An Interpretation of Circular Polarization Measurements
Affected by Propagation Differential Phase Shift

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ABSTRACT

Circularly polarized waves are created by adding horizontally and vertically linearly polarized waves of equal magnitude with a phase difference of 90°. As a transmitted wave propagates through precipitation, which is usually not spherical, the relative phase between the horizontal and vertical components of the forward scattered wave from each particle is shifted slightly from 90°. The addition to the transmitted wave of the forward scattered waves from all the particles causes the net propagating wave to become more and more elliptically polarized. This propagation differential phase shift leads to biases in parameters estimated as though the polarization were still circular and can therefore obscure the correct interpretation of the observations in terms of the characteristics of the precipitation.

In this paper, a method is developed for recovering unbiased circular polarization parameters from measurements biased by propagation differential phase shift. This is accomplished by first expressing the circular polarization quantities as functions of the linear polarization parameters. These equations are combined to eliminate terms affected by propagation differential phase shift. The resulting expressions are then solved to determine the linear polarization parameters. Finally, the circular polarization quantities can be recomputed from these estimates of the linear polarization parameters.

An advantage of this approach is that even when circular polarization measurements are biased by propagation differential phase shift, they can still be transformed into linear polarization parameters unaffected by propagation. These unbiased linear quantities can then be used to characterize the precipitation. It is shown that the standard circular polarization parameters of depolarization ratio ($\Gamma$) and the complex cross-correlation function ($\nu$) provide estimates of the linear polarization differential reflectivity ($\rho_L$) and the magnitude ($\rho_L$) of the linear cross-correlation function ($\nu_L$) which are free of propagation phase effects. The $\rho_L$ may not only lead to improved rainfall estimates from differential reflectivity, but also helps clarifies some puzzling aspects of polarization measurements in the melting layer.

If a radar has the additional capability of switching rapidly between right- and left-handed circular polarizations, the cross-correlation function ($\nu_{RL}$) can be computed between the resulting two copolarizations. It is then shown that even when a propagation differential phase shift is occurring, $|\nu_{RL}|, \Gamma$, and $\text{Re}(\nu_{RL})$ can be transformed into $|\nu_L|, \rho_L$, and $\text{Re}(\rho_{RL})$. This transformation is particularly useful since it provides the measurements necessary for estimating the variance of particle cattng.

1. Introduction

Historically, circular polarization for radars was developed in order to minimize the impact of precipitation upon the detection of targets such as airplanes (e.g., White 1954). It was recognized that when circular polarization is backscattered from spheres, most of the returned power appears in the cross-polarized (opposite sense of polarization from that transmitted) channel. Since, to a first approximation, hydrometeors are often nearly spherical, aircraft could still be detected in the copolarized channel (same sense of polarization as the transmission) even in precipitation "clutter". Precipitation particles are, however, rarely spherical. Consequently, usually some returned signal is in the co-

polarized channel. It was also observed that the strength of this "clutter" signal often increased with increasing penetration into the precipitation. A return to classical optics revealed that this effect could be explained by the phenomenon of propagation differential phase shift.

A transmission with circular polarization is constructed by adding horizontally and vertically linearly polarized waves with a phase difference of 90°. In precipitation, however, forward scattering generates a wave which is then added to the original transmission. Because most precipitation particles are not spherical, this wave has a phase between the horizontal and vertical components slightly shifted from 90°. After the transmitted wave propagates through billions and billions of precipitation particles, the phase between the horizontal and vertical components of the propagating wave can become significantly altered from 90°. As a result, more and more power oozes from the copolarization channel into the cross-polarization one.

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In addition, as the waves propagate through precipitation the magnitudes of their amplitudes are reduced by attenuation. Because most hydrometeors are not spheres, the effect of attenuation on the horizontal and vertical components of circularly polarized waves will not be the same. This is the phenomenon of differential attenuation. Fortunately, at sufficiently long wavelengths (such as 10 cm), attenuation in general and differential attenuation in particular shrink to insignificant levels. Unfortunately, though, even at 10 cm wavelength, propagation differential phase shift is still important.

