Three-Apogee 16-h Highly Elliptical Orbit as Optimal Choice for Continuous Meteorological Imaging of Polar Regions

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ABSTRACT

A highly elliptical orbit (HEO) with a 16-h period is proposed for continuous meteorological imaging of polar regions from a two-satellite constellation. This orbit is characterized by three apogees (TAP) separated by 120°. The two satellites are 8 h apart, with repeatable ground track in the course of 2 days. Advantages are highlighted in comparison to the Molniya 12-h orbit described in detail in a previous study (Trishchenko and Garand). Orbital parameters (period, eccentricity, and inclination) are obtained as a result of an optimization process. The principles of orbit optimization are based on the following four key requirements: spatial resolution (apogee height), the altitude of crossing the trapped proton region at the equator (minimization of radiation dose caused by trapped protons), imaging time over the polar regions, and the stability of the orbit, which is mostly defined by the rotation of perigee. The interplay between these requirements points to a 16-h period with an eccentricity of 0.55 as the optimum solution. The practical range of orbit inclinations that could be maintained during the spacecraft lifetime can vary from a critical value of 63.435° to 70° (subject to the amount of propellant available for orbital maneuvers). In comparison to Molniya, this type of orbit reduces the radiation exposure to high-energy protons by a factor of 10³–10⁴. On the other hand, the main advantage of 16 h versus longer orbital periods up to 24 h is better spatial resolution as a result of a lower apogee height. A two-satellite TAP constellation with an orbital inclination of 66° provides 100% temporal coverage above 60°N, >95% above 55°N, >85% above 50°N, and >75% above 45°N.

1. Introduction

It has been previously shown that the two-satellite system on a Molniya-type highly elliptical orbit (HEO) with a 12-h period can provide continuous observations of the Arctic region above approximately 58° and 38°N for the maximum viewing zenith angles of 70° and 90°, respectively (Trishchenko and Garand 2011). In other words, one or two satellites are seen above the horizon (at 0° elevation) or at 20° elevation at any particular point in time within latitude circles of 38° or 58°N, respectively. The Molniya orbit is a popular choice for communication satellite systems as well as a number of classified missions aimed at continuous surveillance of high latitudes (Chobotov 1991).

The idea of HEO satellite constellation has been recently endorsed by the World Meteorological Organization (WMO) as part of its “Vision for the Global Observing System (GOS) in 2025” adopted by the sixty-first session of the WMO Executive Council (EC-LXI; WMO 2009). According to this concept, the HEO constellation is required to complement the constellation of geostationary (GEO) satellites to achieve continuous global coverage at any time.

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The baseline scenario for HEO satellite constellation so far was mostly based on the Molniya 12-h orbit that has some very unique features.

The basic properties of highly elliptical Molniya orbit are discussed in Kidder and Vonder Haar (1990). Trishchenko and Garand (2011) have extended this study to describe in details the spatial and temporal sampling of polar regions and trade-off space for selecting an effective two-satellite Molniya mission configuration. It has been shown that the Molniya orbit with eccentricity of 0.73–0.74 (and perigee height of 500–800 km) provides imaging geometry resembling quasi-geostationary conditions very well. Advantages of the Molniya orbit include repeatable ground track (RGT) and the stable position of its apogees resulting from an orbit inclination $i = 63.435^\circ$, called the critical inclination. During the time interval of $\pm 3$ h around apogee, the satellite on Molniya orbit stays above 55$^\circ$N with an altitude in the range of 31 000–39 800 km and ground speed $<0.2 \text{ km s}^{-1}$. This orbit offers an excellent imaging capability over the polar regions, similar to GEO satellites; however, this is at the expense of changing altitude and speed relative to the earth.

A significant drawback of the Molniya orbit is the potentially hazardous radiation environment, because the satellite crosses a peak density region occupied by high-energy trapped protons. The importance of this issue for meteorological imaging payload is emphasized by Campbell et al. (2003), who investigated the optical image noise at HEO orbits resulting from proton radiation. Although the Molniya orbit crosses electron and proton radiation belts, it appears clear that the main radiation effects on detectors and electronics are caused by energetic protons (Blake 1992).

The purpose of this paper is to investigate how the HEO orbit could be optimized to reduce the radiation dose without compromising spatial and temporal coverage of the polar regions. The general outline of the elliptical orbit and the definition of the orbital elements are shown in Fig. 1. Several key principles of the optimization approach are formulated below. The main factors include requirements for the radiation conditions, minimal acceptable spatial resolution, orbit maintenance (drift of the apogee), imaging time over the region of interest, and number of satellites. It is demonstrated that an orbit with a 16-h period and an eccentricity around 0.55 represents a solution that meets all key criteria very well. This 16-h orbit has a repeatable ground track that is completed in 48 h with three apogees separated by 120$^\circ$. Because of this symmetry, the constellation of two satellites in one orbital plane 8 h apart will have the same ground track for both satellites. This represents a unique feature of 16-h orbits, because for other orbital periods the two satellites would have to be in differing planes to have the same ground track. The term three apogee (TAP) orbit is proposed for this HEO satellite constellation. It is currently considered as an alternative scenario to a classical Molniya orbit for implementation of the Canadian Polar Communication and Weather (PCW) system that is tentatively scheduled to begin operations in 2017. Combined with a constellation of geostationary satellites, such a system can provide continuous observations over the Northern Hemisphere and for the entire globe, if complemented by a similar system for the Southern Hemisphere, thus contributing to the WMO strategic vision regarding the development of a global satellite observing system.

