Error Sources and Accuracy of Vertical Velocities Computed from Multiple-Doppler Radar Measurements in Deep Convective Storms

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ABSTRACT

Actual data are used in one case to investigate the nature and source of vertical velocity errors resulting from analyses of multiple-Doppler radar measurements. Consistent with earlier analytical works, larger errors are found than would be expected from previous theoretical studies. It is shown that the reconstructed maximum updraft speed in strong updrafts (>20 m s^{-1}) is accurate, on the average, to within about 10% (standard deviation of 10%). Storm advection, incomplete sampling of low-altitude divergence caused by the radar horizon, top boundary errors, and uneven terrain are studied and all are dismissed as dominant sources of error in the case considered here. The inability to determine a dominant error source has important consequences for the formulation of vertical velocity adjustment schemes.

1. Introduction

The difficulties of obtaining the vertical component of motion from multiple-Doppler radar data are significant, as is evidenced by the many papers dealing with vertical velocity errors that have appeared in the literature. The first investigations of error sources dealt with boundary errors and/or Doppler/terminal velocity statistical uncertainty problems (e.g., Bohne and Srivastava, 1975; Doviak et al., 1976; Ray and Wagner, 1976). Later, theoretical calculations and simulated data were used to study many other error sources such as storm advection/evolution, interpolation, etc. (e.g., Clark et al., 1980; Gal-Chen, 1982; Ray et al., 1983). Practical experience, though, has shown that vertical velocity errors are greater than one would expect from these studies (Nelson and Brown, 1982; Carbone and Carpenter, 1983).

There is a need to devise better collection methods and to formulate better adjustment techniques, and these tasks can be accomplished only if the dominant error sources are identified. For example, the foundation of any vertical velocity adjustment method is the a priori specification of the relative distribution of errors in vertical columns. It is typically assumed that error sources are essentially independent of height (Ray et al., 1980; Nelson and Brown, 1982).

Here we take the approach of using *actual data* to study and isolate various error sources. In the following sections, data from one case are used to characterize vertical velocity errors and to isolate and study four error sources—storm advection, incomplete sampling of low-altitude divergence, top boundary errors and uneven terrain.

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2. Analysis techniques and data characteristics

Details of the dual-Doppler synthesis techniques used in this study are described in Brown et al. (1981), Nelson and Brown (1982) and Brown and Nelson (1982), but a short description follows. A three-dimensional grid is constructed by first selecting the grid center to be colocated with the area of interest. Using this point as a basis, a horizontal orthogonal grid is constructed which is the equivalent of making the assumption of a "localized flat plane". This causes the compass direction to be exact only for the center of the grid. For the domain used here (25 × 25 km), however, distortion near the edges of the grid are negligible. In the vertical, the planes are established so that all grid points at a given level are the same distance above a spherical earth's surface. The distortion of these curved surfaces from a true orthogonal grid is negligible for the distances considered here.

The first step in interpolating the data to the grid is to pick a reference time. To minimize advection corrections, a time is selected midway between the first and last times of data collection for the volume scan of interest. The data are interpolated to the grid using a Cressman scheme with each data point time-to-space adjusted to the reference time using a previously determined storm motion vector. For each grid point the weighted average deviation of the data collection time from the reference time is computed. To correct for the aspect angle problem described by Gal-Chen (1982), these times are used to adjust the radar position by the method referred to by Gal-Chen as the "NSSL" technique. For this case, the largest corrections in radar position are 1.5 km and occur for data collected at the lowest and highest altitudes. At and near the midpoints of the scans (mid-levels) the corrections are very small.

For a multiple-Doppler synthesis of deep convective storms the vertical velocity at any level "n" (w_n) usually is calculated from a simplified form of the continuity equation

$$w_n = w_b \frac{\rho_b}{\rho_n} - \frac{1}{\rho_n} \int_{z_b}^{z_n} [\rho(\nabla_h \cdot \mathbf{V}) dz] \qquad (1)$$

where all terms are as defined in the Appendix. The u and v components required for the horizontal divergence typically are calculated either iteratively in a dual-Doppler synthesis (e.g., Brandes, 1977; Brown et al., 1981) or directly in a three-or-more Doppler synthesis (e.g., Ray et al., 1979). In this paper, attention is limited to errors involved in solving Eq. (1), given erroneous boundary values and imperfect measurements of horizontal divergence. Density terms and numerical approximations are assumed to be exact.

