An Approach to Approximate Wave Height from Acoustic Tide Gauges

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ABSTRACT: The demand for nearshore wave observations is increasing due to spatial gaps and the importance of observations for accurate models and better understanding of inundation processes. Here, we show how water level (WL) standard deviation (sigma, σ) measurements at three acoustic NOAA tide gauges that utilize an Aquatrak sensor [Duck, North Carolina, Bob Hall Pier (BHP) in Corpus Christi, Texas, and Lake Worth, Florida] can be used as a proxy for significant wave height (Hₚ). Sigma-derived Hₚ is calibrated to best fit nearby wave observations and error is quantified through RMSE, normalized RMSE (NRMSE), bias, and a scatter index. At Duck and Lake Worth, a quadratic fit of sigma to nearby wave observations results in a R² of 0.97 and 0.83, RMSE of 0.11 and 0.11 m, and NRMSE of 0.09 and 0.22, respectively. A linear fit between BHP sigma and Hₚ is best, resulting in R² 0.62, RMSE of 0.22, and NRMSE of 0.26. Regression fits deviate across NOAA stations and from the classic relationship of Hₚ = 4σ, indicating Hₚ cannot be accurately estimated with this approach at these Aquatrak sites. The dynamic water level (DWL = still WL ± 2σ) is calculated over the historic time series showing climatological and seasonal trends in the stations’ daily maximums. The historical DWL and sigma wave proxy could be calculated for many NOAA tide gauges dating back to 1996. These historical wave observations can be used to fill observational spatial gaps, validate models, and improve understanding of wave climates.

SIGNIFICANCE STATEMENT: There is a large spatial gap in nearshore real-time observational wave data that can provide critical information to researchers and resource managers regarding inundation and erosion, help validate coastal hydrodynamic models, and provide the maritime community with products that help ensure navigational safety. This study utilizes existing infrastructure to help fill the demand for nearshore wave observations by deriving a proxy for wave height at three sites. This work shows spatial variability in the regression fits across the sites, which should be explored at more stations in future work. Multidecadal length time series were also used at the sites to investigate climatological and seasonal trends that provide insight into wave climates and wave driven processes important for coastal flooding.

KEYWORDS: Waves, oceanic; Acoustic measurements/effects; In situ oceanic observations; Spectral analysis/models; distribution; Time series

1. Introduction

There is a lack of wave observations in coastal and estuarine environments. There are nearly 250 wave observation stations nationwide transmitting in real time through NOAA’s National Data Buoy Center (NDBC), the Coastal Data Information Program (CDIP), and/or the U.S. Army Corps of Engineers (USACE). Of these, less than 10% (20/246) are in shallow water, ≤10 m, as defined in the National Operational Wave Observation Plan issued by the NOAA Integrated Ocean Observing System (IOOS) and the USACE (IOOS and USACE 2009, herein referred to as Waves Plan 2009). In comparison to other oceanographic observation time series, e.g., the 165-yr tidal record in San Francisco (NOAA Center for Operational Oceanographic Products and Services 1988; Theberge 2004) or the daily temperature and salinity measurements collected in La Jolla for over a century (Rasmussen et al. 2020), the longest wave observation time series is also relatively short (45 years) and applies to buoys in deep water (>3000 m) (Birkemeier et al. 2012).

The demand for coastal wave observations and high resolution wave models continues to increase (Gibbs et al. 2013). This is partly attributed to the critical need for nearshore wave data in supporting safe navigation in ports and recreational boating, as well as maritime military operations (Fiorentino et al. 2019; Waves Plan 2009). Waves play an important role in storm surge (Jelesnianski et al. 1992), wave setup (Bowen et al. 1968; Longuet-Higgins and Stewart 1964) and runup (Raubenheimer and Guza 1996; Stockdon et al. 2006), coastal inundation, and erosion via sediment transport (Gallagher et al. 1998). Thus, waves need to be monitored for coastal hazard response and mitigation efforts to support coastal resiliency through improved applications and products, such as probabilistic wave runup and rip current models (Dusek et al. 2015).
The global rise of sea level is also adding to coastal communities' risk of more frequent flooding and is projected to continue to increase (Barnard et al. 2019; Elko et al. 2015; Serafin et al. 2019, 2017; Sweet et al. 2018). The addition of wave induced processes, such as runup and setup, on top of rising sea level leads to increased erosion and inundation extending farther inland (Barnard et al. 2019; Lewis et al. 2019; Serafin et al. 2019, 2017). This may be compounded by the projected increase in extreme storm intensity and frequency due to climate change (Bender et al. 2010; Knutson et al. 2020). Wave observations in the coastal environment are needed to test and improve nearshore numerical modeling of waves and coastal hazard risks including flooding (Barnard et al. 2019; Elko et al. 2015).

Observations collected by NOAA tide gauges offer a potential source for wave information that begins to address the need for additional shallow-water wave data. Using existing infrastructure, both mean water level and wave height can be observed simultaneously at a single gauge reducing installation and maintenance costs (Fiorentino et al. 2019). NOAA operates about 310 real-time water level stations across the United States, comprising 210 National Water Level Observation Network (NWLon) stations and approximately 100 short-term stations. These tide gauge stations measure real-time water level using sensors and methods that have evolved over the years, including the acoustic Aquatrak sensor (Fig. 1) that has been utilized at many stations since the mid-1990s (Edwing 1991). At each station, the standard deviation of 1-Hz water level observations is used to quality control the observations prior to calculating a 6-min average water level value. The 6-min standard deviation (sigma, \( \sigma \)) values are stored along with the mean water level measurements for quality control purposes. The approach to calculate wave height from tide gauge observations is an expansion of previous work by Park et al. (2014) and Sweet et al. (2015), who demonstrated that sigma measurements can be used as a proxy for significant wave height (\( H_{\text{rms}} \)). The purpose of this analysis is to assess the use of and quantify the error in using the acoustic Aquatrak tide gauge sigma as a proxy for significant wave height and dynamic water level at three study site locations. The spatial variability of the sigma calibration across NOAA tide gauge stations is also investigated and is primarily attributed to differences in wave climate between stations. This effort will help NOAA to potentially develop an operational product at many coastal stations in order to fill the spatial gap and respond to the need for nearshore wave information.

This paper describes how the NWLon acoustic water level standard deviation at three stations, Duck, North Carolina, Bob Hall Pier in Corpus Christi, Texas, and Lake Worth, Florida (Fig. 2), is utilized to derive a proxy of significant wave height based on comparison to nearby wave observations. These stations were chosen for this analysis based on the locations of the stations along an exposed coastline that is impacted by incident gravity waves as well as the availability of nearby wave reference sensor observations relative to the Aquatrak tide gauges. The methods section describes the quality control processing of the 1-Hz raw water level observations at Duck and Lake Worth as well as the historical 6-min sigma observations available via the NOAA CO-OPS API at Duck, Bob Hall Pier, and Lake Worth. The results section covers the regression analysis between the sigma and nearby wave height measurements and the associated wave error analysis. The discussion includes a comparison of the results at all stations as well as applications of the fitted sigma products, including dynamic water level (DWL) defined by Sweet et al. (2015) as \( \text{DWL} = \text{still water level} \pm 2\sigma \), where still water level is the summation of the mean sea level referenced to a tidal datum, tide, and nontidal residual (e.g., storm surge) and sigma includes oscillations from incident wind waves and swell. Here, the daily maximum DWL, i.e., \( \text{DWL} = \text{still water level} + 2\sigma \), is analyzed over monthly and yearly periods.

