Application of the Lognormal Raindrop Distribution to Differential Reflectivity Radar Measurement (Z_{DR})

GRAHAM FEINGOLD AND ZEV LEVIN

Department of Geophysics and Planetary Sciences, Raymond and Beverley Sackler Faculty of Exact Sciences, Tel Aviv University, Ramat Aviv, Israel 69978

(Manuscript received 18 July 1986, in final form 21 November 1986)

ABSTRACT

Use of the lognormal form of raindrop size distributions in simulations of differential reflectivity (Z_{DR}) measurements is investigated. Using two remotely measured variables and an empirical relation, the three parameters of the lognormal distribution can be deduced and the spectrum integrated to obtain rain rate. This is demonstrated by a simulation of the Z_{DR} method using ground-based drop size distributions. Drop axis ratio and sampling time effects are also investigated and results compared to those obtained using a gamma distribution. It is shown that the lognormal representation is easily adaptable for use in the Z_{DR} method. Using our dataset, we show that the lognormal size distribution provides lower average absolute deviations of theoretically determined rain rates from actual ones (10.7%) than those obtained using either the exponential (41.0%) or gamma distributions (11.8%).

1. Introduction

In a previous paper, Feingold and Levin, (1986 hereafter FL), showed that the lognormal distribution function provides a good fit to raindrop size distributions (DSDs) in Israel and is particularly useful for tracing the evolution of DSDs. In addition, FL, following Ulbrich's (1981) analysis of the suitability of various functional forms in remote measurement techniques, presented preliminary results strongly suggesting that the lognormal presentation would be advantageous in the dual-parameter radar measurement of rainfall. While it is recognized that the effects of variations in the shape of DSDs are by no means the only source of error in measured rainfall parameters, their influence can at least be significantly reduced when a more accurate representation of the DSD is used (see Ulbrich, 1981; Ulbrich and Atlas, 1984). It is with this aim in mind that we investigate here the applicability of the lognormal distribution in differential reflectivity radar measurements of rainfall (Z_{DR}). In the Z_{DR} method, two orthogonally polarized reflectivity components, Z_H and Z_V, are measured and used to determine two parameters of the size spectrum. Once the latter is known, it can be integrated to obtain the corresponding rain rate or any other rainfall parameter. For details of the Z_{DR} technique, the reader is referred to Seliga and Bringi (1976). Since only two parameters are measured by the Z_{DR} radar, the DSD can at best be described by a two-parameter function. Seliga and Bringi (1976) suggested the exponential distribution, whereas Ulbrich (1983a), recognizing the need for a more flexible function, capable of representing curved spectra, proposed use of the modified gamma distribution:

$$N(D) = N_0 D^{\mu} e^{-(3.67+\mu)D/D_0} \quad [\mathrm{m^{-3}\,mm^{-1}}] \qquad (1)$$

which reduces to the exponential distribution when $\mu = 0$. Although the gamma distribution has three parameters, N_0, μ and D_0 (the median volume diameter), it has been used in its two-parameter form with μ empirically related to N_0 by an expression of the form:

$$N_0 = A e^{b\mu} \quad [\mathrm{m^{-3}\,mm^{-1}}] \qquad (2)$$

(Ulbrich, 1983a). Based on their measurements in Israel, FL showed that the lognormal distribution can be reduced to its two-parameter form, thus making it a viable alternative to the exponential and gamma distributions. The lognormal distribution is given by

$$N(D) = \frac{N_T}{\sqrt{2\pi}\,\ln\sigma D} e^{-\ln^2(D/D_g)/2\ln^2\sigma} \quad [\mathrm{m^{-3}\,mm^{-1}}] \qquad (3)$$

with three parameters, N_T, the drop concentration ($\mathrm{m^{-3}}$), D_g, the geometric mean diameter (mm) and σ, the breadth parameter. FL showed that although σ can vary somewhat for any particular rain rate, on average, σ is independent of rain rate and has a value of approximately 1.4. Similar evidence has been given by Markowitz (1976), who found that σ is equal to about 1.39 over a wide range of rain rates (2.5–150 mm $\mathrm{h^{-1}}$).