Figure 1 shows the characteristics of the vertical grid construction used in this case. Mean ground level (z_{bb}, bottom boundary) is defined to be at a height intermediate between the lowest and highest terrain features. Owing to radar horizon and other problems, the lowest collected data are at some distance (ΔG) above z_{bb}. The first grid level (z_1) is chosen to be near this height; subsequent grid levels are evenly spaced at intervals of Δz, ending ideally at a level near the top of the storm (z_t). The upper storm boundary (z_{tb}, top boundary) is assumed to be one-half grid interval above z_t. When the trapezoidal integration approximation is used, (1) becomes

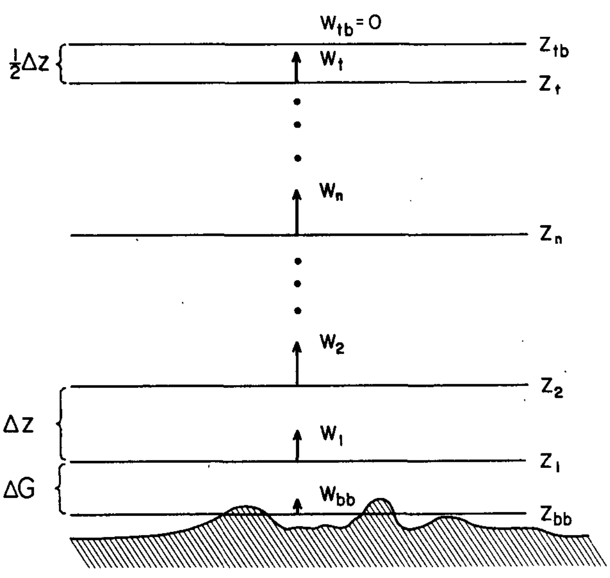

FIG. 1. Vertical grid system used in computing vertical velocities. Plane z_{bb} is positioned at a height roughly equal to the mean ground level. Lowest and highest data are at levels z_1 and z_t, respectively.

$$w_n = w_b \frac{\rho_b}{\rho_n} - \frac{\delta}{\rho_n} \frac{|\Delta z|}{2} \sum_{i=b}^{n-\delta} [\rho_i(\nabla_h \cdot \mathbf{V})_i + \rho_{i+\delta}(\nabla_h \cdot \mathbf{V})_{i+\delta}] \qquad (2)$$

where b = bb and $\delta = +1$ for upward integration, and b = tb and $\delta = -1$ for downward integration. If β is the error associated with estimation of the boundary vertical velocity and ϵ is the error in the computed horizontal divergence, then the relationship between actual and computed vertical velocity (w_n^c) is given by

$$w_n^c = w_n + \beta_b \frac{\rho_b}{\rho_n} - \frac{\delta}{\rho_n} \frac{|\Delta z|}{2} \sum_{i=b}^{n-\delta} [\rho_i \epsilon_i + \rho_{i+\delta} \epsilon_{i+\delta}]. \qquad (3)$$

For this case, downward integration is used; therefore (3) becomes

$$w_n^c = w_n + \beta_{tb} \frac{\rho_{tb}}{\rho_n} + \frac{1}{\rho_n} \frac{|\Delta z|}{2} \sum_{i=tb}^{n+1} [\rho_i \epsilon_i + \rho_{i-1} \epsilon_{i-1}]. \qquad (4)$$

The storm used in this study occurred during the night of 19 June 1980 in central Oklahoma. It is characterized as being moderately severe because it was nontornadic and produced hail (maximum diameters 29 mm) over a relatively small area. Storm motion was from 289° at 10 m s^{-1}. [See Knight et al. (1984) for a more complete description of the storm.] Overall data quality was quite good. Scan times for the Norman and Cimarron (41 km northwest of Norman) radars were moderately long at 345 and 231 s, respectively, but mean spatial resolution was somewhat better than the 1 km spacing used for the three-dimensional grid. The data allowed analyses between z_1 = 0.5 and z_t = 16.5 km (inclusive), the latter being near storm top as evidenced by maximum equivalent radar reflectivity factors of 5 dBZ.