**FIG. 1.** The standard configuration for a NOAA NOS Aquatrak Tide Gauge station (Edwing 1991; NOAA National Ocean Service 2010).
The paper finishes with conclusions and future work for developing an operational tide gauge wave estimate product.

2. Methods

Various datasets are used in this research to compare sigma from NOAA NOS acoustic tide gauge stations to significant wave height (Fig. 2; Table 1). The NOAA water level and sigma datasets include the Aquatrak 1-Hz observations at both Duck (Station ID: 8651370) and Lake Worth (Station ID: 8722670), as well as 6-min observations at Duck, Lake Worth, and Bob Hall Pier in Corpus Christi (Station ID: 8775870) available via the NOAA CO-OPS API. The wave observations datasets include hourly Nortek Acoustic Waves and Currents (AWAC) significant wave height and directional wave spectra collected by USACE in Duck (Station ID: awac02), half-hourly significant wave height derived from an Argonaut current meter pressure sensor collected by Texas A&M University–Corpus Christi Conrad Blucher Institute (CBI) (Station ID: 260, BHPCAL). The 1-Hz water level (WL) and sigma data at Duck and Lake Worth and the wave observations at all three stations span a smaller duration of time relative to the historical 6-min time series available via the CO-OPS API extending back to the mid-1990s for Duck and Bob Hall and back to 2010 for Lake Worth. Additionally, the limited availability of wave observations and/or 1-Hz water level data along with data gaps due to sensor issues and/or the removal of erroneous data through the quality control process restricted the ability to apply regression fits over a full year covering all seasonality. The various datasets and associated sample intervals and durations are listed in Table 1.

a. The NOAA acoustic tide gauge system

The Aquatrak system consists of an acoustic time-of-flight range sensor encased in a vented protective well that sends a
2 kHz acoustic pulse to the water surface through a sounding tube at a 1-Hz rate (Fig. 1; Edwing 1991; Park et al. 2014). The time of flight of the echo of the acoustic pulse from the water surface is used to derive range to the sea surface. The protective well extends below the water surface and concludes with a brass orifice to restrict water mass transport in/out of the well (Edwing 1991; Park et al. 2014). The orifice includes parallel plates on the protective well termination that are necessary to minimize significant draw-down effects related to Bernoulli’s principle, especially in conditions where currents are faster than 0.3 m s\(^{-1}\) and wave heights are greater than 0.6 m (Edwing 1991; Park et al. 2014). The Aquatrak system was originally designed to optimize average 6-min water level measurements by mechanically damping high-frequency (\(\geq 0.2\) Hz or \(\leq 5\) s) wave-induced motion within the protective well (Park et al. 2014).

The water level at NOAA’s primary acoustic Aquatrak gauges is sampled at 1 Hz for 181 consecutive seconds centered every 6 min over which the initial mean water level and associated standard deviation (sigma, \(\sigma\)) are calculated (Park et al. 2014). Raw water level observations are initially quality controlled by removing data that exceed a threshold of three standard deviations from the mean over the 6-min interval (Park et al. 2014). Both the 6-min mean water level and sigma are recalculated, telemetered in real time, archived in the CO-OPS database and available via the API. Since about 1997, the only available window that sigma is routinely collected over is 6 min, so there are no alternative options when utilizing the historic observations.

### Table 1. Station data types, sample rates, sample durations, sample time interval, and the start and end dates spanning the entire analysis period, including data gaps.

<table>
<thead>
<tr>
<th>Station and type</th>
<th>Sample rate</th>
<th>Sigma sample rate</th>
<th>Sigma sample duration</th>
<th>Sigma sample interval</th>
<th>Referred to as</th>
<th>Start date</th>
<th>End date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duck raw water level</td>
<td>1 Hz</td>
<td>181 s (6 min)(^{-1})</td>
<td>Subsampled at top of hour (60 min)</td>
<td>Sigma-subsample ((\sigma_{\text{sub}}))</td>
<td>31 Jan 2013</td>
<td>8 Apr 2014</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2048 s h(^{-1})</td>
<td>60 min</td>
<td>Sigma-2048 ((\sigma_{2048}))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>181 s (6 min)(^{-1})</td>
<td>Mean of several (6) 6-min samples (60 min)</td>
<td>Sigma-mean ((\sigma_{\text{mean}}))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duck AWAC</td>
<td>2 Hz</td>
<td>2048 s h(^{-1})</td>
<td>60 min</td>
<td>North</td>
<td>31 Jan 2013</td>
<td>8 Apr 2014</td>
<td></td>
</tr>
<tr>
<td>Duck API sigma</td>
<td></td>
<td>6 min</td>
<td></td>
<td></td>
<td>1 Jan 1996</td>
<td>31 Dec 2019</td>
<td></td>
</tr>
<tr>
<td>Bob Hall Pier API sigma</td>
<td>6 min</td>
<td>6 min</td>
<td></td>
<td>API-sigma ((\sigma_{\text{API}}))</td>
<td>1 Jan 1996</td>
<td>31 Dec 2019</td>
<td></td>
</tr>
<tr>
<td>Bob Hall Pier Argonaut</td>
<td>1024 s h(^{-1})</td>
<td>30 min</td>
<td>Mean of several (5) 6-min samples (30 min)</td>
<td></td>
<td>BHPCAL</td>
<td>15 Jun 2014</td>
<td>31 Dec 2015</td>
</tr>
<tr>
<td>Lake Worth raw water level</td>
<td>1 Hz</td>
<td>181 s (6 min)(^{-1})</td>
<td>Mean of several (5) 6-min samples (30 min)</td>
<td>Sigma-LW ((\sigma_{\text{LW}}))</td>
<td>14 Aug 2013</td>
<td>19 Nov 2013</td>
<td></td>
</tr>
<tr>
<td>Lake Worth AWAC</td>
<td>2 Hz</td>
<td>2048 s h(^{-1})</td>
<td>30 min</td>
<td>LWA</td>
<td>9 Aug 2013</td>
<td>11 Dec 2013</td>
<td></td>
</tr>
<tr>
<td>Lake Worth API sigma</td>
<td></td>
<td>6 min</td>
<td></td>
<td></td>
<td>1 Jun 2010</td>
<td>31 Dec 2019</td>
<td></td>
</tr>
</tbody>
</table>

The raw 1-Hz range data at Duck were available for several months in 2013 (31 January–7 March and 24 August–13 November) and 2014 (7 January–8 April) and was downloaded directly from the water level station’s datalogger. These data are advantageous relative to the 6-min API time series because it can be used to calculate the power spectral density (PSD) that can be directly compared to the AWAC wave spectrum, as well as calculate 6-min sigma simulating the time series available via the API. The raw 1-Hz data were quality controlled in a similar manner that data via the API are treated where values that exceeded a threshold of three standard deviations from the mean for each 3-min sample window were excluded. The 1-Hz data were then corrected for the sensor and datum offset. As a final quality control step, erroneous WL spikes that exceeded a threshold (\(\pm 3\) m) were also excluded. Gaps within each of the three periods of WL data were linearly interpolated for and were generally less than 30-min duration. The WL data were then detrended prior to calculating the PSD using Welch’s overlapped segment averaging estimator with a Hamming window and 50% overlap applied to the first 2048 samples of the hour (Hayes 1996).