In a simulation of the Z_{DR} method, reflectivity parameters Z_H, Z_V and Z_{DR} [$=10\log(Z_H/Z_V)$] have been calculated from distrometer data sampled in central Israel (4601 1-min samples) and the behavior of these parameters analyzed in terms of their dependences on the lognormal parameters. Using a simple procedure, analogous to that proposed by Seliga and Bringi (1976), it will be shown that the two-parameter lognormal distribution can reduce the inherent errors in rain rate estimation introduced by assumption of the exponential form. Moreover, the lognormal distribution produces results on a par with, or even better than, those reported by workers using the gamma distribution.

2. Theory

Falling drops attain the approximate shape of oblate spheroids due to the balance between hydrostatic, surface tension and dynamic stresses. Drop shapes are therefore parameterized according to their minor to major axis ratios (a/b). Using Rayleigh scattering theory for the case of oblate spheroids, as developed by Gans (1912), it can be shown (see Seliga and Bringi, 1976) that the horizontal and vertical reflectivity components, Z_H and Z_V are given by

$$Z_H = \frac{\Lambda^4}{|K|^2\pi^5}\int_0^{D_{\max}} \sigma_H(a/b)N(D_{eq})dD_{eq} \quad [\mathrm{mm}^6\,\mathrm{m}^{-3}] \tag{4a}$$

$$Z_V = \frac{\Lambda^4}{|K|^2\pi^5}\int_0^{D_{\max}} \sigma_V(a/b)N(D_{eq})dD_{eq} \quad [\mathrm{mm}^6\,\mathrm{m}^{-3}] \tag{4b}$$

where D_{eq} is the equivalent diameter of an oblate spheroid, $D_{\max}$ the maximum drop diameter and $\sigma_{H,V}$, (not to be confused with the breadth parameter, σ), the scattering cross sections for an oblate spheroid at horizontal and vertical polarizations. Here Λ is the radar wavelength and K, a function of the refractive index of water. The differential reflectivity (Z_{DR}) is defined by

$$Z_{DR} = 10\log(Z_H/Z_V). \tag{5}$$

With $N(D)$ given by the exponential distribution (Eq. 1 with $\mu = 0$) we find that Z_{DR} is independent of N_0 and can therefore be used to determine the median volume diameter, D_0. Either of the Z_H or Z_V measurements can then be used to determine the second parameter of the distribution, N_0. With N_0 and D_0 known, the spectrum can be integrated to obtain R:

$$R = \frac{\pi}{6}\int_0^{D_{\max}} N(D_{eq})D_{eq}{}^3 v(D_{eq})dD_{eq}. \tag{6}$$

With the inadequacy of the exponential drop-size distribution now well established, the expression for $N(D_{eq})$ in Eq. (6) should be replaced by either the gamma or the lognormal distributions. We propose substitution of the lognormal distribution (3) with σ = constant into Eqs. (4a) and (4b) as follows:

$$\frac{Z_H}{N_T} = \frac{\Lambda^4}{|K|^2\pi^5}\int_0^{D_{\max}} \frac{\sigma_H(a/b)}{\sqrt{2\pi}\ln\sigma D_{eq}} e^{-\ln^2(D/D_g)/2\ln^2\sigma} dD_{eq} \quad [\mathrm{mm}^6] \tag{7a}$$

$$\frac{Z_V}{N_T} = \frac{\Lambda^4}{|K|^2\pi^5}\int_0^{D_{\max}} \frac{\sigma_V(a/b)}{\sqrt{2\pi}\ln\sigma D_{eq}} e^{-\ln^2(D/D_g)/2\ln^2\sigma} dD_{eq} \quad [\mathrm{mm}^6]. \tag{7b}$$