3. General characteristics of vertical velocity errors

The existence of nonzero vertical velocities at the earth's surface (z_{bb}) after downward integration is assumed to be due to the integrated effects of all error sources in a vertical column. Vertical velocities at z_{bb} are computed by assuming constant divergence from the lowest data level (z_1 = 0.5 km in this case) to the ground[1]. Figure 2 shows the relationship of these errors to the storm's low-altitude (0.5 km) reflectivity field. The error contours show features no smaller than about 2 km, which is to be expected since this is the radius of influence used in the Cressman interpolation scheme. The histogram of errors (Fig. 3a) shows a positive bias (mean + 2.3 m s^{-1}) and a 5.7 m s^{-1} standard deviation (SD). The data in Fig. 3a are not independent, but the large scatter and the positive bias lead us to believe that the errors are not due to Doppler velocity statistical uncertainties alone. The standard deviation,

[1] The assumption's implications to the error fields are discussed in subsections 4b and d.

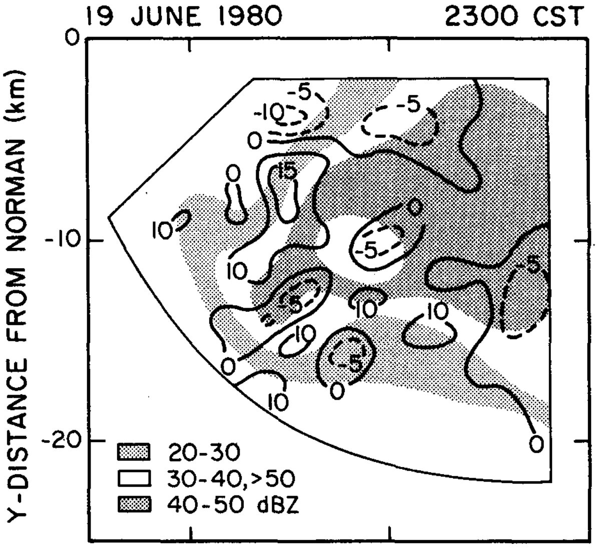

FIG. 2. Vertical velocity errors at the ground; contours are in intervals of 5 m s^{-1}. Shading indicates storm reflectivity fields at 0.5 km AGL.

however, is smaller than that reported by Nelson and Brown (1982), which may be attributable to the current dataset's superior spatial resolution and coverage.

One important question that relates to many research applications is the accuracy of the calculated vertical velocities in active updrafts. Restricting attention to updrafts $\geqslant$ 20 m s^{-1}, we find that they exhibit an even stronger bias than the whole dataset; they have a mean maximum updraft of 27.2 m s^{-1} (SD = 4.8 m s^{-1}) and mean accumulated error at z_{bb} of 5.5 m s^{-1} (SD = 5.9 m s^{-1}), thus yielding an accumulated error of about 20% (SD $\approx$ 20%) of the calculated peak value. Of course, depending on the distribution of the errors, we would expect the percentage error at the height of the maximum updraft to be somewhat less than 20%. For example, the mean height of the maximum updraft in this case is 11 km. Assuming a negligible boundary error, the bias errors are evenly distributed with height and, taking density variations into account, the mean accumulated vertical velocity error at 11 km is about 2.5 m s^{-1}—an error of about 10% (SD $\approx$ 10%) of the maximum updraft. Therefore, unless there is an unusual error distribution, we would expect the unadjusted maximum vertical velocities obtained from downward integration to be accurate within about 10–20% for the scales resolved by the radar data.

4. Investigation of specific error sources

We investigate four possible sources of error (especially bias errors) by looking for specific error patterns and/or characteristics that should exist if any particular error source dominates.

a. Errors due to storm advection

Typically a mean storm motion vector based on reflectivity cores is used to time-to-space adjust the locations of nonsimultaneously collected data. The correct adjustment, though, is one that properly realigns horizontal divergence in vertical columns. The divergence centers probably track close to, though not exactly with, the reflectivity core. Therefore, some error is likely. To test how sensitive the results are to changes in the advection vector, an analysis was performed using the extreme case of allowing no advection correction. As can be seen in Fig. 3b, although the mean and standard deviation of the errors both increase, the changes are minimal overall. One significant difference is the appearance of a few extremely negative errors. Vertical profiles of computed vertical velocity (w^c) and divergence for one point whose error increased from -7 to -20 m s^{-1} are shown in Fig. 4. This particular vertical column is located near a high gradient of divergence aloft. With no correction for advection (Fig. 4b), there is considerably less divergence aloft than before, causing excessive downdraft in the manner de-