Sigma was recalculated based on the quality controlled 1-Hz WL in three different ways to compare results using methods that align with the Nortek AWAC sampling method and data available via the CO-OPS API. The AWAC samples at 2 Hz over the first 2048 s h\(^{-1}\) and uses an acoustic beam to track the...
sea surface elevation from which the hourly significant wave height, period, and direction are derived. To most closely align with the wave sensor sampling scheme, sigma was calculated using the first 2048 WL samples of the hour (herein referred to as sigma-2048, \( \sigma_{2048} \)). To simulate data publicly available via the API, sigma was calculated using 181 samples of 1-Hz WL centered every 6 min and in comparison, the two time series deviated an insignificant amount likely due to manual quality control of the API data. This was then treated in two separate ways: 1) subsamples of the 6-min sigma were taken at the top of the hour to align with the wave height observations (herein referred to as sigma-subsampled, \( \sigma_{\text{sub}} \)) and 2) the mean of six 6-min sigma samples were taken over the half-hour period (0006-0036 UTC) most closely aligned to AWAC sampling and then given an hourly time stamp to align with the AWAC (herein referred to as sigma-mean, \( \sigma_{\text{mean}} \)).

The USACE collected hourly wave observations at two stations located 0.05 km south (Station ID: awac06, herein referred to as South) and 0.45 km north (Station ID: awac02, herein referred to as North) of the NOAA water level station at Duck and lie along approximately the same depth contour (6 m). During the duration of available wave observations (25 October 2012–31 December 2013), the North and South stations observed very similar mean significant wave height (0.96 ± 0.66 m and 0.95 ± 0.64 m, respectively) at the same period (8.86 ± 2.66 s), while the mean wave direction (75.7° ± 20.4° and 128.5° ± 132.3°T, respectively) showed larger variation between the two stations possibly due to effects of the pier. Although the South station was slightly closer to the tide gauge, the North station had a longer time series (approximately 4 months) of overlapping data with the Aquatrak and observed similar conditions as the South station and is therefore used for this analysis. The 1-Hz WL spectral estimates were aligned in time with the available USACE North AWAC spectrum, which included three different time periods across 2013 and 2014 separated by gaps. The WL spectrum was interpolated from 0.002-Hz frequency resolution (or bandwidth) to 0.0075 Hz to match the lower frequency resolution of the USACE North AWAC spectrum and the absolute value of the difference between the two spectra was calculated for comparison.

Following Park et al. (2014), the 1-Hz sigma values (sigma-2048, sigma-subsampled, and sigma-mean) were used to estimate the AWAC significant wave height using the standard spectral relationship: \( H_{\text{sig}} = 4 \sigma \). Linear and quadratic regressions were fit using the 1-Hz sigma and the USACE North AWAC significant wave height for the same time periods that the spectra data were available in 2013/14. The error between the USACE North AWAC significant wave height and the 1-Hz sigma estimates of wave height using a quadratic fit (herein referred to as \( H_{\text{quad}} \)) were calculated using bias, root-mean-square error (RMSE), root-mean-square error normalized by the RMSE of the wave reference taken to be truth (NRMSE), and a scatter index (as in Hanson et al. 2009). The error statistics were binned both per week and based on the AWAC significant wave height distribution.

c. Lake Worth

NOAA CO-OPS temporarily deployed a Nortek AWAC collocated with the Lake Worth Aquatrak tide gauge from 9 August to 11 December 2013 (Fig. 2). The Nortek AWAC deployed at Lake Worth had a fault with the surface tracking beam, and therefore, nondirectional significant wave height was derived every 30 min from the pressure sensor sampling at 2 Hz. Raw 1-Hz range data spanning 14 August to 19 November 2013 were collected directly from the Lake Worth tide gauge datalogger. The raw 1-Hz data were quality controlled similarly to the Duck 1 Hz described above. Water level values that exceeded a threshold of three standard deviations from the mean for each 3-min sample window were excluded and erroneous WL spikes that exceeded a threshold (±2 m) were also excluded. Sigma was calculated every 6 min and spikes above 0.5 m were also removed, which is 5 times greater than the mean of 0.089 m. Five of the 6-min sigma samples were averaged over 30-min periods (i.e., 0006-0030 and 0036-0000 UTC; herein referred to as sigma-LW, \( \sigma_{\text{LW}} \)) and fit to the AWAC significant wave height (LWA) through a quadratic regression. The error statistics between the significant wave height and quadratic fitted sigma wave proxy (herein referred to as \( H_{\text{quad}} \)) were calculated in the same manner as the observations at Duck.

d. Corpus Christi

Texas A&M University–Corpus Christi CBI deployed a Xylem Argonaut 500-kHz 2D current meter (Station ID: 260, BHPCAL, herein referred to as BHPCAL) at the end of the Bob Hall Pier in Corpus Christi, Texas, from 13 June 2014 to 31 December 2015 (Fig. 2). Significant wave height and wave period were derived on 30-min intervals from the pressure sensor every 1024 s (Tissot and Delt 2016). This sensor was collocated with a NOAA NWLON water level station (Station ID: 8775870) and since raw 1-Hz WL observations were not available, the 6-min standard deviation of water level available via the CO-OPS API was used to estimate the significant wave height. The quality controlled sigma (described below) was averaged over 30-min periods (i.e., 0000-0024 and 0030-0054 UTC) and fit to the significant wave height through a linear regression during the BHPCAL deployment period. The error statistics between the significant wave height and linearly fitted API sigma wave proxy (herein referred to as \( H_{\text{lin}} \)) were calculated in the same manner as the observations at Duck.

e. Historical NOAA data

The historical 6-min water level and associated sigma time series at Duck, Lake Worth, and Bob Hall Pier stations were downloaded via the CO-OPS API and, therefore, have undergone the initial quality control processing on board the Aquatrak described above and by Park et al. (2014). The Duck and Bob Hall Pier stations time series spans over two decades (1 January 1996–31 December 2019), while the Lake Worth Pier has a slightly shorter time series reaching nearly a decade (1 June 2010–31 December 2019). The resulting observations relative to mean higher high water (MHHW) were also manually processed and quality controlled based on CO-OPS’s standard procedures (Gill and Schultz 2001). Data that were flagged (flat tolerance exceeded, rate of change tolerance exceeded, and either maximum or minimum expected water level height limit exceeded) were removed. A daily moving window was applied to the sigma
time series to filter out data that exceeded three standard deviations above the mean. Additional noise in the sigma signal was removed using a derivative approach, where any changes in sigma with respect to time that exceeded the mean plus 3 times the standard deviation of the entire time series were excluded. During the time periods the sigma was compared to the nearby wave observations, long periods (e.g., several weeks) of anomalously low sigma (typically \(0.05\)) relative to the climatological (1996–2019) mean of 0.15 m at Duck and 0.09 m at Bob Hall Pier were also removed and likely indicate an error with the measurement that was not flagged by the automated quality control procedure.