Note that N_T can be removed from the integrals so that Z_{DR} is a function of D_g only. It can easily be shown (see e.g., FL) that the median volume diameter (D_0) is related to D_g by the following relation:

$$D_0 = D_g e^{3\ln^2\sigma}. \tag{8}$$

Measurement of Z_{DR} thus enables direct determination of D_g or D_0. N_T can then be calculated from the measurements of either Z_H [Eq. (7a)] or Z_V [Eq. (7b)]. The method is completely analogous to that proposed by Seliga and Bringi (1976), but has the advantage that it avails itself of the superior fit of the lognormal distribution (albeit a two parameter version). Theoretical curves showing the dependence of Z_{DR} and $10\log(Z_H/N_T)$ on D_g are shown in Fig. 1a, b for various values of σ. For the reader more familiar with D_0 as the size parameter, these curves are reproduced in Fig. 2a, b with Z_{DR} and Z_H/N_T this time plotted vs D_0. Superimposed on the curves in Fig. 1a, b are the data points representing two parameter lognormal fits to the measured spectra. The method of fit is one which calculates D_g and N_T from the third and sixth moments of the spectrum (liquid water content and radar reflectivity, respectively), while keeping σ at a fixed value (in this case, 1.4). The curves are obtained by Gaussian-quadrature numerical integration of Eqs. (7a) and (7b) for $D_{\max}$ = 7 mm, a wavelength of 10 cm, a refractive index of water of $m = 8.99 + 1.47i$ and axis ratios of Pruppacher and Pitter (1971). The latter are similar to the equilibrium values calculated by Green (1975). The work of Pruppacher and Beard (1970) and that of Cooper et al. (1983) suggest that drops are more oblate. On the other hand, Jameson and Beard (1982) argue that collision induced oscillations encountered in natural rainfall increase the axis ratios (decrease the oblateness) to values above those at equilibrium. In light of the disagreement in the published literature, we have chosen the Pruppacher and Pitter data which lie within the range of proposed values. For an analysis of the sensitivity of the Z_{DR} method to drop oscillations (periodic variations in the axis ratio), see Seliga et al. (1984). Figs. 1a and 1b show how when the values of Z_{DR} and Z_H/N_T are plotted against D_g for all the data points (each point representing 1 min of rain), the points follow the theoretical curves for σ = 1.4 throughout the range. In fact, in the case of Fig. 1b, there is no discernible deviation of the data points from the theoretical curve.

3. Results and discussion

In order to check the validity of this approach, the DSD data sampled in Hadera, Israel will be used. (For details, see FL). With dual polarization measurements

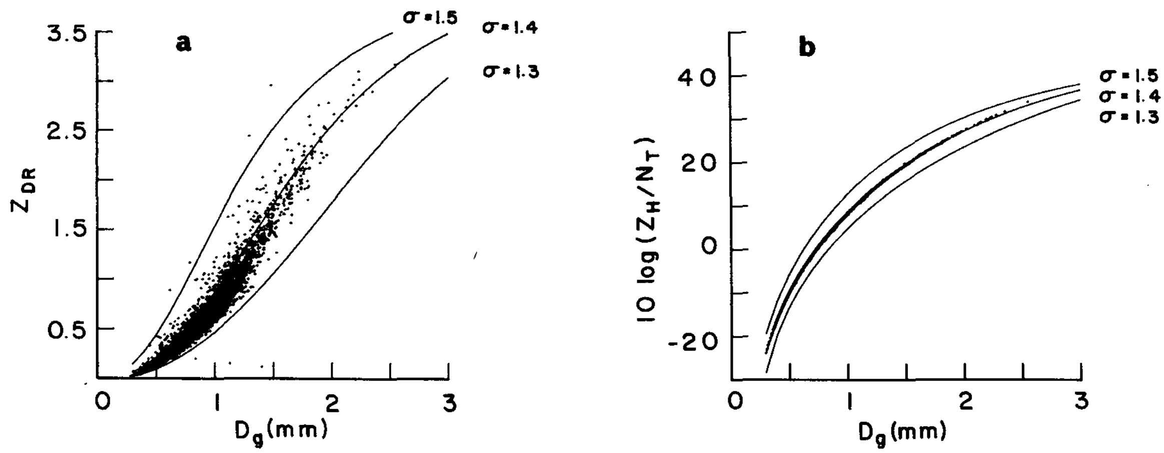

FIG. 1a. Theoretical curves of (a) Z_{DR} and (b) 10 log(Z_H/N_T) vs D_g for various values of σ. Superimposed on the curves are data points representing third and sixth moment weighted fits to the observed spectra. Note that the points in (b) are coincident with the theoretical curve.

