

Radar Backscattering by Large, Spongy Ice Oblate Spheroids

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ABSTRACT

Calculations have been made of the radar backscattering, differential reflectivity, and circular depolarization ratio of large oblate spheroids composed of spongy ice. Results are compatible with laboratory measurements by earlier investigators. As expected, scattering of 10-cm radiation depends to an important extent on the size, water content, and axial ratios of the spheroids. Observations of differential reflectivities close to zero in hailstorms can be explained, as was done by V. N. Bringi and his associates, as resulting from the irregular shapes and tumbling of hailstones. But such observations could also be explained by size-distributed oblate spheroids with equivalent diameters greater than about 3.5 cm, falling with vertical symmetry axes.

1. Introduction

The identification of hail and rain in thunderstorms has practical applications in severe storm forecasting, weather modification, and agricultural meteorology. To infer the type of hydrometeor present, radar observations are often compared with the calculated backscattering properties of particles having known characteristics.

Although Mie theory has been used to calculate the radar backscattering by dry and water-coated ice spheres (e.g., Battan et al., 1970), hailstones are often treated as oblate spheroids (a shape generated by rotating an ellipse about its minor axis). Barge and Isaac (1973) sampled hailstones from thunderstorms in Alberta, Canada, and classified 41% of the hailstones as being oblate spheroids. Matson and Higgins (1980) reported that 81% of the hailstones they observed in the vicinity of northeastern Colorado were ellipsoidal and indicated that the stones were generally oblate.

Hailstones composed of spongy ice (i.e., a mixture of water and ice) can result when large quantities of supercooled water are collected in a relatively short time (Pflaum, 1984). From measurements of the properties of artificial hailstones grown in a wind tunnel, List (1959) concluded that the volume fraction of water in spongy ice hailstones may be as high as 50%. To help interpret radar reflectivity measurements of thun-

derstorms containing hail, more needs to be known about backscattering by spongy ice hailstones.

Although there have been many calculations of radar backscattering by pure ice and water particles, there have been few calculations of scattering by particles composed of spongy ice. This may be attributed, in part, to uncertainties about the appropriate dielectric function of spongy ice. Bohren and Battan (1980) proposed it be calculated by means of an expression originally derived by Maxwell Garnett (1904), which when used in Mie calculations for ice spheres coated with spongy ice produced backscattering cross sections in close agreement with measurements by Joss and List (1963). The complex dielectric function of spongy ice is given by

$$\epsilon_{si} = \epsilon_w \left\{ 1 + \frac{3f \left(\frac{\epsilon_i - \epsilon_w}{\epsilon_i + 2\epsilon_w} \right)}{1 - f \left(\frac{\epsilon_i - \epsilon_w}{\epsilon_i + 2\epsilon_w} \right)} \right\}, \quad (1)$$

where ϵ_i and ϵ_w are the complex dielectric functions of ice and water, respectively, and f is the volume fraction of water in the mixture. Aydin et al. (1984) have used this expression to calculate backscattering by conical spongy ice particles.

This paper presents calculations of the 10-cm backscattering cross sections of oblate, spongy ice hailstones. The calculations have been made by means of the T-matrix method developed by Waterman (1965, 1969) and later modified by Barber and Yeh (1975). In addition to the backscattering cross section, we calculated the differential reflectivity and circular depolarization

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ratio (two radar parameters used for hydrometeor identification which are defined in the next section). These calculations were then compared with data published by other investigators.

2. Schemes for hydrometeor identification

Because horizontally and vertically polarized electromagnetic waves are backscattered differently by nonspherical particles, it is possible to distinguish spherical from nonspherical hydrometeors by means of dual-polarized radar. Seliga and Bringi (1976) introduced the concept of *differential reflectivity* (Z_{DR}), defined as

$$Z_{DR} \text{ (dB)} = 10 \log \frac{Z_H}{Z_V} = 10 \log \frac{\sigma_H}{\sigma_V}. \quad (2)$$

Here Z_H (Z_V) is the coplanar radar reflectivity and σ_H (σ_V) the coplanar radar backscattering cross section for an incident horizontally (vertically) polarized beam. The *circular depolarization ratio* (CDR) may also be used as an indicator of hydrometeor type when the incident beam is circularly polarized (Barge, 1972). This parameter is defined as

$$\text{CDR (dB)} = 10 \log \frac{|S_H - S_V|^2}{|S_H + S_V|^2}, \quad (3)$$

where S_H (S_V) is the backscattered electric field amplitude at horizontal (vertical) polarization. Physically, the denominator (numerator) in Eq. (3) is proportional to the backscattered signal with handedness opposite to (the same as) the incident beam. The quantities Z_{DR} and CDR measure the scatterers' average degree of nonsphericity and orientation, and can be used with Z_H for hydrometeor classification. In the limiting case of spheres or randomly oriented nonspherical particles, $Z_{DR} \rightarrow 0$ and $\text{CDR} \rightarrow -\infty$.