One application of \(H_{\text{Quad}}\) and \(H_{\text{Lin}}\) is DWL, which can be calculated over the full sigma time series dating back to the mid-1990s at many NOAA tide gauge stations and can help better understand the climatology and seasonality of the wave climate at these locations. The DWL was calculated following Sweet et al. (2015) using hourly subsamples of quality controlled water level and sigma observations. The daily maximum DWL was calculated where DWL = still WL + 2\(\sigma\), and is used to derive the climatology (decadal scale) and seasonality (monthly scale) of dynamic water level events at each station.

3. Results

a. Duck

The PSD of the 1-Hz Duck Aquatrak quality controlled water level was interpolated to align with sample times of the USACE AWAC (Fig. 3). In both the water level and wave spectra, the highest energy observed is at frequencies corresponding to the relatively energetic wind sea and swell components of the wave spectrum (0.10–0.25 Hz), which aligns with the PSD of the 1-Hz Lake Worth Aquatrak WL (online supplemental material Fig. S3). The absolute value difference between the North AWAC wave spectrum and the Aquatrak water level spectrum shows the largest differences in energy occur at higher frequencies (>0.25 Hz) which characterize short-period waves (<4 s) (Fig. 3).

From the 1-Hz water level at Duck, sigma-2048, sigma-subsampled, and sigma-mean were calculated and compared to the North AWAC wave observations. Using the classic relationship, \(H_{\text{rel}} = 4\sigma\), the Duck 1-Hz sigma values align well with the AWAC significant wave height time series with a \(R^2\) value of 0.70, RMSE of 2.09 m, and NRMSE of 1.71 (Table 2). However, the 4\(\sigma\) underestimates wave heights. To find the best fit between the sigma measurements and wave observations, sigma-2048 was used as the fairest comparison between the sigma and AWAC sensor sampling interval and duration. The quadratic regression \((H_{\text{Quad}} = 2.12\sigma_{2048}^2 + 3.38\sigma_{2048} + 0.02)\) shows slightly better agreement between sigma and wave heights supported by a higher \(R^2\) (0.97) and lower RMSE (0.11 m) and NRMSE (0.09) compared to the linear regression \((R^2 = 0.96, \text{RMSE} = 0.13 \text{ m, NRMSE} = 0.10)\) or classic 4\(\sigma\) approach \((R^2 = 0.70, \text{RMSE} = 2.09 \text{ m, NRMSE} = 1.71)\) (Fig. 5; Table 2).

The quadratic regression is chosen as the best fit primarily for its ability to more accurately estimate large wave heights (>2 m) as seen by the increase in \(R^2\) values at these wave heights (quadratic = 0.75, linear = 0.67; Table 2). The quadratic regression between the wave heights and both sigma-subsampled and sigma-mean were calculated with \(R^2\) values of 0.94 and 0.97, respectively, RMSE of 0.16 and 0.12 m, respectively, and NRMSE values of 0.13 and 0.09, respectively (Fig. 5; Table 2). Sigma-mean resulted in slightly higher \(R^2\) and lower RMSE and NRMSE values compared to sigma-subsampled and more
Table 2. Statistics associated with various regression fits between the Duck sigma time series with the nearby North AWAC. The equation for the fit is in terms of $H$ and $\sigma$, where $H$ is the estimated wave height based on sigma ($\sigma$). The rows with boldface font indicate best fit based on data type. For the last four rows, a threshold value (2 m) was used to determine the difference in results between small and large wave conditions.

<table>
<thead>
<tr>
<th>Fit, sigma used</th>
<th>Equation, where $H$ is the estimated wave height based on sigma ($\sigma$)</th>
<th>$R^2$</th>
<th>RMSE (m)</th>
<th>NRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear, sigma-2048</td>
<td>$H = 4.77\sigma_{2048} - 0.16$</td>
<td>0.96</td>
<td>0.13</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Quadratic, sigma-2048</strong></td>
<td>$H_{\text{Quad}} = 2.12\sigma_{2048} + 3.38\sigma_{2048} + 0.02$</td>
<td>0.97</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>1:1 line, sigma-2048</td>
<td>$H_{\text{lin}} = 4\sigma_{2048}$</td>
<td>0.70</td>
<td>2.09</td>
<td>1.71</td>
</tr>
<tr>
<td>Linear, sigma-subsampled</td>
<td>$H = 4.61\sigma_{2048} - 0.10$</td>
<td>0.93</td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Quadratic, sigma-subsampled</strong></td>
<td>$H_{\text{Quad}} = 1.68\sigma_{2048} + 3.51\sigma_{2048} + 0.03$</td>
<td>0.94</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td>1:1 line, sigma-subsampled</td>
<td>$H_{\text{lin}} = 4\sigma_{2048}$</td>
<td>0.70</td>
<td>2.09</td>
<td>1.71</td>
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<tr>
<td>Linear, sigma-mean</td>
<td>$H = 4.73\sigma_{\text{mean}} - 0.13$</td>
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<td>0.13</td>
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<td>0.97</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>1:1 line, sigma-mean</td>
<td>$H_{\text{lin}} = 4\sigma_{\text{mean}}$</td>
<td>0.70</td>
<td>2.09</td>
<td>1.71</td>
</tr>
<tr>
<td>Linear ($&lt;2$ m), sigma-2048</td>
<td>$H = 4.77\sigma_{2048} - 0.16$</td>
<td>0.94</td>
<td>0.11</td>
<td>0.11</td>
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<tr>
<td>Quadratic ($&lt;2$ m), sigma-2048</td>
<td>$H_{\text{lin}} = 2.12\sigma_{2048} + 3.38\sigma_{2048} + 0.02$</td>
<td>0.95</td>
<td>0.10</td>
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<tr>
<td>Linear ($\geq2$ m), sigma-2048</td>
<td>$H = 4.77\sigma_{2048} - 0.16$</td>
<td>0.67</td>
<td>0.27</td>
<td>0.09</td>
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<tr>
<td>Quadratic ($\geq2$ m), sigma-2048</td>
<td>$H_{\text{lin}} = 2.12\sigma_{2048} + 3.38\sigma_{2048} + 0.02$</td>
<td>0.75</td>
<td>0.23</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Closely aligned with sigma-2048. This result is the basis for suggesting using a similar approach of calculating the mean of several sigma values available via the API to obtain a wave height proxy in the future when 1-Hz data are not available.