The paper is organized as follows. Section 2 discusses optimization criteria for selecting the HEO constellation. Section 3 presents results of analysis for satellite dwelling time over the polar region for HEO. Section 4 discusses the impact of spatial resolution on the orbit selection. Section 5 analyzes the radiation environment and formulates limitations on the orbital parameters. Section 6 discusses issues related to coverage of the polar region and compares results for the TAP orbit with alternatives, such as the Molniya and a 24-h orbit (a modified Tundra orbit). Section 7 provides further analysis of viewing geometry and other conditions during the imaging period. Section 8 concludes the study. Technical details for the derivation of dwelling time over the polar regions are provided in appendix A.

2. Optimization approach for selecting HEO constellation

Key requirements for orbit optimization approach are presented below. They are numbered from R1 to R4.

R1) A two-satellite system flying on optimized orbit should provide Arctic zone coverage as follows:

![Graphical layout of the elliptical orbit and definition of Keplerian orbital elements.](image-url)
close to 100% above 60°N, >90% above 55°N, >80% above 50°N. These numbers are established based on Molniya orbit sampling properties (Trishchenko and Garand 2011). This requirement is closely linked to the satellite dwelling time over the polar region. It is covered in detail in section 3. The derivation of corresponding equations is provided in appendix A.

R2) The altitude of apogee should be below 45 000 km to ensure adequate spatial resolution with current imaging technology. The threshold value of 45 000 km, which is 25% higher than imaging altitude for geosynchronous satellites (approximately 36 000 km), is selected.

R3) Optimized orbit should avoid the region containing trapped high-energy protons with energy $E > 10$ MeV located near equator. Detailed analysis of this factor is presented in section 5. Here we provide the final conclusion, which is translated into the altitude of semilatus rectum of the optimized orbit above the earth’s surface >15 000 km. It should be understood that any HEO orbit can still be subject to impacts by high-energy solar and cosmic particles resulting from open magnetic lines in the polar region.

R4) The orbit should ensure a stable location of apogees and be easy to maintain.

Some other qualitative factors include good data reception from a single satellite ground receiving station located in Canada and small spacecraft’s speed relative to the earth’s surface during the imaging period to provide quasi-geostationary viewing conditions.

3. Orbital parameters and satellite dwelling time over the polar region

Figure 1 presents the general graphic outline of elliptical orbit and definition of Keplerian orbital elements. It corresponds to the inertial coordinate system (Duboshin 1976; Vallado 2007). Point O denotes the center of the earth. Axis $z$ is directed to the north along the earth’s spin axis. Axis $x$ is directed to the vernal equinox point. The perigee point corresponds to the closest point to the earth’s center; the apogee point corresponds to the farthest point. The distance from the earth’s center to the perigee point is equal to $a(1 - e)$; the distance from the earth’s center to the apogee point is equal to $a(1 + e)$, where $a$ is the semimajor axis; and $e$ is the eccentricity of the orbit. The argument of perigee $\omega$ is the angle between the direction to perigee and the direction from the earth’s center to the ascending node. The angle between direction to the ascending node and the $x$ axis is called the right ascension of ascending node (RAAN) and is denoted as $\Omega$. The position of the satellite on the orbit is determined by the angle $v$, which is called the true anomaly. The angle $i$ between the orbital plane and the equatorial plane is called the orbit inclination. The highest latitude at apogee is achieved when the argument of perigee $\omega = 270^\circ$. In such a case the semilatus rectum for the orbit is equal to the distance from the earth’s center to the ascending node in the equatorial plane denoted in Fig. 1 as $l$.

The dwelling time over the polar region for the satellites moving on circular orbits, that is, with eccentricity $e \approx 0$, is directly proportional to the angular size of the arc encompassing the part of the orbit located in the area of interest, for example, above latitude 45°N. This occurs because of the nearly constant magnitude of the velocity vector. The proportion of time that the satellite spends over the polar region can be higher if the eccentricity of the orbit increases. To good approximation, one can consider the orbit as an ellipse. In such a case, the proportion of time over a particular latitude range can be easily derived from Kepler’s equation. If we denote $P_\varphi$ as the ratio of time interval that the satellite stays above the latitude $\varphi$ to the total period of rotation, then

$$P_\varphi = 1 - \frac{E_\varphi - e \sin E_\varphi}{\pi},$$

(1)

where

$$E_\varphi = 2 \arctan \sqrt{\frac{(1 - e)(\sin i + \sin \varphi)}{(1 + e)(\sin i - \sin \varphi)}},$$

(2)

where $E$ is the eccentric anomaly. The derivation of Eqs. (1)–(2) is provided in appendix A (and the geometry is illustrated in Fig. A1). It is assumed that the argument of
perigee $\omega$ for the elliptical orbit is equal to 270° in order to place the apogee at the highest latitude, such that the true anomaly $v$ for the satellite crossing the equatorial plane is equal to $\pm 90°$.