not at our disposal, the only way of assessing the viability of this method is to change the inverse problem to the forward problem—i.e., Z_{DR} and Z_H will be assumed to be known (their values will be calculated directly from the measured spectra) and the theoretical curves in Figs. 1a and 1b will be used to determine distribution parameters D_g and N_T for a constant value of σ. This approach has the salutary effect of focusing attention on the effects of distribution shape on the measurement of rainfall.

Using the values of Z_{DR} and Z_H computed directly from the spectra, the rain rates will now be calculated. These theoretically estimated rain rates (R_T) will then be compared to the actual rain rates (R_D) as calculated from the distrometer measurements. R_T is plotted vs R_D in Fig. 3. The least-squares fit to the points is represented by the line:

$$R_T = 1.099 R_D + 0.108 \quad (9)$$

with a standard error of estimate (SEE) of 0.581 mm h^{-1}. The absolute average deviation of R_T from R_D, defined by $\Delta = |R_T - R_D|/R_D$, is 22.3%.

The dependence of Δ on the values of D_g and N_T can be considered as follows. From Figs. 1a and 1b it is clear that if D_g is erroneously overestimated by $+\Delta D_g$ then Eq. (7b) will overestimate the value of Z_H/N_T, or underestimate N_T by a value of $-\Delta N_T$. In other words, an error of $+\Delta D_g$ incurs an error of $-\Delta N_T$, and because R_T is directly dependent on both D_g and N_T the errors tend to counter one another. This can be analyzed using the theoretical form of the lognormal distribution in Eq. (6). Dropping the subscript "eq" for brevity, we can write:

$$R = \frac{\pi}{6}\int_0^{D_{\max}} N(D)D^3 v(D)dD = c_n N_T D_g^{\,3.67} \quad [\mathrm{mm\,h^{-1}}] \quad (10)$$

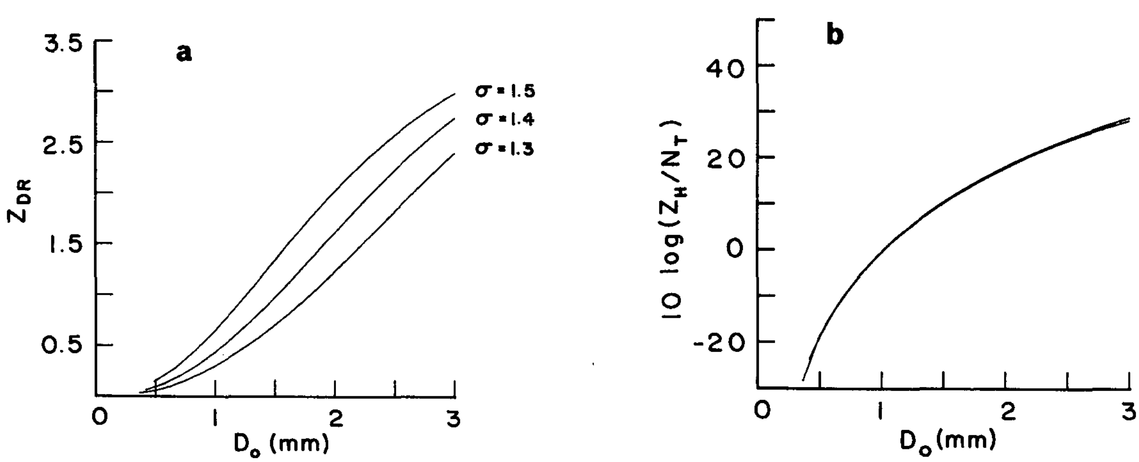

FIG. 2. Theoretical curves of (a) Z_{DR} and (b) 10 log(Z_H/N_T) vs D_0 for the various values of σ. In the case of (b) the curves almost coincide.