The error between $H_{\text{Quad}}$ and the North AWAC wave observations at Duck shows increased RMSE, decreased NRMSE, and a more negative bias with increasing wave height, indicating an underestimation by sigma (Fig. 6). This is consistent with the slight increase in RMSE binned every 7 days during large wave events; however, there are no other significant temporal trends in error (supplemental material Fig. S1). The scatter index remains relatively low with little variation both across wave heights and temporally (Fig. 6 and supplemental material Fig. S1, respectively). Regardless of the differences in calculating sigma, the error between the North AWAC and $H_{\text{Quad}}$ using sigma-2048 is very similar to that of the error between the North AWAC and $H_{\text{Quad}}$ using sigma-mean both temporally and binned by wave height. In comparison, the error between the North AWAC and $H_{\text{Quad}}$ using sigma-subsampled has a higher RMSE, NRMSE, and scatter index both temporally and binned by wave height compared to the other two methods of sampling sigma. This is consistent with the statistical values of the quadratic fit mentioned previously (i.e., sigma-mean has similar $R^2$, RMSE, and NRMSE as sigma-2048 versus sigma-subsampled) and suggests using the mean of several sigma values is the optimal approach.

b. Corpus Christi

Following the approach of averaging sigma values together in order to more closely estimate the wave height with low error, five Bob Hall Pier 6-30 min API sigma values were averaged over 30 min (herein referred to as API-sigma) and compared to the 30-min significant wave height derived from the collocated Argonaut sensor (BHPCAL). Using the classic relationship, $H_{\text{API}} = 4\sigma$, the API-sigma did not align as well to the BHPCAL significant wave height as it did at Duck and is not as accurate as applying a regression fit. The linear regression ($H_{\text{lin}} = 4.40\sigma_{\text{API}} + 0.28$) is chosen as the best fit of the API-sigma to BHPCAL significant wave height at Bob Hall Pier because the linear fit is able to accurately resolve large waves while the error statistics did not significantly differ from the quadratic fit, so the less complex regression is favored. $H_{\text{lin}}$ results in $R^2$, RMSE, and NRMSE values of 0.62, 0.22 m, and 0.26, respectively (Fig. 7; Table 3). Since a linear regression is the best fit between sigma and wave heights at Bob Hall Pier while a quadratic regression is best at Duck, this result indicates that one calibration curve for sigma cannot be used across all CO-OPS Aquatrak stations to estimate wave height.

$H_{\text{lin}}$ has more counts in the midwave height range (0.5–1.0 m) relative to the BHPCAL wave heights at Bob Hall Pier (Fig. 8), leading to slight differences in the probability density functions (PDF) that characterize the observations. This result is important if considering using the PDF to estimate the probability of wave height occurrence or other wave conditions using statistical approaches based on the distribution. The error between $H_{\text{lin}}$ and BHPCAL does not have a linearly increasing trend in RMSE per wave height and instead peaks around 1.3-m wave height (Fig. 8). The NRMSE decreases with wave height (Fig. 8) indicating that although total error is highly dependent on wave height (i.e., larger waves have greater error) the relative fraction of error is not. The bias is positive, indicating an overestimation of wave heights using API-sigma, up until approximately 0.7 m, beyond which the bias is negative, indicating an underestimation of wave height (Fig. 8). The scatter index is relatively consistent, but highest at low wave heights indicating slightly greater variability in these wave conditions (Fig. 8). There is little temporal (per 7 days) variation in RMSE, bias, and scatter index with the exception of high RMSE values and low bias during a small time period near the middle of the deployment (3–31 May 2015), suggesting bad sigma data and/or a poor fit due to specific environmental conditions during this period (supplemental material Fig. S2). With this questionable time period removed, the linear fit improves such that $R^2$ increases (0.71) and RMSE and NRMSE decreases (0.19 m and 0.23, respectively) relative to the linear fit using the full data record (Table 3).
c. Lake Worth

Following the approach of averaging sigma values together in order to more closely estimate the wave height with low error, five Lake Worth 6-min sigma values calculated using 1-Hz water level data were averaged over 30 min (i.e., sigma-LW) and compared to the 30-min significant wave height derived from the collocated AWAC sensor (LWA). The quadratic regression ($H_{\text{Quad}}$) between sigma-LW and LWA significant wave height is determined to be the best fit due to the ability to better resolve the larger wave heights, similar to the Duck fit. $H_{\text{Quad}}$ results in $R^2$, RMSE, and NRMSE values of 0.83, 0.11 m, and 0.22, respectively (Fig. 9; Table 4). The RMSE error does not show a clear trend when binned by wave height (Fig. 10); however, it increases temporally (binned weekly) when large wave events occur at the end of the time series (supplemental material Fig. S4). Similarly, the NRMSE and bias also increased at this time with no other significant temporal trends (supplemental material Fig. S4). The NRMSE decreases with wave height, similar to Bob Hall Pier, as a result of the fraction of error relative to the size of the wave estimated (Fig. 10). The bias is positive indicating an overestimation of height by sigma-LW for low (<0.3 m) and medium (0.85–1.1 m) wave heights (Fig. 10). The scatter index decreases with wave height indicating more variability at lower wave heights, which aligns with the NRMSE (Fig. 10).

d. Historical dynamic water level

The daily maximum dynamic water level (DWL = still WL + 2σ) was calculated at each station using the historical sigma time series (1 January 1996–31 December 2019 for Duck and Bob Hall Pier and 1 June 2010–31 December 2019 for Lake Worth). The resulting box plots show the climatology (annual) and seasonality (monthly) of the influence of waves on the still water level (Figs. 11–13). Wave heights are hindcast over the historical time series using the best fit of sigma and show similar trends as the maximum DWL (Figs. 11–13).