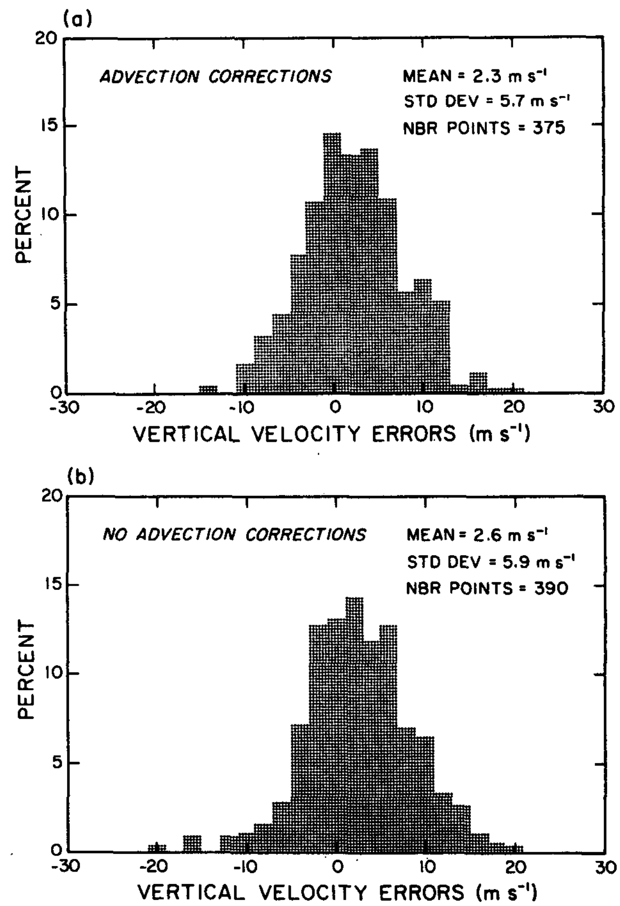

FIG. 3. Histogram of vertical velocity errors. (a) with corrections for advection; (b) without advection corrections.

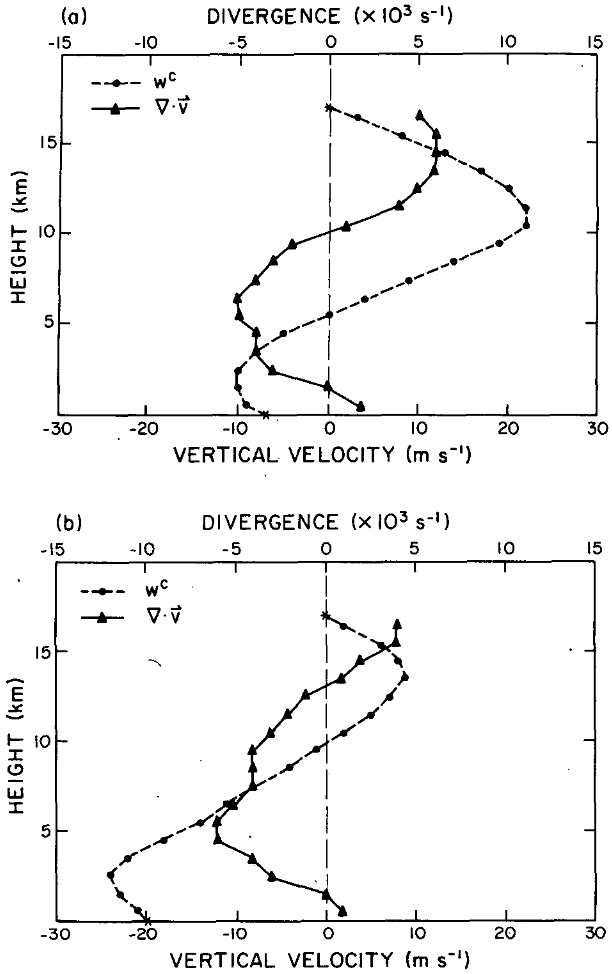

FIG. 4. Vertical profiles of computed vertical velocity (w^c) and divergence for a column located at (−34, −12) in Fig. 2. (a) With advection corrections; (b) without advection corrections.