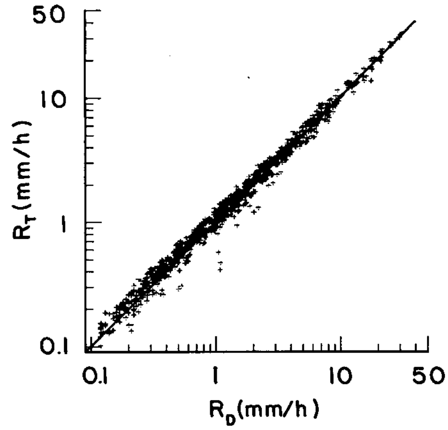

FIG. 3. R_T (the theoretically determined rain rates) plotted as a function of R_D (actual rain rates). Each point represents a 5-min averaged distribution.

where $c_n = 7.1 \cdot 10^{-3} e^{6.73 \ln^2 \sigma}$ and $v(D)$ in Eq. (10) has been approximated by the power law $v(D) = 3.77 D^{0.67}$ (v in m s^{-1} and D in mm) after Atlas and Ulbrich (1977). Following the previous discussion,

$$R_T = c_n (N_T - \Delta N_T)(D_g + \Delta D_g)^{3.67} \quad (11)$$

can then be written as:

$$\Delta = \frac{R_T}{R_D} - 1 = (1 - \Delta N_T/N_T)(1 + \Delta D_g/D_g)^{3.67} - 1. \quad (12)$$

Clearly, Δ is inversely proportional to both D_g and N_T, with a much stronger dependence on D_g. Consequently, Δ is expected to decrease with increasing R (see also Ulbrich and Atlas, 1984). Since the vast majority of our 4601 data points (1 min sampling time) are for rain rates below 5 mm h^{-1}, and only a few points have rain rates greater than 40 mm h^{-1} (see Fig. 1, FL), a value of Δ = 22% is a promising result.

a. Effect of increasing sampling time

In using distrometer measured DSDs in this analysis, we implicitly assume that they are representative of the drop spectra in the volume of air scanned by the radar. However, the sampling volumes of the radar and distrometer differ significantly (km^3 vs m^3, for a sampling time of 1 min). In order to equate these volumes, various workers (see, for example, Austin, 1981) have proposed equating large-volume averaged radar samples with long-time averaged distrometer samples. The results so far presented have been for 1 min time averaged samples. Goddrad and Cherry (1984) have used 30-sec averaged samples, whereas Ulbrich (1983b) used 7-min averages in obtaining his results (quoted later in section 3). The results obtained when a sampling time of 5 min is used on our data are as follows; the equation for the best fit line to R_T vs R_D is given by

$$R_T = 1.033 R_D + 0.057 \quad (13)$$

with an SEE of 0.469. The corresponding value of Δ is reduced to 13.9%. Note that these results are a great deal better than those previously obtained. Clearly, the choice of sampling time has a significant bearing on the results. In fact, if the sampling time is further increased to 7 min then the value of Δ is reduced to 12.1%. At low rain rates, where the number of sampled drops is often small, longer sampling times are required in order to obtain better defined distributions. Thus, use of 5-min averaged distributions in these simulations is justified considering that the majority of rain rates in this dataset are less than 1 mm h^{-1}.