4. Discussion

This research helps to determine the calibration of sigma to best approximate significant wave height using acoustic tide gauges and to quantify the error in that estimate. This is accomplished through regression analysis between the Aquatrak sigma and nearby wave observations at three locations and using the
associated $R^2$, RMSE, and NRMSE values to indicate the goodness of fit. The results show that the calibration curve differed between the three stations. Duck sigma-2048 and Lake Worth sigma-LW are best quadratically fit ($H_{\text{Quad}} = 2.12\sigma_{2048}^2 + 3.38\sigma_{2048} + 0.02$, $H_{\text{Quad}} = 5.27\sigma_{\text{LW}}^2 + 3.00\sigma_{\text{LW}} + 0.08\sigma_{\text{LW}}$) to the North AWAC and LWA wave observations, respectively, with associated $R^2$ values of 0.97 and 0.83, RMSE values of 0.11 and 0.11 m, and NRMSE values of 0.09 and 0.22, respectively (Tables 2 and 4). The Bob Hall Pier API-sigma best linearly fit ($H_{\text{Lin}} = 4.40\sigma_{\text{API}} + 0.28$) to the BHPCAL wave heights ($H_{\text{BHPCAL}} = 4.40\sigma_{\text{API}} + 0.28$) with a $R^2$ of 0.62, RMSE of 0.22 m, and NRMSE of 0.26 (Table 3). The $R^2$ is highest and the NRMSE is smallest at Duck compared to the other two stations indicating slightly greater confidence in the sigma proxy at that location. All three station fits deviated from the linear classic relationship of $H_{\text{api}} = 4\sigma$, indicating that wave heights cannot be accurately estimated using a simple 4σ approach at these station locations using Aquatrak sensors. Additionally, one regression fit will likely not sufficiently represent $H_{\text{api}}$ across all CO-OPS tide gauge stations due to the response of the water level motion within the acoustic well differing across installations and wave climates. The $H_{\text{api}} = 4\sigma$ approach may be suitable at other locations not analyzed here and/or for tide gauge stations that utilize different engineering solutions to measure water levels that do not rely on the Aquatrak stilling wells (i.e., microwave water level sensors).

a. Duck

The Duck 1-Hz water level and North AWAC spectra both show the most wave energy at lower frequencies (0.10–0.25 Hz), which fall into the wind sea and swell wave spectrum frequency band. This characterizes the predominant wave periods (4–10 s) observed along the North Carolina coast which is consistent with the mean period observed by the North AWAC during the analysis period of 8.86 s as well as prior findings of a mean peak spectral period of 8.4 s at Duck observed over an 11-yr period (1981–91) (Kroon et al. 2008). The greatest difference in spectra is seen at high frequencies (>0.25 Hz), which is attributed to sensor and platform design differences. The lower energy observed by the Aquatrak relative to the AWAC at high frequencies is likely due to the protective well dampening effect discussed in Park et al. (2014). This dampening filters out the short-period (<5 s or ≥0.2 Hz) waves leading to differences between the spectra at higher frequencies. Lake Worth 1-Hz water level spectra showed similar results as Duck with the largest energy falling into wind sea and swell wave bands (0.10–0.25 Hz) with lower
energy at higher frequencies (>0.3 Hz) likely attributed to dampening from the Aquatrak stilling well (supplemental material Fig. S3).

For small to moderate waves (<2 m) at Duck, the classic relationship ($H_{\text{max}} = 4\sigma$) and the linear regression ($R^2 = 0.94$, RMSE = 0.11 m, and NRMSE 0.11) between North AWAC and sigma-2048 both perform well (Table 2). However, the quadratic regression ($H_{\text{quad}}$) not only improves accuracy in these small wave conditions with slightly higher $R^2$ (0.95) and lower RMSE (0.10 m) and NRMSE (0.10) (Table 2), but also more significantly improves the accuracy of estimating the wave heights in large wave conditions (>2 m) where the observations deviate from the 1:1 line ($H_{\text{quad}} = 4\sigma$). The RMSE between the wave observations and $H_{\text{quad}}$ for sigma-subsampled (sigma-subsampled, sigma-mean (orange), and sigma-2048 (blue) per wave height.

![Duck Wave Heights](image1)

**FIG. 6.** (left) Probability density functions of the Duck North AWAC wave height (yellow) and $H_{\text{quad}}$ using sigma-2048 (blue) for the full time series. (right) The (top) RMS error, (second) NRMS error, (third) bias, and (bottom) scatter index binned by North AWAC wave heights (≈10% bins), where the dots represent the center of the bin except for the first and last dots, which represent all wave heights below and above those end member values. The dotted lines show the relationship between the AWAC North and $H_{\text{quad}}$ for sigma-subsampled (green), sigma-mean (orange), and sigma-2048 (blue) per wave height.
of the acoustic well mechanically filtering out water level variance based on not only the wave period, but also the wave height as described in Park et al. (2014).

Differences between $H_{\text{Quad}}$ (for all three sigma methods) and North AWAC wave observations at Duck may also partly be due to the 0.45-km distance between the measurements. Park et al. (2014) also mentioned spatial variability between the wave gauge and sigma estimates as a possible factor for differences in their analysis. The mean direction of waves observed by the North AWAC during the analysis period was 71.9°T, which is approximately shore normal, and therefore, obstruction from the pier is not expected. More specifically, the mean wave direction was 66.4°T for large waves (≥2 m) and 72.4°T for small waves (<2 m) and therefore not likely contributing significantly to variations in error with wave height. However, despite the North AWAC and WL tide gauge lying along approximately the same depth contour, Elgar et al. (2001) found the depression in bathymetry below the pier can reach up to 1.5 times greater than depths located 50 m to either the north or south. This variation in bathymetry

![Bob Hall Pier Linear Fit](image)

![Bob Hall Pier Wave Height](image)

**Fig. 7.** (top) Linear regression (black line) between $4 \times \text{API-sigma}$ and the BHPCAL significant wave height (blue circles) along with the 95% confidence intervals (black dotted lines). The 1:1 line is shown in gray and the quadratic fit between $4 \times \text{sigma-2048}$ and North AWAC wave observations at Duck is shown in magenta for reference. (bottom) Time series of the half-hourly significant wave height derived from the pressure sensor on the Argonaut current meter (BHPCAL, blue) and $H_{\text{Lin}}$ using API-sigma at Bob Hall Pier, Corpus Christi.

**TABLE 3.** Bob Hall Pier statistics between the BHPCAL significant wave height and API-sigma averaged every 30 min. The first row (boldface font) indicates best fit (linear).

<table>
<thead>
<tr>
<th>Fit type</th>
<th>Equation, where $H$ is the estimated wave height based on sigma ($\sigma$)</th>
<th>$R^2$</th>
<th>RMSE (m)</th>
<th>NRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$H_{\text{Lin}} = 4.40\sigma_{\text{API}} + 0.28$</td>
<td>0.62</td>
<td>0.22</td>
<td>0.26</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$H = -4.98\sigma_{\text{API}}^2 + 5.88\sigma_{\text{API}} + 0.20$</td>
<td>0.63</td>
<td>0.22</td>
<td>0.25</td>
</tr>
<tr>
<td>Linear, with bad time period removed (3–31 May 2015)</td>
<td>$H = 4.60\sigma_{\text{API}} + 0.23$</td>
<td>0.71</td>
<td>0.19</td>
<td>0.23</td>
</tr>
</tbody>
</table>
leads to an alongshore gradient in wave energy and direction due to shoaling and refraction of southerly waves (Elgar et al. 2001). Therefore, variations in estimated wave height between the North AWAC and sigma may be a result of shoaling and refraction of nonnormally incident waves approaching from the south. At Duck, $H_{\text{Quad}}$ using sigma-mean aligned well with $H_{\text{Quad}}$ using sigma-2048 in the wave error analysis and had a lower error than $H_{\text{Quad}}$ using sigma-subsampled. Perhaps not surprisingly, this demonstrates that the average of six consecutive 6-min sigma values from the API better represents sigma from a continuous 2048 s time series than a single subsampled value, and is the preferred approach to estimating wave height. Similarly, several (five) sigma values were averaged at Bob Hall Pier and Lake Worth and this method will be utilized at future stations analyzed where raw 1-Hz data are not available.

