ABSTRACT: Precipitation is a key process within the shallow cloud life cycle. The Cloud System Evolution in the Trades (CSET) campaign included the first deployment of a 94-GHz Doppler radar and 532-nm lidar. Despite a larger sampling volume, initial mean radar/lidar-retrieved rain rates based on the upward-pointing remote sensor datasets are systematically less than those measured by in situ precipitation probes in the cumulus regime. Subsequent retrieval improvements produce rain rates that compare better to in situ values but still underestimate them. Retrieved shallow cumulus drop sizes can remain too small and too few, with an overestimated shape parameter narrowing the raindrop size distribution too much. Three potential causes for the discrepancy are explored: the gamma functional fit to the drop size distribution, attenuation by rain and cloud water, and an underaccounting of Mie dampening of the reflectivity. A truncated exponential fit may represent the drop sizes below a showering cumulus cloud more realistically, although further work would be needed to fully evaluate the impact of a different drop size representation upon the retrieval. The rain attenuation is within the measurement uncertainty of the radar. Mie dampening of the reflectivity is shown to be significant, in contrast to previous stratocumulus campaigns with lighter rain rates, and may be difficult to constrain well with the remote measurements. An alternative approach combines an a priori determination of the drop size distribution width based on the in situ data with the mean radar Doppler velocity and reflectivity. This can produce realistic retrievals, although a more comprehensive assessment is needed to better characterize the retrieval errors.

KEYWORDS: Cumulus clouds; Marine boundary layer; Precipitation; Cloud microphysics; Drop size distribution; In situ atmospheric observations; Radars/radar observations; Remote sensing; Numerical analysis/modeling

1. Introduction

Precipitation, a pronounced feature of the marine boundary layer, can hasten the stratocumulus-to-cumulus transition within both modeling and observational studies, by encouraging thermodynamically induced decoupling within the boundary layer (Paluch and Lenschow 1991; Sandu and Stevens 2011), aerosol depletion (Yamaguchi et al. 2017; O et al. 2018), and a more pronounced mesoscale organization (Blossey et al. 2021). Since precipitation is typically a poorly modeled quantity, robust observations of precipitation remain necessary for untangling subtle cause and effect relationships using modeling studies (e.g., vanZanten et al. 2011; Blossey et al. 2021). During the Cloud System Evolution in the Trades (CSET; Albrecht et al. 2019) campaign in 2015, precipitating stratocumulus and cumulus clouds were observed by in situ probes as well as by a 94-GHz Doppler radar, and a 532-nm wavelength high spectral resolution lidar (HSRL) over the northeastern Pacific Ocean. Sarkar et al. (2020, hereafter SARKAR20), used these datasets to document the relationship of precipitation to the transition for three clear stratocumulus-to-cumulus transitions.

CSET was the first deployment upon the High-Performance Instrumented Airborne Platform for Environmental Research (HIAPER) Gulfstream V aircraft of both the HIAPER Cloud Radar (HCR) and the HSRL. The incorporation of the HCR into the campaign plan recognizes precipitation’s crucial role within the life cycle of shallow clouds. Cloud radars can sample large volumes of atmosphere and detect the large drops that are few enough to be missed by the in situ probes. Although rain, cloud liquid water and water vapor attenuate 94-GHz (3.2-mm-wavelength) radiation more than for 35-GHz (8.6-mm-wavelength) radars, the smaller 94-GHz antenna is physically more suitable for aircraft, and the shorter wavelength enhances their sensitivity to smaller drops. Aircraft cloud radars, in comparison to space-based radars (e.g., Kalmus and Lesbock 2016), also have the advantage that they can better resolve the near-surface precipitation giving rise to evaporation-driven density currents leading to further mesoscale cloud organization (Zuidema et al. 2017, and references therein). As such, 94-GHz radars have become a standard aircraft instrument for shallow cloud campaigns (Vali et al. 1998; vanZanten et al. 2005; Wood et al. 2011; Dzambo et al. 2019; Schwartz et al. 2019; Stevens et al. 2019; Pincus et al. 2021).

The CSET campaign documented the Lagrangian evolution of stratocumulus-to-cumulus clouds, including their precipitation, by sampling air parcels within the stratocumulus regime.
on the flight from California to Hawaii and resampling them approximately two days later on the way back to California. Near Hawaii, the clouds had evolved into cumuli with tops reaching approximately 2 km capable of supporting short-lived but intense rain burst events (Fig. 1). SARKAR20 document a shift toward larger drop sizes during the transition using two-dimensional cloud (2DC) probe measurements. Drops with diameters as large as 3.2 mm produced 1-s rain rates as high as 10 mm h$^{-1}$ (240 mm day$^{-1}$) during the cumulus rain bursts. This precipitation typically reached the near-surface (150-m) aircraft altitude, indicating the likelihood that cooling through evaporation and downdrafts of drier air from above could further support density-driven gust fronts.