Some results showing values of $P_u$ for various cases are presented in Fig. 2. It displays a set of curves for different values of inclination $i$ equal to 63.435° (critical value), 70°, 80°, and 90°; and values of latitude $\phi$ equal to 30°, 45°, and 60°. Curves are plotted as a function of eccentricity $e$ to emphasize its role on dwelling time. The eccentricity has a very significant impact on the percentage of time the satellite spends at high latitudes. In general, it is required to have significant eccentricity ($e > 0.5$) to employ the HEO advantage and to ensure adequate dwelling time over the high-latitude region. For example, the satellite on orbit with $e > 0.50$ spends $>50\%$ time above 45°N. For the circular orbit, this time will be below 25%, that is, at least twice smaller. It almost triples when eccentricity increases to 0.75. By increasing the inclination $i$, one can partially compensate for the effect of smaller eccentricity. The relative impact is highest when inclination increases from the critical value to 70°. The difference between 80° and 90° inclinations has fairly little effect on dwelling time.

Of course, the altitude of the orbit also impacts coverage: the higher the altitude, the wider the coverage, although this effect is nonlinear and eventually saturates. High-latitude coverage characteristics (requirement R1) can be confirmed once the orbit parameters are defined. At this stage, one can generally conclude that eccentricity $e > 0.5$ is desirable to take advantage of HEO observations in terms of coverage and viewing geometry.

4. Spatial resolution and altitude limit

Requirement R2, restricting the range of altitudes, can be converted directly into the valid range of values for the orbit semimajor axis, as described by

$$H_a = a(1 + e) - R_E < 45\,000\,\text{km},$$

where $H_a$ is the altitude of the apogee point above the earth’s surface, $a$ is a semimajor axis, and $R_E$ is the earth’s radius selected for simplicity to be a constant equal to 6371 km. The nominal orbital period $T$ is directly linked to the semimajor axis as

$$T = \frac{2\pi a^{1.5}}{\sqrt{GM}},$$

where $GM = 3.986\times10^{14}\,\text{m}^3\,\text{s}^{-2}$ is the standard gravitational parameter for the earth.

The graphical representation of expression (3) is displayed in Fig. 3. Figure 3a shows results for the semimajor axis. Figure 3b shows results for the orbital period. Results are shown as a function of eccentricity. Dashed horizontal lines on both panels mark the 24-h orbit. The hatched area shows the valid range of parameters that meet requirement R2. One can see that requirement of the highest altitude being less than 45 000 km for a 24-h orbit limits the eccentricity to $e < 0.22$. The condition $e > 0.50$ can only be satisfied by the orbits with period $T \approx 18$ h. An eccentricity $e > 0.70$ can be achieved only by orbits with period $T < 14$ h (e.g., Molniya has a 12-h orbit). Values of eccentricity near unity are shown for pure illustration because such orbits cannot be practically implemented because the perigee reaches the earth’s surface.

5. Radiation environment: Avoidance of high-energy trapped proton region

The main region that contains a significant amount of high-energy protons is located around the equatorial plane, with peak density around the geomagnetic equator (Fig. 4). Figure 4a displays the distribution of the integral flux of trapped protons with energy $>10$ MeV along the
orbit for the Molniya two-satellite constellation proposed by Trishchenko and Garand (2011). A notable feature of the map shown in Fig. 4a is the region of high-density proton flux in the equatorial zone. An example of the average spectrum of protons and electrons along the orbit for configuration shown in Fig. 4a is presented in Fig. 4b. The Molniya orbit exposes satellites to a significant amount of highly energetic protons with energies between $10$ and $400$ MeV. The European Space Agency (ESA) Space Environment Information System (SPENVIS; online tool available at http://www.spenvis.oma.be/) has been used to produce Fig. 4 and all other results in this paper that concern the space radiation environment. The empirical models AP-8 and AE-8 developed by the National Aeronautics and Space Administration (NASA) for the trapped protons (Sawyer and Vette 1976) and electrons (Vette 1991) in the earth’s radiation belts have been used for the analysis at solar maximum conditions.

An example of altitude distribution of proton flux in the equatorial plane for protons with energies $1, 2, 5, 10$ MeV is presented in Fig. 5. Two solid vertical lines mark the location of the GEO and Molniya orbits. The GEO orbit almost does not have any protons with $E > 1$ MeV, while the Molniya orbit crosses the proton belt at the region of peak density protons with $E > 5$ MeV. The vertical dashed line at the 15 000 km altitude approximately denotes the location of minimal altitude where proton flux for energy $E > 10$ MeV diminishes to less than 10 particles per cubic centimeter per second. This is about $2 \times 10^4$ times smaller flux than for the Molniya orbit.

The above condition can be expressed in terms of the length of the semilatus rectum for the elliptical orbit as

$$l = a(1 - e^2) > 15 000 \text{ km} + R_E. \quad (5)$$

Equation (5) can be also converted into a relation between the orbital period and eccentricity using Eq. (4). Figure 6 shows this new function overlaid on top of Fig. 3b. Together, both relations expressed by Eqs. (3) and (5) limit the valid range of the semimajor axis and eccentricity to the hatched area depicted in Fig. 6. The very remarkable feature of this figure is the existence of wedge-like region that corresponds to the optimum region of maximum eccentricity ($e = 0.55$) and orbital period ($T = 16$ h) that satisfies two key requirements R2 and R3.

