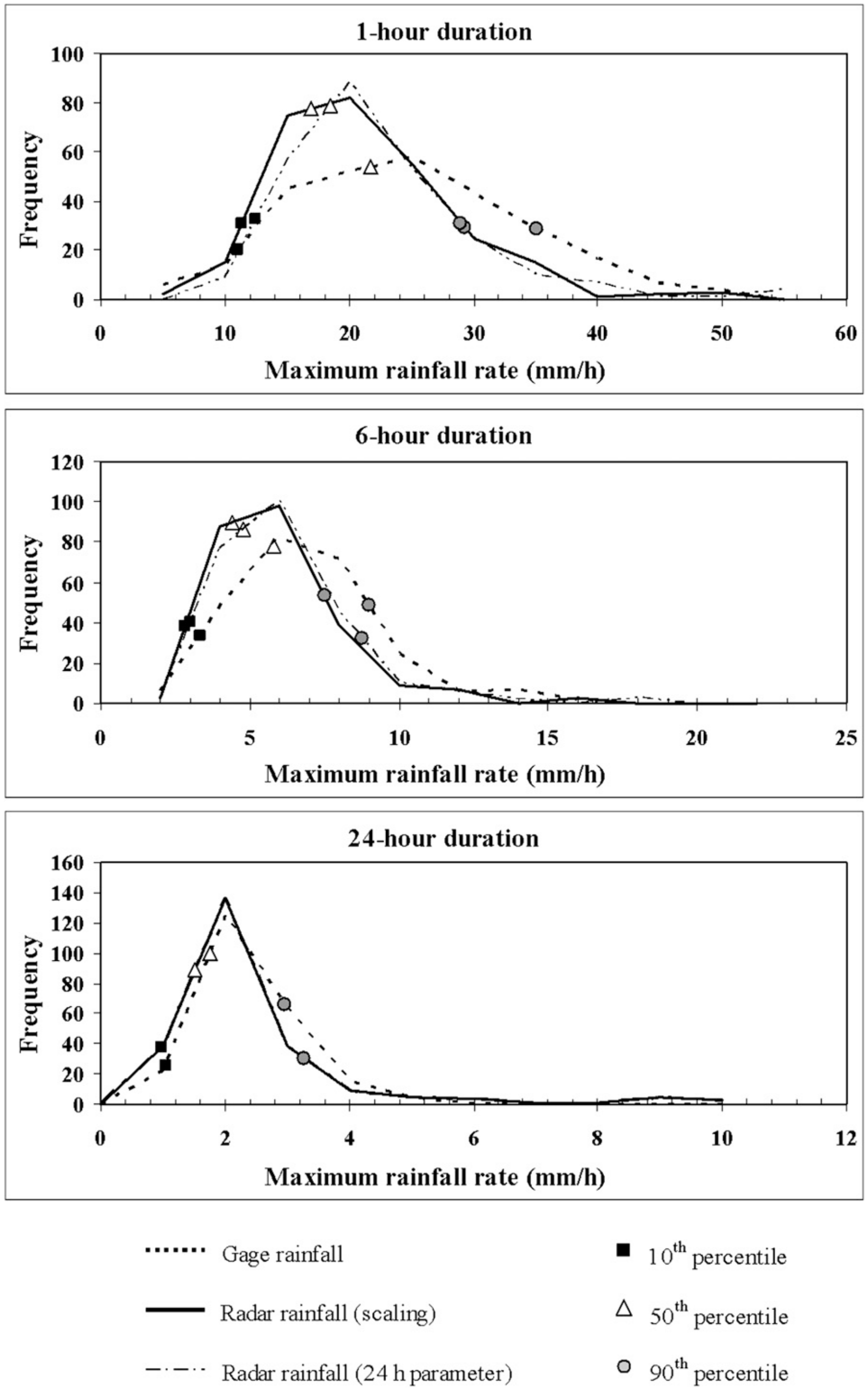

FIG. 6. As in Fig. 5, but for the Brisbane dataset.

24-h A parameter have been calculated, as shown in Table 3. From this result, it is evident that for all rainfall durations of all three radars, errors in extreme radar rainfall of the scaling case are less than the 24-h case. The 24-h A parameter gives higher errors, especially at high rainfall intensities and low temporal resolutions. Using the scaling Z–R relationship can reduce error in extreme radar rainfall, especially at the finer temporal resolution. However, the improvements in radar rainfall estimates, when considering all radar data excluding zero values, are not significant if the scaling Z–R relationship has been used.