b. Corpus Christi

At Bob Hall Pier, a linear relationship ($H_{\text{Lin}}$) was determined to be the best fit between API-sigma and the BHPCL significant wave height; however, the data do not fall along the 1:1 line between $4\sigma$ and $H_{\text{m0}}$, indicating that the classic relationship does not apply well here either (Fig. 7). The RMSE error between the BHPCL significant wave heights and $H_{\text{Lin}}$ did not linearly increase with wave height as it did at Duck and instead peaked around 1.3 m (Fig. 8). The bias was positive at low wave heights indicating an overestimation, and increased in the negative direction up until this point (1.3 m) and remained negative, indicating an underestimation of large wave heights, similar to Duck. Removing the short time window (3–31 May 2015) that indicated suspect sigma data (supplemental material Fig. S2) only slightly improved the...
statistical error estimates and are therefore still included in the linear fit used for calibration at this station.

During the duration that wave data were available (i.e., 13 June 2014–31 December 2015), the Bob Hall Pier mean wave period was 5.9 s and ranged from a minimum of 2.1 s to a maximum of 11.5 s (Tissot and Dell 2016). The mean period at Duck over the time spectral data were available in 2013 and 2014 was 8.86 s with a range between 2.78 to 16.67 s. Park et al. (2014) concluded the Aquatrk orifice system mechanically attenuates signals with periods shorter than 5 s and the protective well induces a nonlinear low-pass filter to the sigma depending on the wave height, period, and water depth. The mean period at Bob Hall Pier (5.9 s) is very close to this frequency cutoff (5 s), which may contribute to the difference in calibration between the two stations. This is supported by differences in wave climates at Duck, which is exposed to the open Atlantic Ocean, versus at Bob Hall Pier, which is located in northwestern Gulf of Mexico, as seen by differences in the wave height distributions (Figs. 6 and 8).

c. Lake Worth

At Lake Worth, the error statistics ($R^2$, RMSE, and NRMSE) did not significantly improve between the linear and quadratic fits; however, the quadratic regression is determined to be the best fit due to the ability to better approximate the large waves (>1.1 m) similar to the determination at

### Table 4. Lake Worth statistics between the LWA significant wave height and sigma-LW averaged every 30 min. The second row (boldface font) indicates the best fit (quadratic).

<table>
<thead>
<tr>
<th>Fit type</th>
<th>Equation, where $H$ is the estimated wave height based on sigma ($\sigma$)</th>
<th>$R^2$</th>
<th>RMSE (m)</th>
<th>NRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$H_{\text{lin}} = 4.40\sigma_{LW} + 0.02$</td>
<td>0.82</td>
<td>0.11</td>
<td>0.23</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$H_{\text{Quad}} = 5.27\sigma_{LW}^2 + 3.60\sigma_{LW} + 0.08$</td>
<td><strong>0.83</strong></td>
<td><strong>0.11</strong></td>
<td><strong>0.22</strong></td>
</tr>
<tr>
<td>1:1 line</td>
<td>$H_{\text{ref}} = 4\sigma_{LW}$</td>
<td>0.61</td>
<td>0.82</td>
<td>1.69</td>
</tr>
</tbody>
</table>
Duck. This can be seen in Fig. 9 where the observations fall nearly entirely above the linear regression line while falling more closely about the quadratic line. Similar to the other two sites, the NRMSE binned by wave height decreases with increasing wave height at Lake Worth (Fig. 10) indicating that the fraction of error is larger when estimating smaller waves. These small waves (\( \leq 0.15\) m at Lake Worth and \( \leq 0.3\) m at Bob Hall Pier) are not resolved well as seen by the PDFs at both Lake Worth and Bob Hall Pier (Figs. 10 and 8, respectively), which may be due to the Aquatrak stilling well dampening high-frequency motion (as seen in the spectra provided in supplemental material Fig. S3), thereby limiting the confidence in using sigma to estimate especially low energy conditions. This is supported by the scatter index showing higher variability in approximations at low wave heights (Fig. 10). The RMSE and bias do not show clear trends with increasing wave heights (Fig. 10); \( H_{\text{quad}} \) overestimates small waves (\( \leq 0.3\) m) and waves ranging from 0.85 to 1.1 m shown by the positive bias while underestimating wave heights ranging from 0.3 to 0.85 m and large heights (>1.1 m) (Fig. 10). Similarly, no clear temporal trends are apparent in the wave error statistics at Lake Worth other than a slight increase in RMSE, NRMSE, and bias at the end of the time series when large waves were observed (supplemental material Fig. S4).

d. Spatial comparison across three sites

The best fit between NOAA tide gauge sigma and nearby wave observations deviated across all three sites, Duck, Bob Hall Pier, and Lake Worth. This may be attributed to different wave climates experienced at each of the sites in addition to data availability at each site during different times of the year leading to seasonal differences in waves observed. The data at Duck were available for over a year capturing two winter seasons; however, either sensor issues and/or erroneous data removed from the quality control process lead to large gaps.

![Lake Worth Wave Heights](image_url)

**Fig. 10.** (left) Probability density functions of the LWA significant wave heights (blue) and \( H_{\text{quad}} \) using sigma-LW (orange). (right) The RMS error (red), NRMS error (magenta), bias (black), and scatter index (green) between the LWA significant wave heights and \( H_{\text{quad}} \) using sigma-LW binned by the LWA wave heights (~10% bins).
during summer months when wave conditions are less energetic. In contrast, Bob Hall Pier collected two summer seasons and had large gaps in early winter and late spring months when cold fronts may lead to large wave events. Lake Worth wave data were available as a result of a temporary test and therefore only captured late summer through early winter months while missing the spring and early summer months. Although a lack of data through all seasons may slightly impact the best fit between sigma and wave observations, the wave error statistics did not show significant temporal trends at any of the sites (supplemental material Figs. S1, S2, and S4), and therefore, additional data may not be necessary at these locations to determine the best fit.

e. Historical dynamic water level

The historical (1 January 1996–31 December 2019) sigma time series can be used to hindcast wave heights and DWL enabling validation of models and providing better understanding of climatological trends in the wave climate, prior storm events, and inundation at spatial locations that currently lack wave information. The historical Aquatrak time series at Duck, Bob Hall Pier, and Lake Worth available via the API have oscillatory climatological trends in the daily maximum dynamic water level (DWL = still WL + 2σ) with many outliers likely due to multiple storm events in a given year (Figs. 11–13). Outliers are values 1.5 times the interquartile range away from the bottom (25th percentile) or top (75th percentile) of the box. The estimated daily maximum
wave height using a quadratic fit of sigma at Duck and Lake Worth (H\textsubscript{Quad}) and a linear fit at Bob Hall Pier (H\textsubscript{Lin}) follow the yearly and seasonal climatology trends seen in the DWL (Figs. 11–13).