This optimal solution with period $T = 16$ h represents the TAP orbit that makes three complete revolutions around the earth in 2 days. Some useful features of the 16-h orbit for communication application have been discussed by Draim (2009) and Kudielka (1997). An example of the TAP orbit ground track for two values of orbit inclination $i = 63.435^\circ$ (critical value) and $i = 70^\circ$, is shown in Fig. 7. The narrow range of meridians encompassing the ground
track is notable. To maximize the viewing of the North American landmass with close to nadir geometry, the corresponding apogee location can be selected at 95°W. This choice determines remaining apogees to be located at 25° and 145°E, which provide very good views over Eurasia.

The examples of TAP orbital parameters are provided in Table 1. They were computed using the approach proposed in Kidder and Vonder Haar (1990). To ensure repeatable ground track, the period of rotation should be properly synchronized with the earth’s rotation, the precession of the perigee, and the precession of the orbital plane. This occurs when the following condition is satisfied:

$$\pi + \omega = k(\Omega - \dot{\Omega}),$$

where $k$ is the number of orbits per sidereal day (e.g., $k = 2$ for the Molniya orbit, $k = 1.5$ for the TAP orbit, $k = 1$ for the 24-h orbit), $\Omega$ is rate of the earth’s rotation in an inertial frame equal to 7.292 115 146 7 3 10⁻² rad s⁻¹, $\dot{\omega}$ and $\dot{\Omega}$ are the rates of change for the argument of perigee and RAAN. The detailed equations are not included here; they can be found in Trishchenko and Garand (2011) and references cited therein. Parameters in Table 1 are provided for three values of inclination $i_{cr} = 63.435°, 66°$, and $70°$. Parameters for values of

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<th>Eccentricity</th>
<th>Semimajor axis $a$ (km)</th>
<th>Perigee height $H_p$ (km)</th>
<th>Apogee height $H_a$ (km)</th>
<th>Period $T$ (min)</th>
<th>RAAN rate $\dot{\Omega}$ (° yr⁻¹)</th>
<th>ECT period [days]</th>
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eccentricity varying between 0.45 and 0.65 are shown. For the reference value of eccentricity ($e = 0.55$, shown as boldface in Table 1), parameters differ very little as function of the inclination, except the RAAN rate. The semimajor axis is approximately equal to 32 175 km, the perigee altitude above the earth’s surface is 8100 km, the apogee altitude above the earth’s surface is 43 493 km, and the period of rotation is 957.29 min (or 15.955 h). The RAAN precession varies from $2^\circ$7 to $2^\circ$17 yr$^{-1}$, which corresponds the Equator Crossing Time (ECT) period between 358 and 348 days.

Figure 8 compares radiation environment conditions for the Molniya, GEO, and TAP orbits. Figure 8a displays the average spectrum of proton flux and Fig. 8b shows the total proton dose in Si after aluminum shielding. No curve for GEO is presented in Fig. 8b, because this orbit has a small concentration of protons with low energy, which are absorbed by very thin layer of aluminum shielding. One can see that the total proton flux for TAP orbit is almost entirely absorbed after approximately 7 mm of Al shielding. The total proton dose drops $10^3$ times after 3 mm of Al shielding.

6. TAP orbit and Arctic coverage

The Arctic coverage requirement R1 can be analyzed now, once all related orbital parameters are defined. The major parameters that determine spatial coverage of the polar region, as discussed in section 3 and appendix A, are semimajor axis, eccentricity, inclination, and argument of perigee. The RAAN and true (or mean) anomaly do not directly affect the coverage of the polar regions. Previous analyses provided the preferable values for semimajor axis (or period) and eccentricity. It is assumed that the argument of perigee should be equal to $270^\circ$ to place the apogee point at the highest northern latitude. It is also advantageous to have the high inclination of the orbit that increases dwelling time over the polar region. However, the issue of stability of the orbit plays an important role here, because orbits with noncritical inclination experience the systematic drift of the argument of perigee.