Previous investigations of stratocumulus precipitation using 94-GHz radars have successfully applied simple power-law relationships between rain rate $R$ and radar reflectivity $Z$ ($Z$-$R$ relationships) (vanZanten et al. 2005; Wood et al. 2011). The CSET campaign understood a priori that these might not apply for the deeper cumulus clouds because larger drop sizes would invalidate the retrievals. A combined radar–lidar retrieval of precipitation was developed for the moving platform by Schwartz et al. (2019), hereafter referred to as the $(Z, \beta, v_1)$ retrieval. The retrieval based on O’Connor et al. (2005), which was originally meant to retrieve rain parameters in stratocumulus regime, may or may not be correctly applicable to deeper precipitating stratocumulus and cumulus clouds as observed during CSET. The validity of using the $(Z, \beta, v_1)$ retrieval technique in a case study from CSET is discussed in this paper. The $(Z, \beta, v_1)$ method uses the ratio of the radar reflectivity ($Z$) to the lidar backscatter intensity ($\beta$), made proportional to the fourth power of the median volume diameter ($D_o$) through a gamma-function drop size distribution assumption, to constrain the retrieved particle size.

The initial precipitation rate estimates based on the lidar/radar retrievals from the subcloud legs were an order of magnitude lower than the in situ values (Fig. 2h) with light precipitation rates not always retrieved (Fig. 2g). This is puzzling because the larger radar sampling volume should mean they are more likely to detect precipitation than the in situ probes (Wood 2005), not less. Subsequent improvements to the retrieval compare better to the in situ values, documented here in section 3, but the retrieved rain rates remain underestimated. This study seeks to identify further causes of the discrepancy. The examined hypotheses include the underlying drop size distribution assumption, rain and cloud liquid attenuation of the radar reflectivity, and Mie dampening of the radar reflectivity, contained in section 4.

An additional retrieval approach examines combining in situ information on the drop size distribution width with the radar...
reflectivity and mean Doppler velocity. The NSF/NCAR HIAPER plane, when deployed with the HCR, will almost certainly be accompanied by the in situ probes, because of plentiful space in the probe canisters and complementary applications. An example is the Organization of East Pacific Convection (OTREC) campaign held in 2019 (Fuchs-Stone et al. 2020). This is explored in section 5.

2. In situ assessment of the radar/lidar retrievals of precipitation microphysics

a. Observational datasets and retrieval methodology

The HCR is sensitive to $-39.6\, \text{dBZ}$ at a range of 1 km and can sample the boundary layer in both upward- and downward-pointing directions, although not simultaneously. The HSRL, operating at a 532-nm wavelength, can also point in either zenith or nadir directions, but not simultaneously. The two remote sensor datasets are placed on a common grid with a time resolution of 0.5 s, corresponding to a horizontal resolution of 50–100 m depending on aircraft speed, and a vertical resolution of 20 m (Schwartz et al. 2019). The most information can be derived from the remote sensors during the subcloud aircraft legs at approximately 150-m altitude, when the small distance to the low clouds reduces gaseous and liquid attenuation and the lidar can detect the cloud base independently of precipitation. The in situ data are gathered during the same subcloud legs and during the neighboring in-cloud-level legs. Both legs are typically 10 min in length, corresponding to a distance of approximately 70 km.

The 2DC optical array probes sample raindrops every second spanning diameters of 75–3200 $\mu\text{m}$, at 25-$\mu\text{m}$ resolution (126 diameter bins total). Each individual image is quality controlled to obtain the drop size capable of producing a particular image [Korolev et al. (2007)], including a visual analysis of the preliminary in-field imagery for evidence of streaking. The rain rate is
calculated using 

\[ R = (\pi/6) \sum_{n=0}^{D_i=3200\text{m}} \frac{n_i(D_i)v(D_i)\Delta D_i}{D_i}, \]

where \( D_i \) is the median diameter within size drop bin \( \Delta D_i \), and \( n(D) \) is the number of raindrops within the corresponding \( \Delta D_i \), and \( v(D) \) is the terminal velocity appropriate for \( D_i \), corrected for air density [Fang et al. 2017, Eqs. (1) and (2)].

The O’Connor–Schwartz retrieval assumes a normalized gamma functional fit for the underlying drop size distribution:

\[ N(D) = N_\infty e^{-(D/D_\infty)^\mu} \frac{(D/D_\infty)^{\mu-1}}{\Gamma(\mu)}, \]

where \( D_\infty \) is the median volume diameter, \( \mu \) is the gamma shape parameter, and \( N_\infty \) is the number concentration normalized so that the liquid water content is independent of \( \mu \). The retrieved in situ diameters are compared using the effective diameter \( D_{\text{effective}} = (\mu + 3)D_\infty/(\mu + 3.67) \). Gamma functions generally represent the oceanic raindrop size distribution well [Geoffroy et al. 2014; Duncan et al. 2019] and modified versions of it are popular within the modeling community because of a slight advantage in analytical ease compared to equally well-performing lognormal distributions (Klain et al. 2015). Reduced forms with \( \mu = 0 \), the exponential distribution, are popular within model microphysical schemes (e.g., the two schemes evaluated within Li et al. 2015) and some space-based retrievals (Duncan et al. 2019). A further modification, in which the exponential distribution is truncated at both an upper and lower bound is applied within the warm rain radar retrieval of Lesbock and L’Ecuyer (2011).