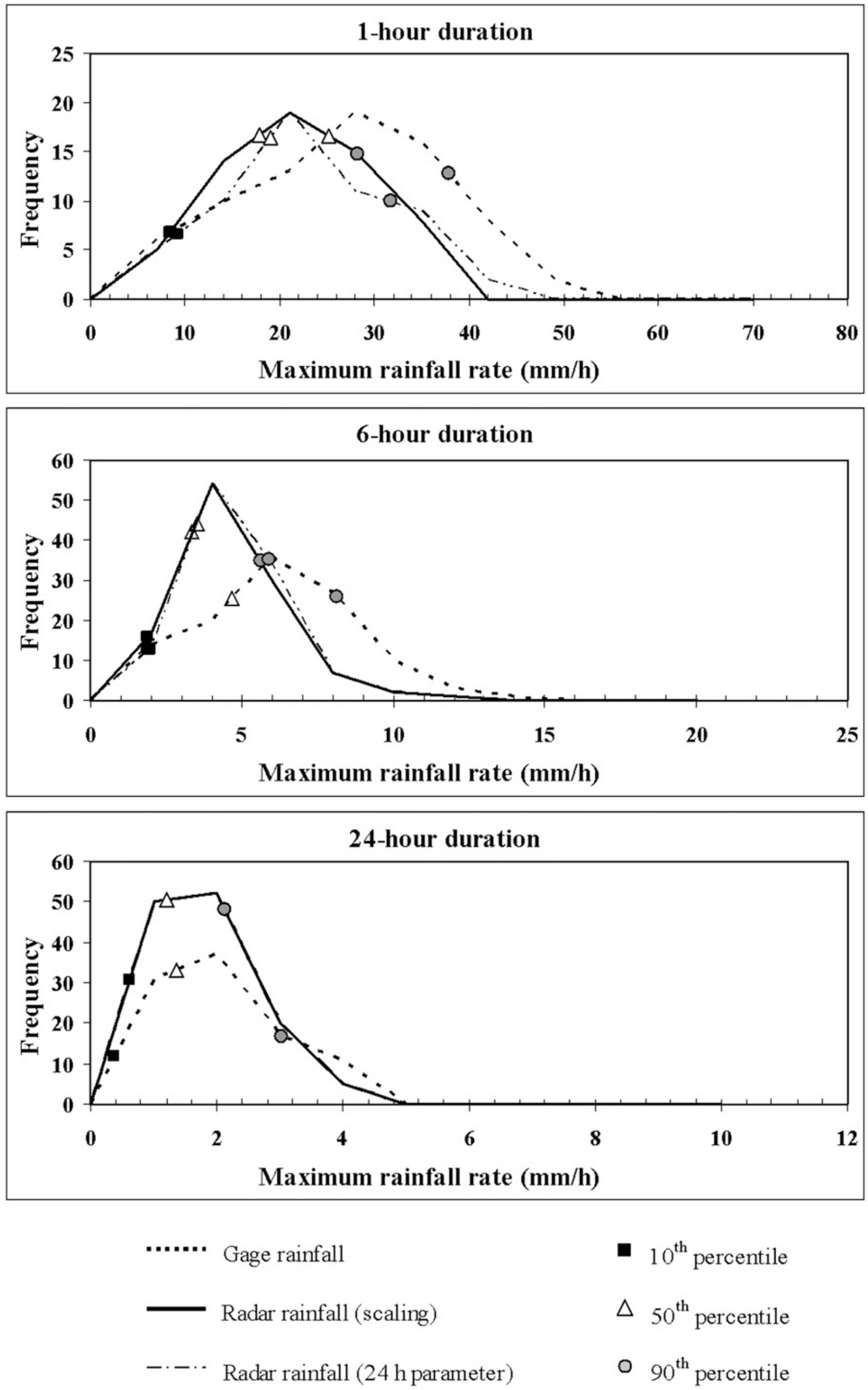

FIG. 7. As in Fig. 5, but for the Bangkok dataset.