Figure 9 shows some basic features of TAP orbit during ±6 h around the central apogee point. Figure 9a displays the variation of altitude. The satellite stays between 34 000 and 43 500 km during ±4 h around the central apogee point. The altitude reduces to approximately 28 000 km at 5 h, and 21 000 km at 6 h. The variation in altitude leads to changes in pixel size from $+5.6\%$ at the apogee to $-17.5\%$ at ±4 h from the apogee, if the reference altitude is selected at the middle point 2 h from the apogee. Relative to GEO the pixel size for the TAP orbit is $+20.8\%$ at apogee, $+14.4\%$ at the 2-h point, and $-5.6\%$ at the 4-h point. The average pixel size for the TAP orbit is 12.1\% larger than that for the GEO orbit during the ±4 h imaging interval, and only 5.1\% larger for an imaging interval ±5 h, 20 min (i.e., for the 16 h day$^{-1}$ imaging for each satellite). Note that pixel size depends on the viewing zenith angle (VZA). For the TAP orbit this effect is similar to the Molniya orbit, and it is more significant than the effect caused by the changing satellite altitude (Trishchenko and Garand 2011). Figure 9b shows the variations of latitude and longitude for the TAP orbit at various inclinations: $i_{cr} = 63.435^\circ$, 66o, and 70o. Longitude 0o is shown for the apogee point. The satellite stays above 46°N within ±4 h around the apogee. Latitude at 5 h from the apogee is about 36°N. It reduces to below ~18°N at 6 h. At a given time from apogee, the latitude is higher if the orbit has a higher inclination. The latitude and altitude position of the satellite outside
of the ±4 h interval makes it impossible to observe the entire Arctic region within the 60°N circle. To improve Arctic coverage, an extended imaging outside of the ±4 h interval is required, which translates in mandatory simultaneous imaging from the two satellites (discussed further in this section). Figure 9c depicts ground speed at the subsatellite point. The orbit with critical inclination has the smallest ground speed. It is slightly above 30 m s⁻¹ at the apogee, reaches 200 m s⁻¹ at ±3 h, 300 m s⁻¹ at ±4 h, >400 m s⁻¹ at ±5 h, and exceeds 700 m s⁻¹ at ±6 h. The ground speeds for the orbits with inclination 66° and 70° are higher, on average, by about 15 and 40 m s⁻¹, respectively. Figure 9d displays the antenna elevation angles for the satellite receiving station located at Yellowknife (62°26’32”N, 114°23’51”W), which is selected as the potential data reception center for the PCW mission. The “best” case corresponds to the orbit apogee centrally positioned over the Yellowknife meridian. The “worst” case corresponds to the apogee located at 25°E (i.e., the central apogee is located at 95°W). The “symm” case corresponds to the apogee of the left- or right-hand side for the orbital configuration with the central apogee positioned over the Yellowknife meridian. In the worst case (i.e., for the apogee located at 25°E), the data reception is limited to within ~4.5 h from apogee.

We now examine factors affecting orbit stability to fulfill criterion R4. Gravitational forces resulting from the earth, moon, and sun have a significant influence on satellites in highly elliptical orbits, and these effects lead to long-term drift and oscillations of the argument of perigee, eccentricity, and inclination (Chobotov 1991; Kudielka 1997; Vallado 2007). The main factor that contributes to the rate of change for the argument of perigee \( \dot{\omega} \) is determined by the second zonal harmonic of the earth’s gravitational field (see, e.g., Duboshin 1976; Vallado 2007). It leads to a systematic drift described by Eq. (7) below,
where \( J_2 \) is the amplitude of the second zonal harmonic, \( r_E \) is the earth’s equatorial radius, \( n \) is the satellite mean motion. The graph computed according to Eq. (7) and showing \( \dot{\omega} \) for TAP orbit as a function of inclination is presented in Fig. 10. The absolute value of \( \dot{\omega} \) increases when the inclination deviates from the critical value \( i = 63.435^\circ \), which makes the factor \( (5 \cos^2 i - 1) \) in Eq. (7) equal to zero. It is around \( 2^\circ \) yr\(^{-1} \) for an inclination of \( 66^\circ \), slightly higher than \( 5^\circ \) yr\(^{-1} \) at \( i = 70^\circ \), near \( 9^\circ \) yr\(^{-1} \) for \( i = 75^\circ \), and reaches about \( 13^\circ \) yr\(^{-1} \) at \( i = 90^\circ \). The perturbations from various forces can increase the rate of perigee rotation depending on the relative position of the orbit, sun, and moon. Even at the critical inclination the \( i = 63.435^\circ \) perigee is undergoing the long-term oscillations as reported by Trishchenko and Garand (2011) from analysis of the orbital data for Molniya satellites. On an annual basis the perigee drift could be from \( 1^\circ \) to \( 2^\circ \). The analysis of the two-line element (TLE) orbital data for the Sirius XM satellites on the 24-h Tundra at critical inclination also reveals the existence of a perigee drift \( < 1^\circ \) yr\(^{-1} \). If not corrected, the drift of the argument of perigee from \( 270^\circ \) will lead to the reduction of coverage for polar regions resulting from the southward movement of the apogee. Therefore, orbit maintenance is a mandatory requirement for HEO Arctic observing system to ensure adequate coverage of the region of interest.

Because of the smaller value of eccentricity for the TAP orbit than for the Molniya orbit, it would be advantageous to increase the orbit inclination above the critical value to improve spatial coverage over the Arctic. As discussed above, the deviation from the critical value causes an additional systematic drift of the argument of perigee that would have to be corrected by orbital maneuvers. The amount of fuel for this maneuver is determined on the basis of required budget of additional velocity (or \( \Delta V \)) required to correct the orbit. Chobotov (1991) provides the following equation for this type of orbital maneuver:

\[
\Delta V = C \Delta \omega, \quad C = \frac{e \sqrt{GM}}{2a(1 - e^2)}, \tag{8}
\]