Partly because of the desire to detect hail (Gershenzon and Schupiatskii 1961; McCormick et al. 1972; Barge 1972, 1974), circular polarization measurements in precipitation quickly dominated polarization research. While a number of unique and potentially interesting measurements in precipitation were collected, especially in hailstorms and severe thunderstorms, there has always been a nagging uncertainty about the magnitude of the bias due to propagation differential phase shift even at long radar wavelengths (Humphries 1974; Seliga et al. 1984). While theory suggests that circular polarization measurements in precipitation potentially contain a wealth of information (McCormick and Hendry 1975; Jameson 1983a; Metcalf 1984, 1986), attempts to analyze actual data immediately resurrected suspicions of obfuscation by propagation differential phase shift. These uncertainties were compounded by computations and measurements (e.g., Humphries 1974) which suggested a rapid degeneration of polarization after even brief encounters with heavy precipitation.

In this paper a method is developed for removing the effects of propagation differential phase shift from circular polarization measurements. This is achieved by first reexpressing the circular polarization quantities in terms of linear polarization parameters. It is then possible to identify and remove those terms in the equations which are affected by propagation differential phase shift. The unbiased quantities can then be used to characterize the precipitation.

2. Circular polarization parameters as functions of linear polarization parameters

a. Autocorrelations and cross correlations for circular polarizations

If \( S_L \) and \( S_C \) denote the backscatter matrices corresponding to linear and circular polarizations, then

\[
S_L = \begin{pmatrix}
S_{HH} & S_{HV} \\
S_{VH} & S_{VV}
\end{pmatrix},
\]

\[
S_C = \begin{pmatrix}
S_{RR} & S_{RL} \\
S_{LR} & S_{LL}
\end{pmatrix},
\]

where

\[
\begin{align*}
S_{RR} &= (S_{VV} - S_{HH} - 2jS_{HV})/2 \\
S_{LL} &= (S_{VV} - S_{HH} + 2jS_{HV})/2 \\
S_{RL} &= S_{LR} = (S_{HH} + S_{VV})/2
\end{align*}
\]

and \( j \) denotes the imaginary number \( \sqrt{-1} \). The first and second subscripts identify the polarization state of the transmitted and received waves, respectively; \( V, H, R, \) and \( L \) denote vertical linear, horizontal linear, right-handed circular, and left-handed circular polarizations, respectively. Since \( S_L \) and \( S_C \) relate to the incident to the scattered electromagnetic fields, the effects of radar elevation angle and particle orientation (e.g., canting) with respect to the plane of propagation are contained within their elements (Atlas et al. 1953; Barge 1972; Jameson 1985a, 1986).

In general, the signals measured by radars result from the superposition of the waves scattered independently by all the hydrometeors in the illuminated volume. At each instant it follows for linear polarizations that

\[
\begin{align*}
E_{HH} &= C \sum_i S'_{HH} \exp[\jmath \psi_i + \Phi_H^i] \\
E_{VV} &= C \sum_i S'_{VV} \exp[\jmath \psi_i + \Phi_V^i] \\
E_{HV} &= C \sum_i S'_{HV} \exp[\jmath \psi_i + \Phi_{HV}^i] \\
E_{VH} &= C \sum_i S'_{VH} \exp[\jmath \psi_i + \Phi_{VH}^i]
\end{align*}
\]

where

\[
\begin{align*}
\Phi_H^i &= 2\phi_H^i + \delta_{H}^i \\
\Phi_V^i &= 2\phi_V^i + \delta_{V}^i \\
\Phi_{HV}^i &= \phi_H^i + \phi_V^i + \delta_{HV} \\
\Phi_{VH}^i &= \phi_H^i + \phi_V^i + \delta_{VH}
\end{align*}
\]

Here \( \phi_k^i \) denotes the propagation phase shift corresponding to polarization \( k \); \( \delta \) denotes the phase shift induced because of the backscatter process; and \( \psi_i = 4\pi n_i/\lambda_i \) is the phase position of the \( i \)th scatterer at range \( r_i \) from the radar operating at wavelength \( \lambda_i \).