Three independent measurements (radar reflectivity \( Z \), lidar backscatter coefficient \( \beta \), and the width of the radar Doppler velocity spectrum \( \sigma_D \)) prescribe the gamma distribution. The radar reflectivities for this study have been updated since SARKAR20 to account for water vapor attenuation using in situ and dropsonde observations; we note that SARKAR20 already included a 4.5-dB calibration offset. The lidar backscatter coefficient is invoked instead of the more physically relevant lidar extinction, because, although the extinction can be measured directly by the HSRL, the requisite averaging is impractical to account for water vapor attenuation using SARKAR20. The comparisons shown within Figs. 17 and 18 of SARKAR20, from a 10-min stratocumulus subcloud leg on 17 July (named module RF06a), and its air mass resampled as precipitating cumulus on 19 July (named module RF07c), are extended with a comparison of \( \gamma’ \) for RF07c in Fig. 2. The examined retrievals are from the same aircraft subcloud leg as the in situ measurements. Due to the radar-dead zone, the first-available radar range gate is at approximately 0.1 to 1 mm (calculation not shown). The \( (Z, \beta, v) \) retrievals shown here include calibration corrections and account for gaseous attenuation, but not for cloud and rain attenuation.

b. Example comparison

The comparisons shown within Figs. 17 and 18 of SARKAR20, from a 10-min stratocumulus subcloud leg on 17 July (named module RF06a), and its air mass resampled as precipitating cumulus on 19 July (named module RF07c), are extended with a comparison of \( \gamma’ \) for RF07c in Fig. 2. The examined retrievals are from the same aircraft subcloud leg as the in situ measurements. Due to the radar-dead zone, the first-available radar range gate is at approximately 0.1 to 1 mm (calculation not shown). The \( (Z, \beta, v) \) retrievals shown here include calibration corrections and account for gaseous attenuation, but not for cloud and rain attenuation.
rain rates is considered, collision–coalescence over 200 m cannot easily explain a discrepancy of an order of magnitude [see, e.g., discussion within Li et al. (2015)].

For the more heavily precipitating cumulus cloud, the retrieved rain frequencies slightly exceed those from the in situ probes, consistent with the smaller sampling volumes of the in situ probes. In contrast, precipitation is retrieved less frequently within the stratocumulus cloud than by the in situ probes. This is attributed to an algorithmic constraint for $Z/\beta$ to exceed 30 dB in the original retrieval scheme, intended to exclude background aerosol.

c. Comparison of 10-min-averaged rain rates

Figure 3 compares the 10-min leg-mean rain rates [derived using the same protocol applied within Mohrmann et al. (2019)] to the $(Z, \beta, v_1)$ retrievals and the in situ values for the same 18 Lagrangian cases as examined within Mohrmann et al. (2019). Some air masses diverge into multiple locations, resulting in 30 cases in total. The retrieved 10-min leg-mean rain rate is constructed using the maximum rain rate within each vertical column, with the rain rate set to zero when no rain is detected. Using this averaging, Mohrmann et al. (2019) report CSET campaign-mean values of 2.2 and 0.7 mm day$^{-1}$ for the California-to-Hawaii (outgoing) and Hawaii-to-California (incoming) flights (their Table 1). The calibration and water vapor attenuation corrections increase these values to 2.8 and 1.36 mm day$^{-1}$. The lower in situ rain rates relative to the retrieved values, for rain rates < 0.01 mm day$^{-1}$, can be explained by undersampling of larger raindrops by 2DC probes. The in situ rain rates continue to exceed the retrieved values for rain rates > 0.01 mm day$^{-1}$ (Fig. 3). The average subcloud in situ precipitation rates for the CA-to-HI and HI-to-CAL flights are 6.3 and 7.9 mm day$^{-1}$, respectively. These increases by factors of 2.25 and 6 over the retrieved values also indicate a slight increase in the 10-min mean precipitation for the deeper clouds sampled closer to Hawaii. Although only reflecting a limited sample size, this also alters the perception of the hydrological cycle. When only those retrieved rain rates exceeding 0.01 mm day$^{-1}$ are considered, in situ rain rates averaged over both CA-to-HI and HI-to-CAL flights increase to 21 mm day$^{-1}$.

3. An improved $(Z, \beta, v_2)$ retrieval (version 2)

A subsequent version of the $(Z, \beta)$ retrieval, hereafter referred to as $(Z, \beta, v_2)$, incorporated several improvements, listed below.

- Improved calibration of the HCR based upon the ocean backscatter for each individual flight, along with an improved correction to the mean Doppler velocity, both done at the National Center for Atmospheric Research (NCAR). These corrections were already incorporated within SARKAR20 ($a + 4.5$ dB offset for RF07c) but are now extended to all of the flights. The calibration application includes a more thorough clutter identification applied to both the HSRL and HCR data, which also removes second trip echoes and sidelobes.
- An accounting for water vapor attenuation using in situ and dropsonde observations following Fairall et al. (2018), based on the original formulation of Liebe et al. (1989), to all flights (also already incorporated within SARKAR20 for RF07c only).