where \( \Delta \omega \) is the difference in argument of perigee to be corrected. It follows from Eq. (8) that the \( \Delta V \) budget is driven by two factors: the shape of the orbit (coefficient \( C \)) and the angular difference \( \Delta \omega \) in argument of perigee. For identical values of eccentricity, orbits with 16- and 24-h periods have the orbital factor \( C \) equal to 0.91 \( C_{12h} \) and 0.79 \( C_{12h} \), respectively, where \( C_{12h} \) is the orbital factor for the Molniya 12-h orbit. The absolute values of \( C \) for the Molniya 12-h orbit with eccentricity \( e = 0.74 \), the TAP 16-h orbit with eccentricity \( e = 0.55 \), and the 24-h orbit with eccentricity \( e = 0.22 \) and \( e = 0.30 \) are given in Table 2. The Molniya orbit with eccentricity \( e = 0.74 \) requires \( \Delta V = 37.2 \) m s\(^{-1} \) for a change of argument of perigee by \( 1^\circ \). The corresponding value for the TAP orbit with eccentricity \( e = 0.55 \) is \( \Delta V = 20.2 \) m s\(^{-1} \). The realistic range of \( \Delta V \) budget determines the possible range of orbit inclination. An inclination up to \( 70^\circ \) for the TAP orbit appears manageable. Other factors that can influence the final selection of orbit inclination include launch vehicle capacity, spacecraft mass, expected mission lifetime, and the efficiency and amount of available fuel for orbital maneuvers.

Zonal mean temporal coverage for various HEO configurations are compared in Table 3 for three types of orbit: TAP, Molniya, and 24 h (Tundra). Results for the TAP orbit are provided for eccentricity \( e = 0.55 \) and values of inclination \( i \) equal to 63.435\(^\circ \), 66\(^\circ \), and 70\(^\circ \). Coverage for the TAP orbit was computed assuming three imaging scenarios. In each scenario satellites are in the same orbital plane with a mean anomaly difference equal to \( 180^\circ \), that is, satellites are 8 h apart. It is assumed that each satellite can observe the entire earth disk during its imaging period.

### Table 2. The values of orbital factor \( C \) (m s\(^{-1} \)at\(^{-1} \)) that determines \( \Delta V \) budget in Eq. (8).

<table>
<thead>
<tr>
<th>Orbit</th>
<th>Eccentricity</th>
<th>Semimajor axis (km)</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molniya, 12 h</td>
<td>0.74</td>
<td>26554</td>
<td>37.2</td>
</tr>
<tr>
<td>TAP, 16 h</td>
<td>0.55</td>
<td>32175</td>
<td>20.2</td>
</tr>
<tr>
<td>24-h HEO</td>
<td>0.30</td>
<td>42163</td>
<td>8.4</td>
</tr>
<tr>
<td>24-h HEO</td>
<td>0.22</td>
<td>42163</td>
<td>6.1</td>
</tr>
</tbody>
</table>
TABLE 3. Zonal mean temporal coverage (% of the time) for various HEO and VZA. The 12- and 16-h notation refers to hours of imaging per day from each satellite in two-satellite constellation. Yk defines scenario for TAP orbit apogees centered at 95°W, 25°E, and 145°E and data reception at Yellowknife (62° 26° 32°N, 114° 23° 51°W) with antenna elevation angle. The 12- and 16-h notation refers to hours of imaging per day from each satellite in two-satellite constellation. Yk defines scenario for TAP orbit apogees centered at 95°W, 25°E, and 145°E and data reception at Yellowknife (62° 26° 32°N, 114° 23° 51°W) with antenna elevation angle.

Temporal coverage is computed as the average value of the proportion of observed time (%) during one sidereal day. Observed time corresponds to the total time when a particular point on the earth’s surface is observed by one or both satellites at a VZA < 70°. The notation of 12 h in Table 3 corresponds to the case of two satellites imaging 12 h day⁻¹, that is, with no overlap. The notation of 16 h implies that the two satellites are imaging 16 h day⁻¹; that is, there are 4 h day⁻¹ of simultaneous imaging. The scenario Yk > 5° for the case of the TAP orbit corresponds to the orbital configuration with apogees located at 95°W, 25°E, and 145°E, and data reception at Yellowknife with an antenna elevation angle > 5°. In this case imaging is not separated equally between the two satellites. The satellite that stays longer in the reception zone with an antenna elevation angle > 5° provides imagery over a longer period of time, unlike the previous two cases with 12- and 16-h imaging intervals that are equal for each satellite (with no consideration of reception capability). The goal of including the Yk > 5° scenario is to make a first assessment of the impact of the location of the ground receiving station on PCW coverage. Note that for the Molniya case, no Yk > 5° scenario is included in the Table 3, because viewing conditions from Yellowknife allow data reception for 16 h day⁻¹ or more for both satellites. Scenarios for the 24-h HEO are similar to the TAP orbit. The position of the two apogees (or location of the central meridian) for the 24-h orbit was assumed to be at 95°W and 85°E. Although the 24-h orbit meets criterion R2 only with eccentricity e = 0.22, we included two cases in Table 3 (e = 0.22 and e = 0.30) to demonstrate the possibility of better coverage from this orbit. In fact, the case e = 0.22 fails the coverage requirement R1.

No other types of HEO configuration were found to be practically useful for PCW Arctic observations, except those listed in Table 3, because of the complexity of ground track, which leads to uneven distribution and insufficient coverage on a daily basis, and unfavorable data reception conditions from Canadian territory, if the satellite dwells for a long time in the Eastern Hemisphere.