For circularly polarized waves

\[
E_{RL} = \frac{C}{2} \sum_i \{S'_{HH} \exp(\jmath \Phi_H^i) \\
+ S'_{VV} \exp(\jmath \Phi_V^i)\} \exp(\jmath \psi_i),
\]

\[
E_{RR} = \frac{C}{2} \sum_i \{S'_{VV} \exp(\jmath \Phi_V^i) - S'_{HH} \exp(\jmath \Phi_H^i) \\
- 2jS'_{HV} \exp[\jmath (\Phi_{HV}^i)]\} \times \exp(\jmath \psi_i),
\]

\[
E_{RL} = \frac{C}{2} \sum_i \{S'_{HH} \exp(\jmath \Phi_H^i) \\
+ S'_{VV} \exp(\jmath \Phi_V^i)\} \exp(\jmath \psi_i),
\]

\[
E_{RR} = \frac{C}{2} \sum_i \{S'_{VV} \exp(\jmath \Phi_V^i) - S'_{HH} \exp(\jmath \Phi_H^i) \\
- 2jS'_{HV} \exp[\jmath (\Phi_{HV}^i)]\} \times \exp(\jmath \psi_i),
\]
\[ E_{LR} = \frac{C}{2} \sum \{ S'_{HH} \exp(j\Phi_{H'}) + S'_{VV} \exp(j\Phi_{V'}) \} \exp(j\psi), \]

\[ E_{LL} = \frac{C}{2} \sum \{ S_{VV} \exp(j\Phi_{V'}) - S'_{HH} \exp(j\Phi_{H'}) \] 

\[ + 2jS_{HV} \exp[j(\Phi_{H'})] \} \times \exp(j\psi), \quad (4) \]

where the summation is over all the particles in the sampling volume. Since \( S_C \) and \( S_L \) depend upon the radar elevation angle as well as the orientation of the scatterers with respect to the plane of propagation, so do all of the backscattered waves. Before and after backscatter, these waves can also be altered by propagation phase shift and differential attenuation. For a short radar pulse length the dependence of intensity on the distance to the center of the sampling volume is essentially constant and is included in the coefficient \( C \). The illumination is assumed uniform for all polarizations.

With time, radar signals fluctuate largely because of motion of the particles with respect to one another which causes random constructive and destructive interference among the waves backscattered from each particle. Polarization parameters, therefore, must be estimated from time averages of elements of autocorrelations and cross correlations of signals. After averaging, the random phases tend to cancel, leaving only the mean values. If the averaging is long, it is often assumed that essentially all realizations of the ensemble of particles have been sampled so that the time average and ensemble average may be equated. From (3) and (4) the circular polarization autocorrelations can be written as

\[ |E_{RR}|^2 = \frac{1}{4} \left\{ |E_{HH}|^2 + |E_{VV}|^2 + 4|E_{HV}|^2 \right\} - 2\Re\langle E_{HH}E_{V'} \rangle + 4\Im\langle (E_{HH} - E_{VV})E_{HV}^* \rangle, \quad (5a) \]

\[ |E_{LL}|^2 = \frac{1}{4} \left\{ |E_{HH}|^2 + |E_{VV}|^2 + 4|E_{HV}|^2 \right\] 

\[ - 2\Re\langle E_{HH}E_{V'} \rangle - 4\Im\langle (E_{HH} - E_{VV})E_{HV}^* \rangle, \quad (5b) \]

while the cross correlations are

\[ |E_{RL}|^2 = |E_{LR}|^2 \]

\[ = \frac{1}{4} \left\{ |E_{HH}|^2 + |E_{VV}|^2 + 2\Re\langle E_{HH}E_{V'} \rangle \right\}, \quad (5c) \]

\[ |E_{RR}|^2 = |E_{LL}|^2 = \frac{1}{4} \left\{ |E_{HH}|^2 + |E_{VV}|^2 \right\] 

\[ + 4|E_{HV}|^2 - 2\Re\langle E_{HH}E_{V'} \rangle, \quad (5a) \]