5. Conclusions

The main conclusions of this paper are summarized below:

1) The A parameters of the Z–R relationship of Sydney, Brisbane, and Bangkok radar stations tend to decrease with a decrease in the rainfall temporal resolution.
2) The decrease in the A parameters can be described through a simple scaling law and a derived scaling transformation function.
3) The scaling exponents for five datasets representing three locations (Sydney, Brisbane, and Bangkok) lie in the vicinity of 0.055. Hence, a climatological scaling law with an exponent equal to 0.055 is proposed.

TABLE 3. Effectiveness of the scaling function for estimation of radar rainfall at varying temporal resolutions for the (a) Sydney, (b) Brisbane, and (c) Bangkok data.

	Time resolution					
(a) MSE $(\mathrm{mm\ h^{-1}})^2$	1 h	2 h	4 h	6 h	12 h	24 h
Maximum radar rainfall (optimal)	296.817	107.452	45.571	24.363	12.487	5.078
Maximum radar rainfall (scaling)	307.367	107.987	45.652	24.373	12.488	5.078
(% error between scaling and optimal)	(3.55%)	(0.50%)	(0.18%)	(0.04%)	(0.01%)	—
Maximum radar rainfall (24-h parameter)	386.367	121.687	46.338	24.524	12.536	5.078
(% error between 24-h parameter and optimal)	(30.17%)	(13.25%)	(1.68%)	(0.66%)	(0.39%)	—
Radar rainfall excluding zeroes (optimal)	11.5848	5.0740	2.0544	1.3587	0.7758	0.4762
Radar rainfall excluding zeroes (scaling)	11.5901	5.0755	2.0550	1.3600	0.7760	0.4762
(% error between scaling and optimal)	(0.05%)	(0.03%)	(0.03%)	(0.10%)	(0.03%)	—
Radar rainfall excluding zeroes (24-h parameter)	11.7100	5.1100	2.0703	1.3633	0.7780	0.4762
(% error between 24-h parameter and optimal)	(1.08%)	(0.71%)	(0.77%)	(0.34%)	(0.28%)	—

	Time resolution					
(b) MSE $(\mathrm{mm\ h^{-1}})^2$	1 h	2 h	4 h	6 h	12 h	24 h
Maximum radar rainfall (optimal)	167.695	89.512	19.222	7.963	5.309	1.496
Maximum radar rainfall (scaling)	167.949	90.332	19.359	7.963	5.350	1.496
(% error between scaling and optimal)	(0.15%)	(0.92%)	(0.71%)	(0.00%)	(0.77%)	—
Maximum radar rainfall (24-h parameter)	180.257	96.681	20.229	8.008	5.400	1.496
(% error between 24-h parameter and optimal)	(7.49%)	(8.01%)	(5.24%)	(0.57%)	(1.71%)	—
Radar rainfall excluding zeroes (optimal)	0.7741	0.5893	6.3236	0.9900	0.5708	0.2431
Radar rainfall excluding zeroes (scaling)	0.7742	0.5893	6.3240	0.9900	0.5723	0.2431
(% error between scaling and optimal)	(0.01%)	(0.00%)	(0.01%)	(0.00%)	(0.26%)	—
Radar rainfall excluding zeroes (24-h parameter)	0.7789	0.5907	6.3330	0.9974	0.5750	0.2431
(% error between 24-h parameter and optimal)	(0.62%)	(0.24%)	(0.15%)	(0.75%)	(0.74%)	—

	Time resolution					
(c) MSE $(\mathrm{mm\ h^{-1}})^2$	1 h	2 h	4 h	6 h	12 h	24 h
Maximum radar rainfall (optimal)	376.737	104.925	24.499	22.215	6.483	2.407
Maximum radar rainfall (scaling)	376.899	105.030	24.529	22.236	6.493	2.407
(% error between scaling and optimal)	(0.04%)	(0.10%)	(0.12%)	(0.09%)	(0.15%)	—
Maximum radar rainfall (24-h parameter)	387.450	107.049	25.189	22.889	6.503	2.407
(% error between 24-h parameter and optimal)	(2.84%)	(2.02%)	(2.82%)	(3.03%)	(0.31%)	—
Radar rainfall excluding zeroes (optimal)	95.4651	28.6070	8.6592	3.3065	1.3842	0.3518
Radar rainfall excluding zeroes (scaling)	95.4861	28.6096	8.6597	3.3069	1.3842	0.3518
(% error between scaling and optimal)	(0.02%)	(0.01%)	(0.01%)	(0.01%)	(0.00%)	—
Radar rainfall excluding zeroes (24-h parameter)	95.7830	28.6354	8.6795	3.3177	1.3852	0.3518
(% error between 24-h parameter and optimal)	(0.33%)	(0.10%)	(0.23%)	(0.34%)	(0.07%)	—