scribed by Carbone (1981). We can see from this that some points are affected significantly by not allowing for advection, but overall the errors are very similar.

b. Errors due to incomplete sampling of low-altitude divergence

Because of radar horizon and other problems, it is not possible to obtain direct Doppler measurements of divergence in the lowest few hundred meters (i.e., between z_{bb} and z_l). In the case presented here the measurements permit an analysis at z_l = 0.5 km AGL. As mentioned in section 1, vertical velocities at the earth's surface are obtained by assuming divergence to be constant from the lowest analysis level to the ground, but this can cause errors if the divergence between the two levels is significantly higher or lower than the measured values at z_l. Determination or refutation of this problem as the main error source is of great importance, for, *if this is the major cause of errors for downward integration, then no adjustment should be made to calculate vertical velocities because all errors are outside the analysis region.*

As an example, Fig. 4a shows a surface vertical velocity error of −7 m s^{-1} with measured divergence at 0.5 km of 2 × 10^{-3} s^{-1}. An average divergence of 18 × 10^{-3} s^{-1} in the lowest 0.5 km (necessitating a surface divergence value of 34 × 10^{-3} s^{-1} for the trapezoidal integration rule) would bring the −9 m s^{-1} vertical velocity at 0.5 km to zero at the earth's surface. Although such values may be reasonable for small scales, it is doubtful that they exist in this storm over the distances averaged in the interpolation scheme. In fact, such values exceed any of those measured in the storm's upper level outflow.

Additionally, if underestimation of divergence is a problem, then (assuming that the sign of divergence does not change between 0.5 km and the ground) negative and positive errors would occur only in divergent and convergent regions, respectively. However, the data in Fig. 5 clearly indicate that there are roughly equal areas of positive and negative errors in both the convergent and divergent regions near the ground. Therefore, underestimation of divergence does not stand out as a dominant error source in this case.

c. Top boundary errors

Even though z_t should be at or above storm top, in practice this is not always the case. Consider, for example, a situation where the top of the storm (that is, the top of the updraft) is well above the top grid level.

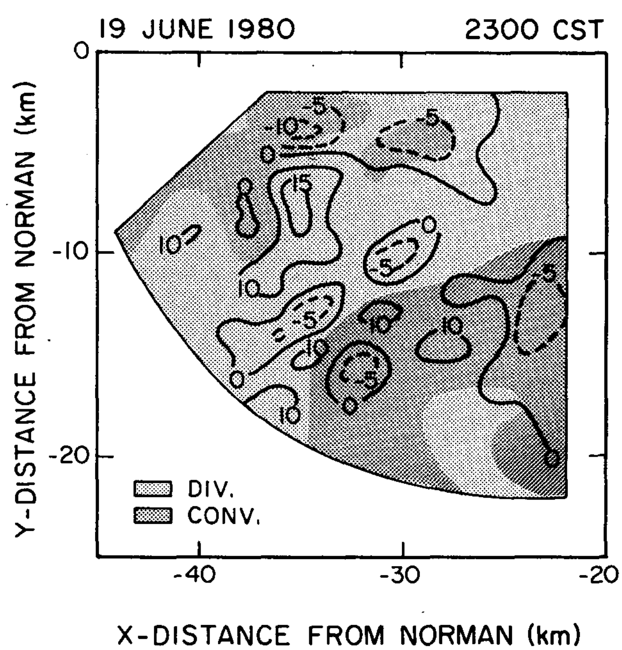

FIG. 5. Contours of vertical velocity errors at the ground superimposed on measured divergence field at 0.5 km AGL.

In reality, the vertical profile of divergence corresponds to a vertical profile of w that is zero at the ground, increases to a maximum value at middle altitudes then decreases part way toward zero at the top grid level. If one erroneously assumes that w is zero one-half grid interval above z_t (boundary condition for downward integration), the vertical velocity profile shifts toward negative (less positive) values. This shift produces negative w values at the ground. Thus, if vertical velocity boundary values are a major contributor to errors, there should be a negative correlation between upper-altitude divergence and w errors at the ground. The data, however, show exactly the opposite: divergence aloft has a weak positive correlation (correlation coefficient ≈ 0.4) with the vertical velocity errors. On the average, therefore, we can dismiss erroneous boundary values as a major source of error in this case.