![Fig. 3. The 10-min-mean in situ rain rates within the near-surface 150-m-level legs for the same 18 Lagrangian cases examined within Mohrmann et al. (2019) vs the corresponding $(Z, \beta, v_1)$ retrieved rain rates computed following Mohrmann et al.’s (2019) algorithm, wherein the maximum retrieved rain rates within each vertical column are extracted for each 2-Hz sample and averaged for the 10 min. A one-on-one ratio line is included. The radar reflectivities have been corrected for $a + 4.5$ dB calibration offset and for water vapor attenuation, so that the remotely retrieved rain rates are slightly higher than those documented within Mohrmann et al. (2019). Some air masses of the nominal 18 Lagrangian cases diverge into two or more locations, resulting in 30 samples total.]

- A clearer articulation of the $\gamma$ using smaller increments in the modal diameter and shape parameter, leading to a better match to Fig. 3 in O’Connor et al. (2005).
- A normalized drizzle number concentration $N_d$ calculated from the measured radar reflectivity after accounting for Mie-to-Rayleigh reflectivity ratio $\gamma$ [missing in Eq. (5) within Schwartz et al. (2019)] according to

$$N_d = \gamma Z \frac{(3.67 + \mu_g)^{2+\mu_s}}{2DV_{\text{d}}(7 + \mu_g)}$$  \hspace{1cm} (4)

- to which the total number concentration $N_t$ corresponds to as

$$N_t = \frac{N_d}{2DV_{\text{d}}} \frac{(\mu_g + 1)}{(3.67 + \mu_g)^{2+\mu_s}},$$  \hspace{1cm} (5)

$$f = \frac{6}{3.67^4} \frac{(3.67 + \mu_g)^{4+\mu_s}}{\Gamma(\mu_g + 4)},$$  \hspace{1cm} (6)

- The contribution of the aircraft motion to the width of the Doppler spectra (contained within the turbulence term $\sigma_r$) is now the moving average based on approximately 20 samples of the aircraft speed (a distance of 1–2 km, better matching the boundary layer depth), as opposed to one only [Eq. (1) of Schwartz et al. 2019]. This recognizes the approximate scaling of the turbulence contribution with the boundary layer depth.
A mistake in the range gate start value was rectified, resulting in a reduced radar "dead zone" of 300 m, from a previous 400 m. The first usable height remains the dead zone + the GV altitude.

The improved retrievals \((Z, \beta, v_2)\) produce rain rates that are higher than those from the original \((Z, \beta, v_1)\) values, though still less than the in situ values (Fig. 4a). The \((Z, \beta, v_2)\) median rain rate is 16 mm day\(^{-1}\) for the same subcloud RF07c leg shown in Fig. 2. The comparable median rain rate is 2 mm day\(^{-1}\) for the \((Z, \beta, v_1)\) retrieval, and 44 mm day\(^{-1}\) based on the in situ data. The raindrop number concentration also matches the in situ measurements more closely than the original retrieval (Fig. 4b), if still biased low. However, the retrieved \(D_{\text{eff}}\) has decreased significantly, worsening the comparison (Fig. 4c). Consistent with the \(D_{\text{eff}}\) underestimate, \(g_0\) remains overestimated (Fig. 4d). The retrieved shape parameter is too large (Fig. 4e), overly narrowing the raindrop size distribution compared to the in situ size distribution.

4. Potential causes for the retrieval bias

The improvements incorporated into \((Z, \beta, v_2)\) produce improved rain rates, but systematic biases remain. Here we discuss three potential reasons: 1) the underlying assumption of a gamma distribution, 2) attenuation by rain and liquid, and 3) difficulty in fully incorporating \(\gamma'\) within the iterative retrieval.

a. Assessment of the gamma functional fit

Gamma distribution fits are applied to the in-cloud and subcloud drop size distributions of both stratocumulus and cumulus clouds of four to six cloud modules (Fig. 5, also examined in Fig. 12 of SARKAR20). Inverse exponential fits truncated at 76 \(\mu\text{m}\) and 3.2 mm are also shown. The gamma functions adequately represent each of the stratocumulus drop size distribution, and the in-cloud cumulus drop size distribution. The median volume diameter \(D_0\) increases from 0.5 mm within stratocumulus to 0.8 mm below the cloud. \(D_0\) within cumulus is larger at 1.1 mm and increases to 1.3 mm below the cloud. The shape parameter \(\mu_g\) is more negative for stratocumulus (-1.9 at 150 m and -2.5 at in-cloud level) than for the cumulus clouds (0 at 150 m and -1.2 at in-cloud level), indicating a shift to a narrower size distribution centered at bigger drop sizes for cumulus. This is consistent with Geoffroy et al. (2014), who derived \(\mu_g\) (their \(v - 1\)) values of 0–2 from in situ microphysical measurements from the Rain in Cumulus over Ocean campaign (Rauber et al. 2007; Zuidema et al. 2012b). The gamma fit to the subcloud cumulus cloud drop size distribution, the condition with the most intense shallow rain, is weaker than for the other three cases. Here, the truncated exponential fit performs best. This suggests the truncated exponential might produce larger rain rates and \(D_{\text{eff}}\) values overall, if
applied within the retrieval, and supports its application for the
global CloudSat precipitation retrievals (Lebsock and L’Ecu-
yer 2011). The lognormal fit [also applied within vanZanten et al.
(2005)] is also adequate and arguably provides the best com-
promise for depicting both light and more intense shallow
precipitation.