\[ \langle E_{RR}E_{RL}^* \rangle = (|E_{RR}|^2|E_{RL}|^2)^{0.5}\nu_{RC}, \]

\[ = (|E_{RR}|^2|E_{RL}|^2)^{0.5}\rho_c \exp(j\Phi_c), \]

\[ = \frac{1}{4} \left\{ |E_{VV}|^2 - |E_{HH}|^2 - 2 \right\] 

\[ \times \Im\langle (E_{HH} + E_{VV})E_{HV}^* \rangle 

\[ + 2j\Im\langle E_{HH}E_{V'} \rangle 

\[ + \Re\langle (E_{HH} + E_{VV})E_{HV}^* \rangle \right\}, \quad (5d) \]

\[ \langle E_{LL}E_{RL}^* \rangle = (|E_{LL}|^2|E_{RL}|^2)^{0.5}\nu_{LC}, \]

\[ = \frac{1}{4} \left\{ |E_{VV}|^2 - |E_{HH}|^2 + 2 \right\] 

\[ \times \Im\langle (E_{HH} + E_{VV})E_{HV}^* \rangle 

\[ + 2j\Im\langle E_{HH}E_{V'} \rangle 

\[ - \Re\langle (E_{HH} + E_{VV})E_{HV}^* \rangle \right\}, \quad (5e) \]

\[ \langle E_{RR}E_{LL}^* \rangle = (|E_{RR}|^2|E_{LL}|^2)^{0.5}\nu_{RL}, \]

\[ = \frac{1}{4} \left\{ |E_{HH}|^2 + |E_{VV}|^2 - 4|E_{HV}|^2 - 2 \right\] 

\[ \times \Re\langle E_{HH}E_{V'} \rangle + 4j \]

\[ \times \Re\langle (E_{HH} - E_{VV})E_{HV} \rangle \right\}, \quad (5f) \]

where \( \langle \rangle \) denotes time averaging; \( |E|^2 = \langle EE^* \rangle \); and the complex number \( \nu \) denotes various cross-correlation functions.

Since \( E_{HH} \) and \( E_{VV} \) (or \( E_{RR} \) and \( E_{LL} \)) cannot be observed simultaneously, their measurement must be separated by one or more interpulse periods \( \tau \). The cross correlations \( \langle E_{HH}E_{V'} \rangle \) and \( \langle E_{RR}E_{LL}^* \rangle \) must be formed, therefore, from proper sequencing of alternating copolarizations (Jameson and Mueller 1985; Sachidananda and Zrnić 1986) to eliminate the phase shift due to the motion of the particles during \( \tau \). For simplicity, \( E_{V'} \) is replaced by \( E_{HV} \) as permitted by the reciprocity theorem (Saxon 1955).

Simplification of (5) is possible. First, \( |E_{RR}|^2 \) must equal \( |E_{LL}|^2 \) since \( |S_{RR}|^2 = |S_{LL}|^2 \) so that \( \Im\langle (E_{HH} - E_{VV})E_{HV}^* \rangle \) must be zero. Second, regardless of particle canting, the cross correlations \( \langle E_{HH}E_{HV}^* \rangle \) and \( \langle E_{VV}E_{V'} \rangle \) will probably be very small, at least for Rayleigh–Gans scatterers, since the large number of scatterers in a typical radar sampling volume will tend to guarantee that the contributions to the cross polarization will be nearly symmetrically distributed with respect to the local true vertical. It follows then that
\[ \nu_C = \nu_{RC} = \nu_{LC}, \]
\[ = \frac{|E_{VV}|^2 - |E_{HH}|^2 + 2j \text{Im}(E_{HH}E_{VV}^*)}{(|E_{HH}|^2 + |E_{VV}|^2 + 4|E_{HV}|^2 - 2 \text{Re}(E_{HH}E_{VV}^*)) \times (|E_{HH}|^2 + |E_{VV}|^2 + 2 \text{Re}(E_{HH}E_{VV}^*))^{0.5}}, \]  
(6b)

\[ \nu_{RL} = \frac{|E_{HH}|^2 + |E_{VV}|^2 - 4|E_{HV}|^2 - 2 \text{Re}(E_{HH}E_{VV}^*)}{|E_{HH}| + |E_{VV}| + 4|E_{HV}|^2 - 2 \text{Re}(E_{HH}E_{VV}^*)}. \]  
(6c)