Seliga and Bringi (1978) calculated the differential reflectivities (at $\lambda = 10$ cm) of oblate spheroids composed of dry ice and oriented with their symmetry axes vertical. Particles having equivolume sphere diameters less than about 3 cm yielded positive values of Z_{DR} while larger particles yielded Z_{DR} values that were negative and as small as -6 dB. To the extent that comparisons are possible, these results are in agreement with the experimental data of Harper (1962) and of Atlas and Wexler (1963). Seliga and Bringi (1978) proposed that a negative Z_{DR} might be used to indicate the presence of large, dry hailstones.

Warner (1978) also calculated the radar backscattering cross sections of oblate ice particles and found them to be in good agreement with the laboratory measurements of Atlas and Wexler (1963). Warner's (1978) calculations of the CDR of oblate ice particles were in general agreement with corresponding measurements by Allan and McCormick (1978).

Aydin et al. (1984) calculated the Z_{DR} of oblate dry and wet models of hailstones having equivolume sphere diameters between 0.1 and 2.7 cm. Water-coated ice

particles produced greater Z_{DR} values than did dry particles of the same size. Oblate ice spheroids with an axial ratio of 0.6, a water coating of 0.5 mm, and diameters between about 1.7 and 2.1 cm yielded values of Z_{DR} exceeding 10 dB. Aydin et al. proposed that Z_{DR} values outside the range associated with rainfall, from 0.2 to 3.5 dB, might be used to indicate the presence of ice hydrometeors, particularly hail. They presented radar observations showing some Z_{DR} values outside the normal range associated with rain, and suggested that they might have been caused by hail.

In a recent paper, Bringi et al. (1984), on the basis of radar observations of two hailstorms, reported that although radar reflectivity could be high for both raindrops and hailstones, Z_{DR} in hailstone shafts was slightly above or below zero ($-0.5 \leq Z_{DR} \leq 0.5$ dB), while in rainfall regions usually $Z_{DR} \geq 1.0$ dB. Bringi et al. surmised that the difference in Z_{DR} signals results from oblate raindrops falling mostly with their minor axes vertical, whereas hailstones have irregular shapes and often tumble.

McCormick et al. (1979) attributed to "Russian and other earlier workers" a possible criterion for hail identification: that the *cancellation* (defined as $-\text{CDR}$) be less than 10 dB. They noted that experimental values of cancellation less than 10 dB were found only for large ice oblate spheroids. Furthermore, McCormick et al. reported observations of cancellations as low as -1 dB, possibly indicating the presence of hail.

3. Backscattering by spongy ice particles

This section presents calculations of the backscattering of 10-cm radiation by spongy ice hailstones in the shape of oblate spheroids. Oblate spheroids having typical hailstone minor/major semiaxis ratios of 0.8 and 0.6 were considered. In calculating the dielectric function of spongy ice [with Eq. (1)], we used the same dielectric functions of ice ($\epsilon = 3.15 + 0.001i$) and water ($\epsilon = 79.98 + 21.44i$) as those used by Aydin et al. (1984). We assume that the symmetry axes of the spheroids are vertical.

Figures 1a and 1b give the normalized (with respect to πc^2 , where c is the semiaxis length along the axis of symmetry) horizontal radar backscattering cross section σ_H for water volume fractions f of 0%, 10%, 20%, 30% and 40%. The abscissa is size parameter ($2\pi c/\lambda$) and equivalent diameter D_{eq} given by

$$D_{eq} = 2a(c/a)^{1/3}, \quad (4)$$

where c/a is the minor/major semiaxis ratio. These figures indicate that the horizontal reflectivity for spongy ice varies greatly as D_{eq} changes, more than it does for pure ice. This variability tended to increase for a given f as the spheroid became more oblate. For particles having D_{eq} less than about 3.5 cm, σ_H of spongy ice spheroids exceeded that of pure ice spheroids. For most volume fractions of water, there were minima in σ_H between D_{eq} of 4.0 and 5.5 cm. Consequently, in this region, the reflectivity of spongy ice