b. Assessment of attenuation from cloud and rain

Another hypothesis for the retrieval bias within cumulus
clouds is a lack of correction for radar attenuation by liquid
and rainwater, potentially reducing radar sensitivity to the scat-
tering by the target of interest (Lhermitte 1989; Pujol et al.
2006; Matrosov 2007; Chandra et al. 2015; Fairall et al. 2018;
Oh et al. 2020). The attenuation can be notable for millimeter-
wave radars (Matrosov 2007), and downward-looking W-band
radars often show a reduction in reflectivity near cloud
bases from either attenuation, or raindrop evaporation, cre-
a ting an ambiguity (Mason et al. 2017). The path-integrated
attenuation is large enough to underpin its own rain-rate re-
trieval (L’Ecu yer and Stephens 2002; Fairall et al. 2018).

Effects of attenuation by both cloud liquid and rain are ana-
lyzed. Although a precise approach for estimating cloud liquid
attenuation would require a dual-wavelength radar (e.g.,
Hogan et al. 2005), an attenuation estimate can be made for the cumulus
case shown here using the mean in-cloud liquid water content and
mean cloud thickness. The two-way liquid water attenuation co-
efficient in dB km$^{-1}$ is proportional to the cloud liquid water
content (LWC) (Atlas and Ulbrich 1977; Pujol et al.
2006):
\[
a_{\text{cloud}} = 2 \times 0.4343 \int_{D_{\text{min}}}^{D_{\text{max}}} N(D) Q_t(D) dD = k \times \text{LWC}
\]
where $Q_t(D)$ is the total extinction cross section of the cloud
drops. $Q_t(D)$ is estimated using the Mätzler (2002) algorithm
and Lhermitte (1989), for a radar wavelength of 3.2 mm and an
ambient temperature of 20°C. The leg-mean $k$ for the RF07c in-
cloud leg is 7.5 dB km$^{-1}$ (g m$^{-3}$)$^{-1}$; multiplied by the leg-mean
LWC of 1.5 $\times$ 10$^{-3}$ g m$^{-3}$, this produces an attenuation of
0.01 dB km$^{-1}$. For a 1.5-km-thick cloud as RF07c, the two-way

![Figure 5](image-url)
The rain attenuation is larger, but nevertheless remains on par with the uncertainty of 1–2 dB associated with the radar reflectivity (Romatschke et al. 2021). This is demonstrated through an estimation based on the vertical profiles of radar reflectivity. For the subcloud legs with an upward-viewing radar, any reduction in reflectivity between the radar and the altitude of maximum reflectivity (typically below 400-m altitude) is assumed to be from raindrop evaporation only. Reduction in the reflectivity above this altitude is assumed to be from signal attenuation. An attenuation correction is only applied when the column maximum reflectivity exceeds 10 dBz and the Doppler velocity is higher than 1 m s⁻¹, at or below 1-km altitude. These thresholds are determined from the 1-Hz in situ rain rates greater than 1 mm h⁻¹, but are similar to those applied within Chandra et al. (2015) and Oh et al. (2020); 70% (257 out of 371) of the radar profiles for RF07c satisfied this criteria. The two-way attenuation is thereafter calculated following Fairall et al. (2018): \( Z_r(h) = Z_{raw}(h) \exp[0.2 \ln(10)] \int_0^h \gamma_{rain}(s) ds \), where \( Z_r(h) \) and \( Z_{raw}(h) \) are the corrected and attenuated radar reflectivity (mm⁶ m⁻³), respectively, at altitude \( h \). \( \gamma_{rain} \), the gradient of the reflectivity (dBZ km⁻¹), defines the attenuation by rain at distance \( s \) from the radar, over a length of \( ds \), through \( \gamma_{rain} = 0.5[dBZ_{raw}]/dh \); \( h \) and \( s \) differ primarily by the distance of the radar “dead zone.”

For the RF07c cumulus example shown here, the attenuation correction increases the reflectivity by 0.5–1 dBZ in the lower 1 km of the cloud, and by 1–3 dBZ near the cloud tops (not shown). The RF07c cumulus module was selected in part because of its intense rain shower. These attenuation values represent upper limits on the potential impact from rain attenuation upon the measured radar reflectivity. Given the radar reflectivity is uncertain by 1–2 dB, attenuation by rain can be ruled out as a significant contributor to the retrieval bias, particularly below the cloud.