Consider the profile of vertical velocity in Fig. 4a, where there is a w error of -7 m s^{-1} at the ground. The question arises as to what boundary error ($\beta_{\rm tb}$) would produce a -7 m s^{-1} error at $z_{\rm bb}$. Assuming no errors in the integrated divergence and that $w_{\rm bb}$ is zero, Eq. (4) yields the following relationship between the top boundary vertical velocity error and the computed vertical velocity at $z_{\rm bb}$:

$$\beta_{\rm tb} = w_{\rm bb}{}^c \frac{\rho_{\rm bb}}{\rho_{\rm tb}}.$$

The density ratio in this case is about 6.7, thus requiring that our assumed upper boundary value of zero is in error by over 45 m s^{-1}. A 45 m s^{-1} boundary value near the radar-determined storm top ($\approx$17 km) does not appear to be reasonable. Thus, for a second reason, we believe that the choice of a realistic boundary value (for downward integration) does not have a significant influence on w errors at the ground.

d. Uneven terrain

In all of the preceding, it has been assumed that any deviation of $w_{\rm bb}$ from zero is indicative of an error. As Fig. 1 illustrates, because of uneven terrain it is possible, for example, for there to be an actual finite value for $w_{\rm bb}$. As a first step, a visual inspection compared a topographical map with the vertical velocity error pattern, but no discernible relationship was seen.

This problem is essentially the same as the incomplete sampling of low-altitude divergence described in section 4b, with the additional problem that we are unsure not only about the correct divergence value, but also about the correct depth over which it applies. In this case, the difference in height between $z_{\rm bb}$ and the actual ground is typically a few tens of meters, which is much less than the distance between $z_{\rm bb}$ and z_1 ($\Delta G \approx 500$ m). To the extent that the low-level divergence is a problem, the uneven terrain will not contribute much additional error. This, of course, is true only for regions whose terrain is relatively even.

5. Discussion

In the quest for more accurate multiple-Doppler computed vertical velocities, much has been written on error sources and the need for better collection techniques and adjustment schemes. Most of the previous works have been theoretical or have used simulated data from numerical cloud models. It has been shown here and in other empirical works, however, that the errors are typically somewhat larger than expected from these theoretical studies. In this one case we show that an overall bias error of about 2.3 m s^{-1} exists and that in updrafts ≥ 20 m s^{-1} the maximum values are probably accurate to within about 10%.

We isolate likely error sources by assuming that if a single error source dominates, certain characteristics will be discernible in the data. With a good deal of confidence we believe that, in this case, all four sources studied can be eliminated as *dominant* error mechanisms. This finding is especially important with respect to low-altitude divergence for, if it were the major cause of errors, vertical velocity adjustment schemes for downward integration should not be used since all errors would be outside the analysis domain.

Of course there are many error sources not treated here. Most notable are storm evolution, misaligned beams and side lobe contamination (e.g., Knupp, 1983). These could be treated in a similar manner. For example, with regard to storm evolution one could compare data taken in a RHI mode with data taken in a sector scan mode. Since the RHI scan would provide the better time continuity between low- and upper-altitude divergence, it should help address the evolution question.

The authors do not wish to imply that the error sources treated heretofore are insignificant. Indeed, the specific example shown in Fig. 4b clearly demonstrates the value of storm advection corrections. The important point is that it is imperative to know if there is a predominant error source so that a general adjustment technique can be properly formulated. On the basis of this one case study we see no compelling reason to deviate from the usual assumption that error sources are uniform with height. In any event, we believe that error analyses that use actual data are important for improving the synthesis of vertical velocities from multiple-Doppler data, and they should be used on a case-by-case basis.