From (5) and (6), it is possible to formulate circular polarization parameters in terms of those for linear polarizations. Since all the backscattered waves are functions of particle canting and radar elevation angle, so are all the parameters in (6) (for example, see Jameson 1987).

b. Circular polarization parameters expressed as functions of linear polarization parameters

Circular polarization parameters can be written in terms of three linear polarization quantities, namely the differential reflectivity \( \xi \), the linear depolarization ratio \( L \), and the linear copolarization cross correlation \( \nu_L \). These are defined as

\[ \xi = \frac{|E_{HH}|^2}{|E_{VV}|^2}, \]  
(7a)

\[ L = 10^{L_{DR/10}}, \]  
(7b)

\[ \nu_L = \rho_L \exp(j\Phi_L), \]
\[ = \frac{\langle E_{HH}E_{VV}^* \rangle}{(|E_{HH}|^2|E_{VV}|^2)^{0.5}}. \]  
(7c)

Two common circular polarization parameters are the circular depolarization ratio (\( \Gamma \)), defined by

\[ \Gamma = \frac{|E_{RR}|^2}{|E_{RL}|^2} \]
\[ = \frac{|E_{LL}|^2}{|E_{LR}|^2} \]
\[ = 10^{L_{CDR/10}}. \]

and \( \rho_c \), the magnitude of the circular cross-correlation function defined in (5d). Using (5c) and (6a), \( \Gamma \) can be written as a function of \( \xi \), \( L \), and \( \nu_L \), namely

\[ \Gamma = \frac{\xi + 1 + 4\xi L - 2\xi^{0.5} \text{Re}(\nu_L)}{\xi + 1 + 2\xi^{0.5} \text{Re}(\nu_L)}. \]  
(9)

Similarly from (6b), the magnitude of \( \nu_c \) becomes

\[ \rho_c = \frac{(1 - \xi)^2 + 4\xi \text{Im}(\nu_L))^{0.5}}{(1 + \xi + 4\xi L - 2\xi^{0.5} \text{Re}(\nu_L)) \times (1 + \xi + 2\xi^{0.5} \text{Re}(\nu_L))^{0.5}}. \]  
(10)

In (9) and (10), the only quantities which are influenced by propagation differential phase shift are \( \text{Re}(\nu_L) \) and \( \text{Im}(\nu_L) \) since \( \xi \) and \( L \) are derived from power (amplitude squared) measurements. Since \( \rho_L = |\nu_L| \), it is also not affected by phase. The effect of propagation differential phase shift on \( \rho_c \) and \( \Gamma \) must therefore be felt through \( \Phi_L \), the argument of \( \nu_L \). For circular polarizations, the propagation differential phase shift must also appear in the argument (\( \Phi_c \)) of \( \nu_c \). These observations suggest a relation between \( \Phi_c \) and \( \Phi_L \), and, indeed, this is the case.

Using (6b) the tangent of \( \Phi_c \) is given by

\[ \tan \Phi_c = \frac{\text{Im}(\nu_L)}{\text{Re}(\nu_c)} \]
\[ = \frac{2 \text{Im}(E_{HH}E_{VV}^*)}{|E_{VV}|^2 - |E_{HH}|^2}, \]  
(11a)

or, conversely,

\[ \text{Im}(\nu_L) = \frac{(1 - \xi) \tan \Phi_c}{2\xi^{0.5}}, \]  
(11b)

where

\[ \text{Im}(\nu_L) = \rho_L \sin \Phi_L. \]

Using (11b) in (10), and combining (9) and (10) to eliminate \( \text{Re}(\nu_L) \),

\[ \frac{\rho_c \Gamma}{(1 + \Gamma^2)(1 + \tan^2 \Phi_c)} = \frac{(1 - \xi)^2}{[2(1 + \xi) + 4L]\xi}. \]  
(12)

Since the right-hand side contains terms only of measured powers at linear polarizations, it is completely unaffected by propagation differential phase shift. Hence, the left-hand side must also be independent of propagation differential phase shift. Therefore, if \( \Phi_c \) is measured along with \( \Gamma \) and \( \rho_c \), it should be possible to create an estimator of \( \xi \) unbiased by propagation differential phase shift provided the linear depolarization ratio is known.