### Underacknowledgement of Mie dampening within the \((Z, \beta)\) retrieval

The need for an additional constraint on particle size is a key motivation for incorporating the independent lidar backscatter measurement into the \((Z, \beta)\) retrievals; the Mie-induced dampening (Mie 1908) of the reflectivity \( Z \) for the larger raindrops may dominate the rain rates. Figures 6 and 7 reinforce the inordinate impact of the frequently occurring large raindrops (>200 \( \mu \)m) upon the radar reflectivity. A neglect of Mie scattering in an algorithm. This contrasts with some previous work on stratocumulus clouds, for which traditional \( R = aZ^b \) relationships can increase the sensitivity of reflectivity-derived rain-rate estimates to any errors in the radar reflectivity (Fig. 6), and underestimate rain rates by 50%–90% for drops larger than 200 \( \mu \)m [Fig. 7; calculated using the Mätzler (2002) algorithm]. This contrasts with some previous work on stratocumulus clouds, for which traditional \( R = aZ^b \) relationships, with \( a \) and \( b \) empirically determined using microphysical datasets, work well (vanZanten et al. 2005; Wood et al. 2011). For those campaigns, the stratocumulus rain rates were much lighter, at approximately 0.5 mm day⁻¹, than those measured during CSET.

The O’Connor et al. (2005) retrieval of rain rate was originally intended for the stratocumulus regime, for which Mie dampening of the reflectivity is typically not significant. Perhaps because of the original application, the \( \gamma \) parameter is determined after \( D_{eq}, \mu_g, \) and \( N_w \) have been established iteratively. Figure 4a indicates the upper limit to the retrieved rain rates coincides with too-small \( D_{eq} \), producing a \( \gamma \) that is barely modulated from its initially assumed value of 1. We argue that additional lack of information on this constraint, which is difficult to estimate from the remote measurements, contributes to the underestimated rain rates shown in Fig. 4a.

### A retrieval approach based on \( Z \) and \( \overline{v} \), constrained by in situ data

The suite of instrumentation typically deployed upon the NSF/NCAR Gulfstream V for precipitation studies will
include both the HCR and the in situ microphysical probes, because the instruments complement each other and the space is readily available upon the GV for the probes. This suggests another retrieval approach for constraining the particle size, by combining the in situ droplet size distribution information with the radar reflectivity $Z$ and the mean Doppler velocity $\bar{v}$. The latter have been used for cloud microphysical retrievals by, for example, Williams (2002) for an ultra-high-frequency radar operating at a wavelength of 32.75 cm, and by Mason et al. (2017), also for an airborne 94-GHz Doppler radar. The mean Doppler velocity is sensitive to the reflectivity-weighted terminal fall speed of raindrops, and as such is a function of the raindrop size. Although Mie dampening still affects the radar-perceived mean Doppler velocity, the Mie influence is less severe than it is on $Z$, as the terminal fall velocity increase with drop size provides a stronger weighting to larger drop sizes than is contained within $Z$. A comparison of the reflectivity-weighted raindrop fall speeds, calculated from the in situ data with and without an accounting for Mie scattering, indicated a negligible Mie dampening effect (not shown).

A full exploration of the relative strengths of a rain-rate retrieval based on $(Z, \sigma)$ as opposed to $(Z, \beta)$ requires a formal error characterization that includes an optimal estimation (e.g., Mason et al. 2017). Bayesian approaches relying on in situ data for cloud remote sensing retrievals (McFarlane et al. 2002, Evans et al. 2002) calculate a priori probability distributions for the desired drop size distribution moments independent of any assumed fits (e.g., McFarlane et al. 2002). Here, we explore a preliminary approach, in which the parameters from a lognormal fit to the in situ in-cloud-level raindrop size distributions from three flights (19 and 29 July and 9 August, shown in Fig. 5d), constrain a $(Z, \sigma)$ retrieval for the RF07c cumulus module shown in Fig. 1. 15% of all the mean Doppler velocities are positive (Fig. 4f), indicating upward movement with drops that are not big enough to be detectably falling. These are not included within the rain retrieval. A lognormal fit, justified by Fig. 5, is applied to the remaining samples, truncated at 0.075 and 3.2 mm.

The radar mean Doppler velocity $(\bar{v})$ is the sum of the vertical air velocity $(w)$, and the mean $Z$-weighted droplet terminal velocity $(\nu_d)$, or

\[
\bar{v} = w - \nu_d = w - \frac{n(D)d^6\gamma(D)\gamma(D)\gamma(D)dD}{n(D)d^6\gamma(D)dD},
\]

where $\gamma$ is the ratio of Mie and Rayleigh backscatter coefficients. The downward radar Doppler velocities shown in Fig. 1 exceed 5 m s$^{-1}$ within the cumulus downdrafts. The stratocumulus cloud downward radar Doppler velocities are gentler but remain capable of reaching 3 m s$^{-1}$. The 10-min mean aircraft-determined updrafts $w$ over the raining samples within the cumulus at ~600- and at 150-m altitude are $-0.3$ and $-0.2$ m s$^{-1}$, respectively (Fig. 4f), significantly less than the radar Doppler velocities, indicating these are dominated by the falling raindrops. Toward reducing the number of unknowns, $w$ is assumed to be zero within the retrieval, so that $\bar{v} = \nu_d$, following Mason et al. (2017).

The $Z$-weighted mean terminal fall velocity $(\nu_{t,\text{lookup}})$ is obtained from the in situ data in terms of the lognormal distribution $N(D) = (N/D\sqrt{2\pi\sigma^2})e^{-(\ln D - \mu)^2/(2\sigma^2)}$ where $\mu$ is related to the geometric mean diameter $D_g$ through $D_g = e^\mu$ and $N_t$ is the total raindrop concentration. The standard deviation or width of the lognormal distribution $\sigma$ increases from 0.5 within stratocumulus cloud to 0.6 below stratocumulus and within cumulus and 0.7 below cumulus for the distributions shown in Fig. 5.