Results shown in Table 3 illustrate that imaging beyond 12 h day⁻¹ per satellite is mandatory to meet the coverage requirement R1. The limitations imposed by satellite ground station availability have no impact on the Molniya system and a very marginal impact on the TAP orbit statistics (0.3%–0.5%, i.e., a less than 10-min difference in coverage per day) above the 55°N circle. The impact of data reception is much more significant for the 24-h HEO system. It may lead to a reduction of coverage by nearly 6%, that is, close to 90-min gaps in data over a specific area. Figure 11 compares the zonal mean coverage for the above three types of orbits over the extended range of latitudes. Results are shown for the TAP orbit with 66°
and 70° inclinations, for the Molniya orbit, and for the 24-h orbit with $e = 0.22$ and $e = 0.30$. Figure 11a displays results for 16-h imaging per satellite for (a) two-satellite constellation and (b) with a restriction of antenna elevation at Yellowknife > 5°.

The spatial map of temporal coverage (% of the time) for the TAP orbit (a) at 66° inclination with two satellites operating 16-h day$^{-1}$ and for (b) the resulting percentage of available dual views.

16-h imaging scenario for each satellite and condition VZA < 70°, which is considered a practical limit for quantitative data processing. The 100% coverage zone aligns quite well with the 60°N parallel. The regions along satellite apogee central meridians located at 95°W, 25°E, 145°E show slightly better temporal coverage by approximately 5%. The coverage zone with VZA < 90° normally goes approximately 20° farther south; that is, a 100% line would move to 40°N for VZA < 90° (Trishchenko and Garand 2011). Temporal distribution of dual views presented in Fig. 12b shows that they are available approximately one-third of the time around the North Pole, and the availability decreases by a factor of 2 (i.e., to ~15%) between 60° and 65°N. The probability of dual views drops below 2% for the central apogee meridians south of 50°N.

Figure 13 provides some additional details about 24-h HEO with eccentricity 0.30, which potentially can meet the coverage requirement R1. Figures 13a–d show the distribution of altitude, ground track, ground speed, and antenna elevation angles, respectively. Comparison with similar parameters presented in Fig. 9 for the TAP orbit reveals that the 24-h HEO orbit provides less favorable imaging conditions than the TAP and Molniya HEO.
7. Analysis of viewing geometry for the TAP orbit

An important feature of HEO orbits is the diversity of viewing geometry, in contrast to polar-orbiting low Earth observing (LEO) and GEO satellites. Figures 14a,b display the map of minimum VZA and average VZA for the region above 45°N for the TAP orbit with an inclination of 66°. Near-nadir views are observed along the satellite ground track. The North Pole region can be observed approximately at 25° minimum viewing zenith angles. The spatial map of average VZA (averaged over the entire imaging period VZA < 70°) shows that it varies from 35° to slightly less than 42°. Maximum values are observed between the apogee central meridians and minimums are located along these lines.

A comparison of zonal mean values of average VZA for TAP, Molniya, and 24-h orbit is shown in Fig. 15. North of 73°N the 24-h system provides viewing at smaller VZA angles (difference < 4°). South of 70°N the Molniya and TAP orbits have smaller VZA. The difference can reach nearly 10° for the region from 45° to 52°N, thus favoring the Molniya and TAP orbits for...
various land surface applications in the Northern Hemisphere. Performance of TAP relative to the Molniya system is fairly close: on average 2° higher for the TAP orbit.

Additional insight on viewing geometry from the TAP orbit, including solar illumination conditions, is provided by Fig. 16, which shows a temporal sequence of the earth views. The red box on each image marks the location of the Yellowknife ground receiving station. The red circle denotes the 50°N parallel, and the North Pole is marked by a red dot. Images are displayed for the apogee point (0 h), ±2 h, ±4 h, and ±6 h from apogee. The angular size of the earth’s disk varies from 14.5° at apogee, to 15.3° at ±2 h, to 17.9° at ±4 h, and to 26.2° at ±6 h. The position of the solar terminator is also shown as the line separating bright section of the image and dimmed section of the image. As noted by Trishchenko and Garand (2011), the viewing geometry from the HEO system provides quite a unique opportunity to obtain diverse angular sampling to study the anisotropy of clouds, atmospheric, and surface phenomena. Unlike GEO, the HEO system allows observations of the same point at different VZA. Because of the changing position of sun, this leads to varying solar zenith angles and relative azimuth angles in a wide range during the course of 1 day. The geometry is also changing slowly from day to day resulting from motion of the earth around the sun. The total period of the 24-h rotation for the Equator Crossing Time (ECT) is close to 354 days as it follows from the results presented in Table 1.

8. Conclusions

An optimized HEO system for Arctic meteorological imaging is proposed. The optimization is based on four key requirements: Arctic coverage, spatial resolution, minimization of the proton radiation dose, and orbit maintenance. The optimal solution leads to a 16-h HEO orbit with eccentricity of 0.55. This type of orbit reduces the radiation dose resulting from high-energy protons by a factor of 10³–10⁴ in comparison to the Molniya orbit. The 16-h orbit, referred to as TAP, is characterized by three apogees separated by 120°. Analysis of orbital stability and ΔV budget required to maintain the orbit parameters and Arctic region coverage suggest that orbit inclination could range from critical value 63.43° to 70°. The TAP orbit with an inclination of 66° was analyzed in detail to demonstrate that it provides similar or even superior viewing conditions over the region of interest from 45° to 90°N relative to the Molniya orbit. This orbit has apogee height above the earth’s surface around 43 500 km, perigee near 8100 km, and semilatus rectum height more than 16 000 km. This allows the spacecraft to stay above the peak density–trapped proton region. In comparison to a GEO orbit, the pixel size would be smaller by 5.6% ±4 h from apogee, and larger by 20.8% at apogee. A two-satellite constellation in a single plane using that optimized orbit provides 100% temporal coverage above 60°N, >95% above 55°N,
>85% above 50°N, and >75% above 45°N. While the orbit selection for the PCW satellite mission is not yet definite, current understanding points to a 16-h TAP orbit as the best solution.