b. Comparison with a two parameter gamma distribution

Widespread use has been made of the gamma distribution of Z_{DR} measurements, and this discussion would not be complete without a comparison of the results obtained for a gamma distribution with those quoted above for the lognormal representation. We use a gamma distribution [Eq. (1)] with μ = constant rather than the $N_0 - \mu$ relationship [Eq. (2)] since it has been shown by FL as well as Chandrasekhar and Bringi (1986) that such a relation may lead to inaccuracies, especially for large μ. Fitting the gamma distribution to our data using the weighted moment method suggested by Ulbrich (1983a) and then plotting the derived values of μ as a function of R_D, it was found that a least-squares fit to the data points yielded a value of μ = 6.5. Using the latter and a sampling time of 5 min, we find that

$$R_T = 0.868 R_D + 0.118 \quad (14)$$

with a SEE of 1.157 and Δ = 12.3%. Table 1 summarizes the values of Δ and the parameters of the least-squares regression lines (A, the slope parameter and B, the intercept parameter) for both the lognormal and gamma distributions for sampling times of 1 and 5

TABLE 1. A comparison of the values of Δ (average absolute deviation), slope and intercept parameters of the regression lines (A and B, respectively) and standard error of estimate (SEE) for the lognormal and gamma distributions using sampling times of 1 min and 5 min and the axis ratios of PP and PB.

Sampling time (min)	A	B	Δ %	SEE	Axis ratio
		Lognormal (σ = 1.4)			
1	1.086	0.003	15.5	0.716	PB
5	1.027	0.005	10.2	0.383	PB
1	1.099	0.108	22.3	0.581	PP
5	1.033	0.057	13.9	0.469	PP
		Gamma (μ = 6.5)			
1	0.934	0.127	19.1	1.310	PB
5	0.869	0.106	11.1	1.150	PB
1	0.934	0.163	22.1	1.280	PP
5	0.868	0.118	12.3	1.157	PP

min. Also included is a comparison of the results produced by the Pruppacher and Beard (1970) (henceforth PB) axis ratios with those obtained using Pruppacher and Pitters' (1971) (henceforth PP) data. The following points are evident from Table 1.

(i) Although the value of Δ obtained using the gamma distribution is a little lower than that of the lognormal (12.3% as opposed to 13.9% for a sampling time of 5 min), the parameter A of the former deviates from the ideal by −13% (i.e., underestimates R_D) whereas that of the latter deviates by only +3% (overestimates R_D). In the case of the more oblate PB axis ratios, however, the value of Δ obtained using the lognormal is lower than that for the gamma distribution (10.3% as opposed to 11.1%) while the A parameters are similar to those found for the PP axis ratios.

(ii) For both distributions lower values of Δ are obtained when 5-min averaged distributions are used rather than 1-min averages.

(iii) Regardless of which axis ratios are employed, the lognormal values of SEE tend to be lower than those of the gamma distribution.

(iv) The more oblate a/b ratios of PB produce better results than those obtained using the PP data. This can be ascribed to the fact that the success of the Z_{DR} method rests entirely on the degree of oblateness of the drops. Beard (1984) has suggested that axis ratios are in fact a function of rain rate. This premise is based on the assumption that higher rain rates are characterized by more frequent drop collisions and more substantial deviations from equilibrium axis ratios. Adopting this premise, the drop distributions with low rain rates which constitute the majority of this dataset should therefore exhibit axis ratios close to their equilibrium values, thus enhancing the success of the Z_{DR} method.

(v) The parameter A of the least-squares fit can be "corrected" by adjusting the values of μ to 4.5 and σ to 1.37 (see Table 2). In this case (using the PP axis ratios), we find that Δ (lognormal) = 10.7% and Δ (gamma) = 11.8% (i.e., the lognormal presentation performs better. Table 2 also includes a comparison with the values obtained using an exponential distribution). Clearly, the latter produces unacceptably high values of Δ (41.0%).

(vi) The results presented in Table 2 seem to suggest that for application to Z_{DR}, the weighted moment fit to DSDs proposed by Ulbrich (1983a) places too much emphasis on the large drop component of the spectrum and overestimates μ (i.e., produces distributions which are too narrow). Decreasing this value of 4.5 (broadening the spectrum) corrects the gamma distribution's consistent underestimation of rain rate evident in Table 1. Similarly, Table 2 also suggests that the method of weighted fit proposed by FL, which places homogeneous weighting throughout the spectrum, tends to overestimate the rain rate deduced by these Z_{DR} simulations by overestimating σ (i.e., producing distributions which are too broad). The above discussion underlines the importance of choosing a correct value of the breadth parameter (be it σ or μ) if satisfactory results are to be obtained.