The a priori incorporation of these $\sigma$ values effectively reduces the dependence of $\nu_{t,\text{lookup}}$ to just $\mu_t$, as $N_t$ cancels out. A $\nu_{t,\text{lookup}}$ lookup table is constructed based on $\mu_t$ values ranging from log(0.01 mm) to log(3.2 mm). Terminal fall velocities are calculated for each drop size based on Beard (1985). The retrieved $\mu_t$ then corresponds to the minimum in $(\bar{v} - \nu_{t,\text{lookup}})^2$. This retrieved $\mu_t$ is used to forward model $Z_{\text{lookup}}$ according to $Z_{\text{lookup}} = (N_t/\sqrt{2\pi\sigma^2})\sum D^6\gamma(D)e^{-(\ln D - \mu)^2/(2\sigma^2)}$ where $N_t$ is varied logarithmically from $10^{-3}$ to $10^{10}$ m$^{-3}$ for 1000 values of $N_t$. The retrieved $N_t$ then corresponds to the minimum in $(Z - Z_{\text{lookup}})^2$.

In Figs. 8 and 9, $\sigma$ is held constant at 0.6, and a small (<1 dBZ) attenuation correction is also applied to the reflectivity. The rain rate, $D_{\text{eff}}$, raindrop concentration and $y'$ are computed using the retrieved $\mu_t$ and $N_t$ through integrating over the moments of the lognormal distribution $N(D)$. The $(Z, \sigma)$ retrieval, only applied to time periods with negative Doppler velocities (85% of the time), is compared to those based on the $(Z, \beta, v_2)$ retrievals in Figs. 8 and 9. Figure 8 also includes in situ values from a descent profile through a raining cloud made immediately prior to the subcloud leg values shown in Fig. 9.

The $(Z, \sigma)$ rain rates exceed those based on $(Z, \beta, v_2)$ (Fig. 8a), with $(Z, \sigma)$ retrieved raindrops more numerous. 

![FIG. 7. Rain rates estimated from a $Z$–$R$ relationship ($R_{\text{estimated}}$) vs in situ rain rate ($R_{\text{in-situ}}$) for the Hawaii-to-California flights on 19 Jul, 29 Jul and 9 Aug. Empirical parameters $a$ and $b$ within $R_{\text{estimated}} = aZ^b$ are derived for each flight, for the subcloud (blue) and in-cloud (red) legs. The $Z$–$R$ relationship is then applied to an in situ calculated radar reflectivity that also accounts for Mie scattering. The Mie scattering is accounted for using Matzler’s (2002) algorithm.](image-url)
Fig. 8. Longitude–height cross section for \((Z, v_d)\) and \((Z, \beta, v2)\) are shown for (a),(b) rain rate, (d),(e) raindrop number concentration, (g),(h) \(D_{eff}\), and (j),(k) \(\gamma^*\) for the 19 Jul RF07c cumulus example. The aircraft position is indicated as a black dashed line. The 10-min leg-mean retrievals for (c) rain rate, (f) raindrop number concentration, (i) effective diameter, and (l) \(\gamma^*\), and in situ values from the nearest vertical leg [location shown in inset within (b)]. Dotted lines are cloud-only (>40 dBZ) averages and full lines include both cloudy and clear samples.

(Fig. 8b) and larger (Fig. 8c), reducing \(\gamma^*\) (Fig. 8d), compared to \((Z, \beta, v2)\). At the lowest detected range gate (≈340 m), rain rate, number concentration, effective diameter and \(\gamma^*\) from \((Z, \tau)\) method are 20 mm day\(^{-1}\), 5 × 10\(^5\) m\(^{-3}\), 375 µm, and 0.97, compared to 10 mm day\(^{-1}\), 9 × 10\(^5\) m\(^{-3}\), 325 µm, and 1.03, from the \((Z, \beta, v2)\) method. Some of the increase in the drop size is expected through the application of a nonvarying drop size distribution width. The in situ drop size distribution width during the aircraft descent leg was slightly narrower, at \(\sigma = 0.5\), than the assumed width within the retrieval. The in situ vertical profile values, which are not collocated in time but are sampled next to the RF07c horizontal leg, indicate slightly higher rain rates with lower raindrop concentration and larger diameters, while also serving to indicate that the retrieved values are reasonable.
in situ, \((Z, \beta)\) v2 and \((Z, \sigma)\) retrievals for the subcloud leg in Fig. 9 affirms conclusions drawn from Fig. 8. The leg-mean rain rate, total number concentration, effective diameter and \(\gamma\) calculated over the cloudy samples only using \((Z, \sigma)\) method are 23 mm day\(^{-1}\), \(4 \times 10^3\) m\(^{-3}\), 317 \(\mu\)m, and 0.8, respectively, compared to 20 mm day\(^{-1}\), \(6 \times 10^3\) m\(^{-3}\), 261 \(\mu\)m, and 1 for \((Z, \beta, v2)\) retrievals. The in situ rain rates are the highest (53 mm day\(^{-1}\)), with those based on the \((Z, \sigma)\) retrieval comparing slightly better to the in situ values than those based on the \((Z, \beta, v2)\) retrieval. This reflects a retrieved \((Z, \sigma)\) raindrop number concentration that, perhaps fortuitously, is closer to the measured in situ concentrations than those from the \((Z, \beta, v2)\) retrieval.