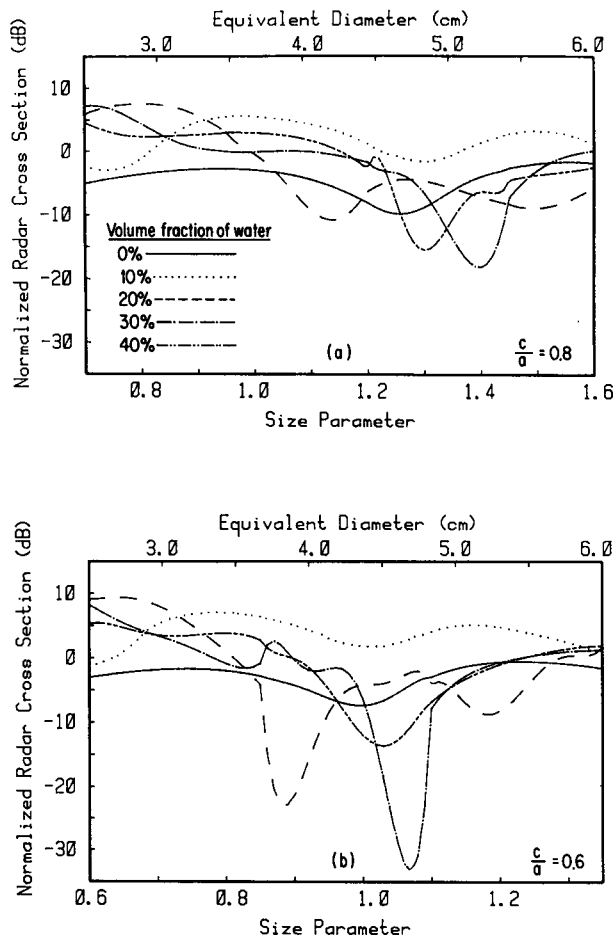


FIG. 1. The normalized radar backscattering cross sections (σ_H) of spongy ice oblate spheroids vs size parameter: (a) $c/a = 0.8$, (b) $c/a = 0.6$.

was mostly less than that of pure ice (for $f = 20\%$, 30% and 40%).

Values of differential reflectivity for semi-axis ratios of 0.8 and 0.6 are given in Figs. 2a and 2b. Our calculations of Z_{DR} agree with those of Aydin et al. (1984) for pure ice spheroids with c/a equal to 0.6 and a D_{eq} of 2.5 cm. For D_{eq} less than about 4 cm, spongy ice oblate spheroids generally have lower Z_{DR} than pure ice. At larger equivalent diameters, there is a gradual reversal, that is, the Z_{DR} of spongy ice exceeds that of pure ice for D_{eq} greater than 5.5 cm in Fig. 2a and D_{eq} greater than 5.0 cm in Fig. 2b. Beyond a D_{eq} of 3.5 cm, Z_{DR} oscillates between positive and negative values with a period (ΔD_{eq}) varying with water content. Also, the magnitude of these oscillations increases with oblateness.

In general, as shown in Figs. 3a and 3b, CDR increases for a given f as oblateness increases. It is highly variable over the range of D_{eq} considered, especially for $f = 30\%$ and 40% . For a given semi-axis ratio, the CDR of spongy ice is generally greater than that of pure ice.

4. Discussion and interpretation

Our calculations of Z_{DR} (Figs. 2a and 2b) for spongy ice spheroids with equivalent diameters between 2.5 and 4 cm and volume fractions of water equal to 20%, 30% and 40% are marginally closer to the measured values of Z_{DR} reported by Bringi et al. (1984) than calculated Z_{DR} of pure ice (for the same equivalent diameters). That is, if such oblate spongy ice hailstones were falling with their symmetry axes vertical, the Z_{DR} would be closer to zero than for pure ice spheroids.

As mentioned earlier, Seliga and Bringi (1978) calculated backscattering by large oblate ice hailstones (D_{eq} greater than 4 cm) oriented with their axes of symmetry vertical, and suggested that a negative Z_{DR} could be used as a criterion for their detection. Based on our spongy ice calculations, it cannot be concluded that $Z_{DR} \leq 0$ for all large preferentially oriented oblate hailstones. The very large negative and positive values of Z_{DR} we have calculated for spongy ice spheroids having equivalent diameters between 3.5 and 6 cm have never been reported in field studies. We note, however, that small changes in equivalent diameter can lead to large changes in Z_{DR} , and therefore a counterbalancing of