TABLE 2. A comparison of the *optimum* values of Δ (average absolute deviation), slope and intercept parameters of the regression lines (A and B, respectively) and standard error of estimate (SEE) for the lognormal, gamma and exponential distributions, using a 5 min sampling time and the axis ratios of PP.

	A	B	Δ %	SEE
Lognormal $\sigma = 1.37$	0.990	0.028	10.7	0.425
Gamma $\mu = 4.3$	0.996	0.002	11.9	0.418
Exponential	1.289	0.117	41.0	0.621

Note that two factors have been isolated in the above discussion, one being the effect of drop distribution shape and the other, the effect of the drop axis ratios on Z_{DR} measurements. If we use Δ as a measure of the success of the method, then DSD effects show (section 3a) that Δ is indirectly dependent on rain rate and hence that better results would be achieved under high rain rate conditions. The effect of the axis ratios works in exactly the opposite direction. If one adopts Beard's (1984) assumption that the greater the rain rate, the more frequent the collisions and the less oblate the drops, then one would expect the effectiveness of the Z_{DR} method to be reduced under such conditions. On the other hand, recent measurements by Caylor and Illingworth (1986) suggest that large Z_{DR} signals (up to 6 dBZ) may be obtained at high rain rates. To explain this phenomenon, they conclude that large drops are more oblate than their equilibrium values. Clearly, more measurements of the shapes of natural rain drops at various rain rates are required to resolve this issue.

We can compare the aforementioned results to those found by other workers. Ulbrich (1983) has used a three-parameter form of the gamma distribution and the empirical relation relating N_0 to μ [Eq. (2)] and found a value of Δ = 11.9%. Using a two-parameter gamma distribution with a constant value of $\mu = 2$, Ulbrich (1983b) found a deviation of 13.5%. Goddard and Cherry (1984) used a gamma distribution with a value of $\mu = 5$ and applied it to their ground-based distrometer measurements of spectra. They found that the systematic offset of R_T from R_D is 1%, with a standard deviation of 14%. Using a gamma distribution and an $N_0 - \mu$ relationship, Goddard and Cherry found an offset of 6% and a deviation of the order of 14%. It should be noted that both these datasets appear to have proportionately higher numbers of points at high rain rates than did our dataset. Thus, the values of Δ obtained using the lognormal distribution function compare very favorably (especially for 5-min averaged distributions) with these aforementioned results.

4. Conclusions

The evidence produced here of the applicability of the lognormal distribution to dual parameter remote measurement techniques corroborates our preliminary results as discussed in FL. We show here how a two-parameter lognormal function can be adapted for use in the dual parameter Z_{DR} method. With a constant value of σ in Eq. (7a, b), Z_{DR} measurements can be used to calculate D_g. Then N_T can in turn be calculated from Z_H. Rain rates determined using Eq. (6) and the deduced values of D_g and N_T are shown to deviate from distrometer measured rain rates by between 13.9% and 22.3%, for 5-min and 1-min averaged distributions, respectively, with a standard error of estimate of about 0.5 mm h^{-1}. These values compare favorably with those obtained using the gamma size distribution representation. If, on the other hand, the value of σ is adjusted to 1.37 (i.e., a little lower than the empirically determined value), Δ is reduced to only 10.7% (using the PP axis ratios and a sampling time of 5 min). This result was not equaled using a gamma distribution with μ = constant.

The results of these simulated Z_{DR} methods in which measured DSDs are used to deduce Z_H, Z_V and Z_{DR} values rather than using the real radar measured reflectivity parameters are sensitive to: (i) the relative number of events at high rain rates, (ii) the distribution sampling time, (iii) the drop axis ratios (a/b) and (iv) the choice of the distribution breadth parameter.