The seasonality at Duck shows max DWLs and H\textsubscript{Quad} in the months of October–March (Fig. 11), likely due to the last two months of hurricane season and the onset of winter storms. The lowest DWLs and H\textsubscript{Quad} at Duck are in the summer months of June and July (Fig. 11), which are typically characterized by low winds and a less energetic wave climate. This supports previous findings of wave height data collected over an 11-yr period (1981–91) using a Waverider buoy located seaward of the North AWAC station at Duck in 18-m depth (Kroon et al. 2008). Kroon et al. (2008) found smaller waves occur in the summer months with the lowest average monthly wave height maximum of 1.4 m in July, while the largest wave heights occur in the winter months with the largest average monthly wave height maximum of 3.5 m in February.

At Lake Worth, the max DWLs and H\textsubscript{Quad} occur in September and October and remain high throughout the winter months while the lowest DWLs and H\textsubscript{Quad} occur in the summer months of June and July. This seasonality is very similar to what is observed at Duck with an increase in storms during September and October and a slackening of winds and, therefore, a less energetic wave climate throughout the summer.

In contrast to Duck and Lake Worth, lower magnitude seasonality is observed at Bob Hall Pier in the DWL and H\textsubscript{Lin} with the maximum occurring in September (Fig. 12). As large wave events are expected during storms, this aligns with September having the largest number of storms occurring per 100 years observed in the Atlantic basin, including the Gulf of Mexico, and characterizing the peak of official hurricane season that runs from 1 June to 30 November (NOAA National Hurricane Center 2011). This also aligns with a 30-yr (1979–2008) wave hindcast model of the Gulf of Mexico that shows the extreme wave heights have increased in the months of September and October over the decadal period attributed to the increase in cyclone intensity (Appendini et al. 2014). This is supported by work done by Calderón-Vega et al. (2013) showing two seasonal patterns in the maximum wave height observed by ten NOAA buoys throughout the Gulf of Mexico recording for at least 17 years. Calderón-Vega et al. (2013) concluded the maximum wave heights observed between August and October are due to hurricanes while the maximum wave heights occurring between February and April are due to cold fronts. Appendini et al. (2014) also found seasonal variability in the mean wave climate attributed to winter cold fronts that is not as apparent here in the seasonal DWL or H\textsubscript{Lin} at Bob Hall Pier.

5. Conclusions

There is a large spatial gap in nearshore wave observations and wave height estimates from existing NOAA tide gauge installations can potentially help fill this gap. Three CO-OPS acoustic Aquatrak water level stations (Duck, Bob Hall Pier in Corpus Christi, and Lake Worth) standard deviation (sigma) is compared to nearby wave observations to determine the best relationship to use sigma as a proxy for significant wave height. The results show that Duck has the smallest NRMSE and highest \( R^2 \) using a quadratically fitted sigma \( (H_{\text{Quad}}) \), while Bob Hall Pier has a linear best fit \( (H_{\text{Lin}}) \) between BHPCAL significant wave height and API-sigma and Lake Worth has a quadratic best fit \( (H_{\text{Quad}}) \) between LWA significant wave height and sigma-LW. The best fit was determined through the statistical error (RMSE, NRMSE, and \( R^2 \)) between the observations and the regression curves as well as the error between the observations and the fitted sigma binned by wave height and temporally (weekly). This supports the idea that local and/or regional calibrations are required instead of using the same fit across all NOAA Aquatrak stations. The
appropriate calibration applied to quality controlled sigma at select stations can be used to better understand the historical wave climate and will provide insight into nearshore wave processes, such as storm surge, wave runup, coastal inundation, and erosion in areas that currently lack observations.

CO-OPS is assessing the requirements and feasibility of providing users with nearshore wave products to help fill the critical spatial gap in observations. This includes possibly developing an operational tide gauge wave estimate product that relies on the standard deviation of water level. CO-OPS has begun transitioning from acoustic Aquatrak water level sensors to microwave sensors at NWLON stations starting in 2012 based on several benefits including reduced cost and maintenance (Fiorentino et al. 2019; Park et al. 2014). The microwave sensor technology allows for a direct measurement of the sea surface allowing for simultaneous water level and wave height measurements without the acoustic well dampening effects (Fiorentino et al. 2019; Park et al. 2014). Therefore, real-time wave products would likely best be estimated using microwave sensors; however, the acoustic sensors provide a historical time series from which climatological and seasonal wave data can be derived. The historical wave climate data are useful for model hindcast validation and for application of sigma proxies across large model spatial grids. Further research is required to determine whether regional calibrations of acoustic sigma are acceptable or whether calibration needs to be calculated locally at each station. Additionally, further investigation is required to better understand the anomalously low sigma periods that were removed here at two of the three stations in the quality control process. To provide a sigma derived wave proxy and/or DWL product, a robust automated quality control procedure will need to be implemented in order to flag suspect data, such as the anomalously low sigma periods. This study determined the best method to estimate wave heights is to average several quality controlled 6-min API sigma values spanning half-hourly intervals. By using acoustic sigma for wave products, CO-OPS will help fill the critical spatial gap in nearshore wave products using existing infrastructure and provide historical wave climate data to various users.

Acknowledgments. We thank those who helped in this analysis, including the NOAA CO-OPS OSTEP team, specifically Bob Heitsenrether, who provided the 1-Hz Duck and Lake Worth Aquatrak data and gave valuable feedback throughout the analysis and writing process. Additionally, we thank the USACE at the Duck FRF, specifically Kent Hathaway, for collecting and sharing the USACE AWAC observations and spectra. Funding for the acquisition of the Texas wave data was provided by Nueces County Parks and as part of an R&D 2014–15 Funding Cycle grant from the Texas General Land Office (TGLO). Both are gratefully acknowledged. The views expressed herein are those of the authors and do not necessarily reflect the views of TGLO or Nueces County.

Data availability statement. The 6-min water level and sigma time series data dating back to the 1990s at the NOAA Duck (Station ID: 8651370) and Bob Hall Pier (Station ID: 8775870) stations and dating back to the 2010s at the NOAA Lake Worth (Station ID: 8722670) station are openly available via the CO-OPS API (https://api.tidesandcurrents.noaa.gov/api/prod/). The raw 1-Hz range data at the NOAA Duck and Lake Worth stations as well as the NOAA AWAC data collected at Lake Worth are stored on an internal NOAA CO-OPS network drive and available upon request. The USACE North AWAC data are openly available via the Field Research Facility (FRF) data portal (https://frfdatat portal.erdc.dren.mil/). The CBI Argonaut derived wave height at Bob Hall Pier (Station ID: 260, BHPICAL) is openly available through an online data query (http://lighthouse.tamucc.edu/pq).

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