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APPENDIX

Definition of Terms

b Subscript denoting a quantity at the top (tb) or bottom (bb) boundary of the Doppler data

bb Subscript denoting quantities at "ground" level as defined by grid construction
n Subscript denoting a quantity at the "nth" height level (relative to lower boundary)
t Subscript denoting a quantity at the top data level with respect to the ground
tb Subscript denoting quantities at "storm top" as defined by grid construction
u West–east component of air velocity, **V**
v South–north component of air velocity, **V**
V Three-dimensional air velocity
w^c Vertical velocity computed from multiple-Doppler radar analysis
w Vertical component of air velocity
z Vertical Cartesian direction
ΔG Distance between "ground" level and first data level $(z_1 - z_{bb})$
$|\Delta z|$ Absolute value of vertical grid spacing
$\nabla_h \cdot \mathbf{V}$ Horizontal divergence
β Boundary error at initial grid level when vertical integration is used to compute vertical velocity
δ Index. $\delta = 1$ for upward integration and $\delta = -1$ downward integration
ϵ Error in multiple-Doppler radar computed horizontal divergence
ρ Air density

REFERENCES

Bohne, A. R., and R. C. Srivastava, 1975: Random errors in wind and precipitation fall speed measurement by a triple Doppler radar system. Lab. for Atmos. Probing Rep. No. 37, University of Chicago, 44 pp.

Brandes, E. A., 1977: Flow in severe thunderstorms observed by dual-Doppler radar. *Mon. Wea. Rev.*, **105**, 113–120.

Brown, R. A., and S. P. Nelson, 1982: Multiple Doppler radar derived vertical velocities in thunderstorms. II: Maximizing areal extent of vertical velocities. NOAA Tech. Memo. ERL NSSL-94, 11–21. [NTIS PB83-152-553].

——, C. R. Safford, S. P. Nelson, D. W. Burgess, W. C. Bumgarner, M. L. Weible and L. C. Fortner, 1981: Multiple Doppler radar analysis of severe thunderstorms: Designing a general analysis system. NOAA Tech. Memo. ERL NSSL-92, National Severe Storms Laboratory, Norman, 21 pp. [NTIS PB82-114-117].

Carbone, R. E., 1981: Selected comments on multiple Doppler analysis. *Preprints, 20th Conf. Radar Meteor.*, Amer. Meteor. Soc., Boston, 490–494.

——, and M. J. Carpenter, 1983: Rapid scan Doppler radar development considerations. I: Sampling requirements in convective storms. *Preprints, 21st Conf. Radar Meteor.*, Amer. Meteor. Soc., 278–283.

Clark, T. L., F. I. Harris and C. G. Mohr, 1980: Errors in wind fields derived from multiple-Doppler radars: Random errors and temporal errors associated with advection and evolution. *J. Appl. Meteor.*, **19**, 1273–1284.

Doviak, R. J., P. S. Ray, R. G. Strauch and L. J. Miller, 1976: Error estimation in wind fields from dual Doppler radar measurements. *J. Appl. Meteor.*, **15**, 868–878.

Gal-Chen, T., 1982: Errors in fixed and moving frame of references: Applications for conventional and Doppler radar analysis. *J. Atmos. Sci.*, **39**, 2279–2300.

Knight, N. C., S. P. Nelson and S. V. Vasiloff, 1984: Wet growth of hailstones: Integration of multiple Doppler data and hailstone structure analysis in a kinematic model. *Preprints, Ninth Int. Cloud Physics Conf.*, Tallinn, 267–269.

Knupp, K., 1983: Doppler radar analysis of the structure of a highly sheared multicell convective storm. *Preprints, 21st Conf. on Radar Meteor.*, Amer. Meteor. Soc., 64–69.

Nelson, S. P., and R. A. Brown, 1982: Multiple Doppler radar derived vertical velocities in thunderstorms. I: Error analysis and solution techniques. NOAA Tech. Memo. ERL NSSL-94, 1–10. [NTIS PB83-152-553].

Ray, P. S., and K. K. Wagner, 1976: Multiple Doppler radar observations of storms. *Geophys. Res. Lett.*, **3**, 189–191.

——, J. J. Stephens and K. W. Johnson, 1979: Multiple-Doppler radar network design. *J. Appl. Meteor.*, **18**, 706–710.

——, C. L. Ziegler, W. C. Bumgarner and R. J. Serafin, 1980: Single- and multiple-Doppler radar observations of tornadic storms. *Mon. Wea. Rev.*, **108**, 1607–1625.

——, J. Klemp, R. Wilhelmson and K. Johnson, 1983: Testing of multiple-Doppler analysis procedures from the synthesis of simulated radar scans. *Preprints, 21st Conf. Radar Meteor.*, Amer. Meteor. Soc., 547–552.