A more comprehensive application to the full CSET radar dataset would benefit from incorporation of a vertically varying value for \(\sigma\) that evolves with boundary layer depth, to better capture size sorting effects on the distribution width. The propagation of errors upon the retrieval uncertainties through an optimal estimation framework also remains to be done. A more complex retrieval could also incorporate the Doppler spectrum width.

6. Summary and discussion

In situ rain rates often exceeded 1 mm h\(^{-1}\) (24 mm day\(^{-1}\)) in cumulus clouds of 2-km height during CSET (SARKAR20), significantly higher than the initial rain rates retrieved using a combination of the HCR and HSRL (Schwartz et al. 2019; Bretherton et al. 2019; Mohrmann et al. 2019). The combination of radar and lidar measurements for rain-rate retrievals was originally proposed by O’Connor et al. (2005) for light stratocumulus drizzle regime, with rain rates below 1 mm h\(^{-1}\). However, this method is incorporated by Schwartz et al. (2019) for the deeper stratocumulus and cumulus precipitation observed during CSET, and it poses certain underestimates in the rain-rate retrievals as shown in the present study. Subsequent improvements to the Schwartz et al. (2019) retrieval increase the retrieved rain rates, if still less than the in situ rain rates. Three potential sources for the retrieval bias are explored: the gamma function fit for the drop size distribution, attenuation by rain and cloud water, and treatment of Mie dampening of the radar reflectivity within the retrieval. An exponential fit truncated for drops larger than 75 \(\mu\)m and a lognormal fit provides an improved fit to the drop size distribution beneath the cumulus cloud example of an intense, short-lived shower, although its implementation into the retrieval is not explored here. Attenuation by rain and cloud liquid is either negligible or comparable to the measurement uncertainty of the radar, keeping in mind that the comparisons are only applied to the subcloud legs. Last, the retrieval does not account for the dampening of the radar reflectivity by Mie scattering well, with the modeled raindrop size distribution typically too narrowly peaked (as indicated by a too-large \(\mu_g\)) around too-small drop sizes. Also, \(\gamma\) becomes an additional free parameter that is difficult to constrain well with the remote measurements.

An alternative approach is explored based on the mean radar Doppler velocity \(\sigma\), which is less affected by Mie scattering, and a priori information on the drop size distribution width, along with the radar reflectivity. The standard deviation of the lognormal distribution \(\sigma\) is estimated from the in situ drop size distribution, reducing the retrieval to just two parameters \((N_l, \mu_l)\); \(\mu_l\) (related to the geometric mean diameter), is obtained from the radar Doppler velocity and \(N_l\) from reflectivity. In the cumulus cloud RFO7c example, the \((Z, \sigma)\) retrieval produces slightly higher rain rates than the \((Z, \beta)\) retrieval, primarily because the retrieved raindrop number concentrations are larger. An advantage of this approach is that only the HCR is needed, increasing space availability upon the Gulfstream V for other instrumentation and lending itself well to campaigns lacking a lidar. A disadvantage is the
higher-order dependence on particle size. A more comprehensive assessment is needed before robust conclusions can be drawn on the differences between the two retrieval methods.

The NSF/NCAR Gulfstream V is a permanent platform for the requestable HCR and HSRL. The NSF/NCAR Gulfstream V research plane carried the HCR and HSRL in situ probes for the Southern Ocean Clouds, Radiation, Aerosol Transport Experimental Study (McFarquhar et al. 2021). The plane’s deployment for the Organization of East Pacific Convection campaign included the HCR and the 2D cloud probes, but not the HSRL (Fuchs-Stone et al. 2020). A small microwave radiometer was initially incorporated into the CSET instrument payload, to constrain the column liquid water (Zuidema et al. 2012a). Although the radiometer did not function on this deployment, it can provide an additional independent piece of information helpful to the retrievals. Further effort can also be placed into improving the lidar discrimination of the rain from the aerosol below cloud, using a priori information to better estimate the relative contributions to the backscattered intensity. The NSF/NCAR Gulfstream V plane is one of NSF’s premier platforms for atmospheric research, and the commitment to a repeated use of the same instrumentation indicates the value of deepening the characterization of their dedicated retrievals.

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Data availability statement. The version 2 merged, quality-controlled radar–lidar 2 Hz dataset placed on a uniform coreferenced grid and (Z, β) v2 retrievals are available at https://doi.org/10.26023/WB9j-MFRK-QY0Z. More details on the retrievals are also available at https://data.eol.ucar.edu/datafile/nph-get/487.047/drizzle_ret_data_writeup.pdf. Details on the calibration offsets are available at https://doi.org/10.5065/D6CJ8BV7. All other datasets are available through the NCAR EOL archive at https://data.eol.ucar.edu/master_lists/generated/cset/.

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