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APPENDIX

Derivation of Dwelling Time above Selected Latitude for Satellite on Elliptical Orbit

For the satellite moving along the elliptical orbit, the time it spends within certain latitude range can be determined analytically using Kepler’s equation that relates mean anomaly $M$ with eccentric anomaly $E$ (see, e.g., Duboshin 1976; Vallado 2007),

$$M = E - e \sin E,$$  \hspace{1cm} (A1)

where $e$ is the eccentricity of the orbit. The $M$ changes linearly with time $t$, that is, $M = nt$, where $n$ is the satellite mean motion. Therefore, the time interval between two points in time $t_2$ and $t_1$, where the satellite crosses the selected latitude $\phi$ during ascending and descending parts of the orbit, is linearly proportional to the difference between values of mean anomaly $M_2 = M(t_2)$ and $M_1 = M(t_1)$. Because one orbit corresponds to a $2\pi$ angle change for the mean anomaly, the relative proportion $P$
of satellite time it spends above certain latitude \( \phi \) is equal to

\[
P = \frac{2\pi - 2M_\phi}{2\pi} = 1 - \frac{M_\phi}{\pi}, \quad (A2)
\]

where \( M_\phi \) is the value of the mean anomaly when the satellite crosses latitude \( \phi \) during the ascending node at the time \( t_1 \). It is assumed that the argument of perigee \( \omega \) for the elliptical orbit is equal to 270° to place the apogee at the highest latitude, such that the true anomaly \( v \) for the satellite crossing the equatorial plane is equal to ±90°. Note, that mean anomaly equals \(-M_\phi\) for the moment \( t_2 \). Substituting (A2) into (A1) leads to

\[
P = 1 - \frac{E_\phi - e \sin E_\phi}{\pi}, \quad (A3)
\]

where \( E_\phi \) is the corresponding value of eccentric anomaly for the moment \( t_1 \) when the satellite crosses the latitude \( \phi \) during the ascending part of the orbit. To express \( E_\phi \) through latitude \( \phi \), one can use the relationship between eccentric anomaly \( E \) and true anomaly \( v \) (see Duboshin 1976, p. 222), and then relate \( v \) and \( \phi \) as

\[
\tan \frac{\nu}{2} = \frac{1 + e}{1 - e} \tan \frac{E}{2}. \quad (A4)
\]

If we define the intermediate angle \( \psi \) shown in Fig. A1 as

\[
\psi = v - \frac{\pi}{2}, \quad (A5)
\]

then from (A4) and (A5) it follows that

\[
\tan \frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}} \tan \frac{\nu}{2} = \sqrt{\frac{1 - e \sin \psi}{1 + e \cos \psi}} + \cos \frac{\psi}{2} \quad (A6)
\]

From the geometry shown in Fig. A1 it follows that

\[
\begin{align*}
(SC) &= R \sin \phi \\
(SC) &= (SB) \sin i, \\
(SB) &= R \sin \psi
\end{align*} \quad (A7)
\]

where parentheses denote the length of the segment. Therefore,

\[
\sin \phi = \sin \psi \sin i. \quad (A8)
\]

Using (A8) and (A5) one can obtain

\[
\cos \nu = \frac{\sin \phi}{\sin i}. \quad (A9)
\]

Using standard trigonometric relationships

\[
\begin{align*}
\cos \nu &= 1 - \tan^2 \frac{\nu}{2} = -\frac{\sin \phi}{\sin i}, \\
\sin i &= \frac{1 - \cos^2 \psi}{\sin^2 \phi} = \frac{1 - \cos^2 \psi}{\sin^2 \phi}
\end{align*} \quad (A10)
\]

From (A10) it follows that

\[
\tan \frac{\nu}{2} = \frac{\sin i + \sin \phi}{\sin i - \sin \phi}, \quad (A11)
\]

and with (A11) Eq. (A6) becomes

\[
\tan \frac{E}{2} = \sqrt{\frac{(1 - e)(\sin i + \sin \phi)}{(1 + e)(\sin i - \sin \phi)}} \quad (A12)
\]

Therefore, the value of eccentric anomaly \( E_\phi \) in (A3) is

\[
E_\phi = 2 \arctan \sqrt{\frac{(1 - e)(\sin i + \sin \phi)}{(1 + e)(\sin i - \sin \phi)}} \quad (A13)
\]

Note, that the range of valid solutions in (A13) is \( |\phi| \leq |i| \) assuming that the inclination satisfies condition

\[
-90^\circ \leq i \leq +90^\circ. \quad (A14)
\]

This means that the satellite in our case (\( \omega = 270^\circ \)) cannot reach latitudes beyond the apogee and perigee points.
REFERENCES