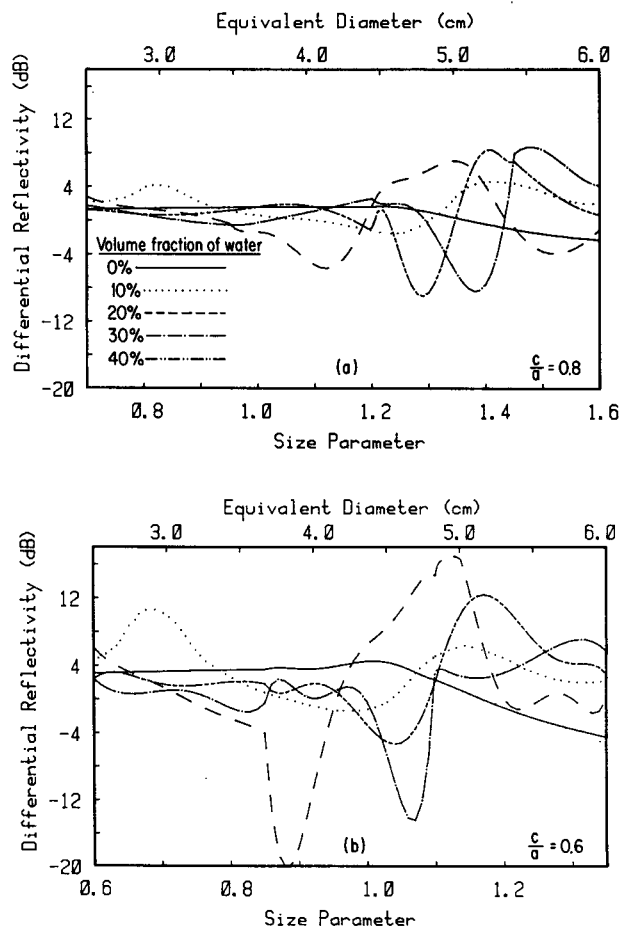


FIG. 2. The differential reflectivity of spongy ice oblate spheroids vs size parameter: (a) $c/a = 0.8$, (b) $c/a = 0.6$.

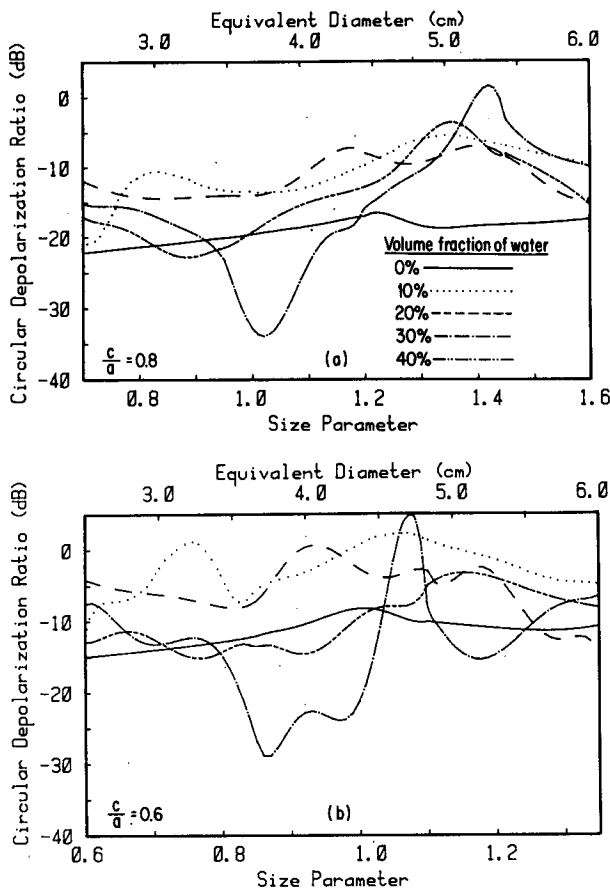


FIG. 3. The circular depolarization ratio of spongy ice oblate spheroids vs size parameter: (a) $c/a = 0.8$, (b) $c/a = 0.6$.

the Z_{DR} may be occurring for hailstones of nearly the same size. That is, for a size distribution of large spongy ice hailstones, Z_{DR} could still be near zero, even if the hailstones were falling with their axes of symmetry vertical. Nevertheless, based on the available evidence, it is more likely that the small Z_{DR} associated with hailstones can be explained by their irregular shapes and tumbling which was concluded by Bringi et al. (1984).

Measurements with a 10-cm radar by McCormick et al. (1979) of the CDR of pure ice oblate spheroids are for size parameters (expressed as $\pi D_{eq}/\lambda$) of 1.0 to 4.5 (hence, D_{eq} between 3.2 and 14.3 cm). For pure ice spheroids with equivalent diameters of 3.2 to 6 cm, our calculations of CDR (Figs. 3a and 3b) are similar to their experimental values; both are between -15 and -10 dB for a minor/major semiaxis ratio of 0.6 and from -22 to -17 dB for a minor/major semiaxis ratio of 0.8. In comparison, calculations of CDR for spongy ice oblate spheroids, with water volume fractions of 10% and 20% are greater than the measurements by McCormick et al. The calculations in Figs. 3a and 3b show that, at some axial ratios and water contents, spongy ice oblate spheroids having diameters between 2.5 and 6.0 cm can lead to CDRs greater than -10 dB (and hence a cancellation less than 10 dB).

This result is consistent with the statement by McCormick et al. (1979): "Cancellations < 10 dB reported by Russian and other earlier workers were interpreted as indicating the presence of hail." This implies that, in some instances, hailstones are spongy ice oblate spheroids having their symmetry axes mostly in the vertical.

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