The lognormal distribution has a wide range of applications in rainfall studies and, as shown here, can also be successfully employed in the Z_{DR} and other dual-parameter techniques. Although our results show that both the gamma and lognormal distributions perform admirably well, and are certainly superior to the exponential form, we feel, on the basis of these results and those previously presented by Feingold and Levin (1986), that the lognormal distribution is to be preferred.

REFERENCES

Atlas, D., and C. W. Ulbrich, 1977: Path and area-integrated rainfall measurement by microwave attenuation in the 1–3 cm band. *J. Appl. Meteor.*, **16**, 1322–1331.

Austin, P. M., 1981: On deducing rainfall from radar reflectivity measurements. *Conf. Radar Meteorology,* Boston, Amer. Meteor. Soc., 200–207.

Beard, K. V., 1984: Oscillation models for predicting raindrop axis and backscatter ratios. *Radio Sci.*, **19**, 67–74.

Caylor, I. J., and A. J. Illingworth, 1986: Observations of the growth and evolution of raindrops using dual-polarization radar. *Conf. Radar Meteorology,* Snowmass, Amer. Meteor. Soc., 88–91.

Chandrasekhar, V., and V. N. Bringi, 1986: Estimators of dropsize distribution parameters from raindrop sampling devices. *Conf. Radar Meteorology and Cloud Physics,* Snowmass, Amer. Meteor. Soc., JP154–JP157.

Cooper, W. A., V. N. Bringi, V. Chandrasekhar and T. A. Seliga, 1983: Analyses of raindrop parameters using a 2-D precipitation probe with application of differential reflectivity. *Conf. Radar Meteorology,* Edmonton, Amer. Meteor. Soc., 488–493.

Feingold, G., and Z. Levin, 1986: The lognormal fit to raindrop spectra from frontal convective clouds in Israel. *J. Climate Appl. Meteor.*, **25**, 1346–1364.

Gans, R., 1912: Über die form ultramikroskopischer goldteilchen. *Ann. Phys.* (Leipzig), **37**, 881–900.

Goddard, J., and S. Cherry, 1984: Quantitative precipitation measurements with dual linear polarisation data. *Conf. Radar Meteorology,* Zurich, Amer. Meteor. Soc., 352–357.

Green, A. W., 1975: An approximation for the shapes of large raindrops. *J. Appl. Meteor.*, **14**, 1578–1583.

Jameson, A. R., and K. V. Beard, 1982: Raindrop axial ratios. *J. Appl. Meteor.*, **21**, 257–259.

Markowitz, A., 1976: Raindrop size distribution expressions. *J. Appl. Meteor.*, **15**, 1029–1031.

Pruppacher, H. R., and K. V. Beard, 1970: A wind tunnel investigation of the internal circulation and shape of water drops falling at terminal velocity in air. *Quart. J. Roy. Meteor. Soc.*, **96**, 247–256.

——, and R. L. Pitter, 1971: A semi-empirical determination of the shape of cloud and raindrops. *J. Atmos. Sci.*, **28**, 86–94.

Seliga, T. A., and V. N. Bringi, 1976: Potential use of radar-differential reflectivity measurements at orthogonal polarizations for measuring precipitation. *J. Appl. Meteor.*, **15**, 69–76.

——, K. Aydin and V. N. Bringi, 1984: Differential reflectivity and circular depolarization ratio radar signals and related drop oscillation and propagation effects in rainfall. *Radio Sci.*, **19**, 81–89.

Ulbrich, C. W., 1981: Effect of size distribution variations on precipitation parameters determined by dual-measurement techniques. *Conf. Radar Meteorology,* Boston, Amer. Meteor. Soc., 276–281.

——, 1983a: Natural variations in the analytical form of the raindrop size distribution. *J. Climate Appl. Meteor.*, **22**, 1764–1775.

——, 1983b: An empirical method of accounting for variations in the form of the raindrop size distribution in dual measurement techniques. *Conf. Radar Meteorology,* Edmonton, Amer. Meteor. Soc., 317–322.

——, and D. Atlas, 1984: Assessment of the contribution of differential polarization to improved rainfall measurements. *Radio Sci.*, **19**, 49–57.