4) Attenuation might have some effects on the climatological scaling component for C-band radar at far range from the radar site if no corrections are made for attenuation in the data; however, the scaling hypothesis appears to be valid for the S-band radar.
5) The proposed scaling transformation equation with scaling exponent 0.055 exhibits significant improvements in estimating extreme rainfall, especially at fine temporal resolutions, in contrast to the accuracy obtained when using the A parameter, which is based on 24-h rainfall. For the extreme rainfall, this accuracy decreases as one proceeds to higher resolutions.
6) The proposed scaling relationship is consistent across multiple locations but exhibits variations with range in radars where attenuation is a significant issue. The use of this relationship is promising, especially in locations with limited short-duration rain gauge measurements and attenuation-corrected radar measurements.

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REFERENCES

Atlas, D., 1964: Advances in radar meteorology. *Advances in Geophysics,* Vol. 10, Academic Press, 318–478.

Austin, P., 1987: Relation between measured radar reflectivity and surface rainfall. *Mon. Wea. Rev.,* **115,** 1053–1071.

Battan, L. J., 1973: *Radar Observation of the Atmosphere*. The University of Chicago Press, 324 pp.

Burrows, D. R., and S. S. Attwood, 1949: *Radio Wave Propagation*. Academic Press, 219 pp.

Chumchean, S., A. Sharma, and A. W. Seed, 2003: Radar rainfall error variance and its impact on radar rainfall calibration. *J. Phys. Chem. Earth,* **28,** 27–39.

——, A. Seed, and A. Sharma, 2004: Application of scaling in radar reflectivity for correcting range-dependent bias in climatological radar rainfall estimates. *J. Atmos. Oceanic Technol.,* **21,** 1545–1555.

——, ——, and ——, 2006a: Correcting of real-time radar rainfall bias using a Kalman filtering approach. *J. Hydrol.,* **317,** 123–137.

——, A. Sharma, and A. Seed, 2006b: An integrated approach to error correction for real-time radar-rainfall estimation. *J. Atmos. Oceanic Technol.,* **23,** 67–79.

——, ——, and ——, 2008: An operational approach for classifying storms in real-time radar rainfall estimates. *J. Hydrol.,* **363,** 1–17, doi:10.1016/j.jhydrol.2008.09.005.

Collier, C. G., 1996: *Applications of Weather Radar Systems: A Guide to Uses of Radar in Meteorology and Hydrology*. John Wiley & Sons, 383 pp.

Collinge, V. K., and C. Kirby, 1987: *Weather Radar and Flood Forecasting*. John Wiley & Sons, 296 pp.

Doelling, I. G., J. Joss, and J. Riedl, 1998: Systematic variations of *Z-R* relationships from drop size distributions measured in northern Germany during seven years. *Atmos. Res.,* **47–48,** 635–649.

Doviak, R. J., and D. S. Zrnic, 1992: *Doppler Radar and Weather Observation*. Acadamic Press, 545 pp.

Fabry, F., A. Bellon, M. R. Duncan, and G. L. Austin, 1994: High resolution rainfall measurements by radar for very small basins: The sampling problem reexamined. *J. Hydrol.,* **116,** 415–428.

Fulton, R. A., J. P. Breidenbach, D.-J. Seo, D. A. Miller, and T. O'Brannon, 1998: The WSD-88D rainfall algorithm. *Wea. Forecasting,* **13,** 377–395.

Gupta, V. K., and E. C. Waymire, 1990: Multiscaling properties of spatial rainfall and river flow distribution. *J. Geophys. Res.,* **95,** 1999–2009.

Hagen, M., and S. E. Yuter, 2003: Relations between radar reflectivity, liquid-water content, and rainfall rate during the MAP SOP. *Quart. J. Roy. Meteor. Soc.,* **129,** 477–494.

Hildebrand, P. H., 1978: Iterative correction for attenuation of 5 cm radar in rain. *J. Appl. Meteor.,* **17,** 508–514.

Hitchfeld, W., and J. Bordan, 1954: Errors inherent in the radar measurement of rainfall at attenuating wavelengths. *J. Meteor.,* **11,** 58–67.

Joss, J., and A. Waldvogel, 1970: A method to improve the accuracy of radar measured amounts of precipitation. Preprints, *14th Radar Meteorology Conf.,* Tucson, AZ, Amer. Meteor. Soc., 237–238.

Klazura, G. E., 1981: Differences between some radar-rainfall estimation procedures in a high rain rate gradient storm. *J. Appl. Meteor.,* **20,** 1376–1380.

Krajewski, W. F., and J. A. Smith, 2002: Radar hydrology: Rainfall estimation. *Adv. Water Resour.,* **25,** 1387–1394.

Mandelbrot, B. B., 1982: *The Fractal Geometry of Nature*. W. H. Freeman, 460 pp.

Mapiam, P. P., and N. Sriwongsitanon, 2008: Climatological *Z-R* relationship for radar rainfall estimation in the upper Ping river basin. *ScienceAsia,* **34,** 215–222.

Marshall, J. S., and W. M. Palmer, 1948: The distribution of raindrops with size. *J. Meteor.,* **5,** 165–166.

Menabde, M., A. Seed, and G. Pegram, 1999: A simple scaling model for extreme rainfall. *Water Resour. Res.,* **35,** 335–339.

Rinehart, R. E., 1991: *Radar for Meteorologists*. University of North Dakota Press, 333 pp.

Russo, F., F. Napolitano, and E. Gorgucci, 2005: Rainfall monitoring systems over an urban area: The city of Rome. *Hydrol. Processes,* **19,** 1007–1019.

Seed, A. W., J. Nicol, G. L. Austin, C. D. Stow, and S. G. Bradley, 1996: The impact of radar and raingauge sampling errors when calibrating a weather radar. *Meteor. Appl.,* **3,** 43–52.

——, L. Siriwardena, X. Sun, P. Jordan, and J. Elliott, 2002: On the calibration of Australian weather radars. Australian Bureau of Meteorology Tech. Rep. 02/7, 40 pp.

Sivakumar, B., and A. Sharma, 2007: A cascade approach to continuous rainfall generation at point locations. *Stochastic Environ. Res. Risk Assess.,* **22,** 451–459, doi:10.1007/s00477-007-0145-y.

Smith, J. A., and W. F. Krajewski, 1993: A modeling study of rainfall rate–reflectivity relationships. *Water Resour. Res.,* **29,** 2505–2514.

Smith, P. L., Jr., D. E. Cain, and A. S. Dennis, 1975: Derivation of an *R–Z* relationship by computer optimization and its use in measuring daily areal rainfall. Preprints, *16th Radar Meteorology Conf.*, Houston, TX, Amer. Meteor. Soc., 461–466.

Steiner, M., and J. A. Smith, 2000: Reflectivity, rain rate, and kinetic energy flux relationships based on raindrop spectra. *J. Appl. Meteor.,* **39,** 1923–1940.

——, R. A. Houze, and S. E. Yuter, 1995: Climatological characterization of three-dimensional storm structure from operational radar and rain gauge data. *J. Appl. Meteor.,* **34,** 1978–2007.

——, J. Smith, S. J. Burges, C. V. Alonso, and R. W. Darden, 1999: Effect of bias adjustment and rain gauge data quality control on radar rainfall estimation. *Water Resour. Res.,* **35,** 2487–2503.

Sun, X., R. G. Mein, T. D. Keenan, and J. F. Elliott, 2000: Flood estimation using radar and raingauge data. *J. Hydrol.,* **239,** 4–18.

Uijlenhoet, R., 2001: Raindrop size distributions and radar reflectivity-rain rate relationships for radar hydrology. *Hydrol. Earth Syst. Sci.,* **5,** 615–627.

Ulbrich, C. W., 1983: Natural variations in the analytical form of the raindrop size distribution. *J. Climate Appl. Meteor.,* **22,** 1764–1775.

Vieux, B. E., 2003: Combined use of radar and gauge measurements for flood forecasting using a physics-based distributed hydrologic model. *Flood Warning Systems, Technologies, and Preparedness Fifth National Conf. and 14th Conf. of the Southwestern Association of Alert Systems,* Dallas, TX, National Hydrologic Warning Council.

Wilson, J. W., and E. A. Brandes, 1979: Radar measurement of rainfall—A summary. *Bull. Amer. Meteor. Soc.,* **60,** 1048–1058.