Fortunately, except perhaps in the melting layer, \( L \) is usually sufficiently small so that it can be set to zero in (12), provided

\[ L \ll \frac{(1 + \xi)}{2\xi}. \]  
(13)
For $\xi = 1$ this condition will be met as long as $L \approx 1$, while for $\xi = 4$, $L$ must be $< 0.625$. Since $L$ is usually $< 0.01$, the $4L^2\xi$ term in (12) can usually be ignored. Because (12) is quadratic in $\xi$, there are two possible solutions, one for $\xi < 1$ ($10 \log \xi < 0 \text{ dB}$) and one for $\xi \geq 1$ ($10 \log \xi \geq 0 \text{ dB}$). Since it is anticipated that $\xi \geq 1$ in most cases, the appropriate solution is given by

$$
\xi(\xi) = \frac{1 + 2N^{0.5}}{1 - 2N^{0.5}}\quad (14)
$$

where

$$
N = \frac{\rho e^2 \Gamma}{(1 + \Gamma)^2(1 + \tan^2 \Phi_e)}.
$$

Note that (6b) implies that if $\xi < 1$, then Re$(\nu_e) > 0$. Hence, when Re$(\nu_e) > 0$, the alternative solution for $\xi < 1$ can be calculated by inverting the right-hand side of (14).

This is not the first estimator of $\xi$ to be derived from circular polarization measurements; that was formulated by McCormick (1979). That estimator, however, is biased by a propagation differential phase shift as demonstrated by Bebbington et al. (1987). They proposed using McCormick's estimator after first correcting for a propagation differential phase shift using the degree of polarization and ellipticity of the scattered field. The approach in this paper has the advantage that the extraction of additional linear polarization parameters is reasonably transparent.

Recent data reported by Hendry et al. (1987) permit an initial test of (14). In heavy rain they measured $\text{CDR} = -13.75$ dB, $\rho_e = 0.873$, and Re$(\nu_e) = -0.8035$. Combining $\rho_e$ and Re$(\nu_e)$, it follows that $|\text{Im}(\nu_e)| = 0.3414$ and $|\Phi_e| = 23^\circ$. Inserting these values in (14), $\xi(\xi) = 1.93$. From independent linear polarization observations, Hendry et al. measured $\xi = 1.94$. Similarly, in snow they observed $\text{CDR} = -22$ dB, $\rho_e = 0.28$, and Re$(\nu_e) = -0.2518$ which means that $|\Phi_e| = 25.9^\circ$. Equation (14) implies $\xi(\xi) = 1.08$, which is exactly the value Hendry et al. measured using linear polarizations.

Alternatively, (9) and (10) can be combined to estimate Re$(\nu_L)$. Although this estimate is affected by propagation differential phase shift, it can be combined with (11b) to estimate $\rho_L^2 = \text{Re}^2(\nu_L) + \text{Im}^2(\nu_L)$. This estimate, of course, is unbiased by propagation differential phase shift. In particular the appropriate solution for $\xi \geq 1$ is

$$
\text{Re}(\nu_L) = \frac{(\xi - 1) - M^{0.5}(\xi + 1)}{2(M\xi)^{0.5}}\quad (15)
$$

where

$$
M = \frac{\rho e^2 \Gamma}{(1 + \tan^2 \Phi_e)}.
$$

Obviously $\Phi_L$ can then be derived from $\text{arctan}[\text{Im}(\nu_L) / \text{Re}(\nu_L)]$. When Re$(\nu_e) > 0$ then $\xi < 1$ and Re$(\nu_L)$ may be found by replacing $(\xi - 1)$ by $(1 - \xi)$ in the numerator of (15).

Three measurements ($\rho_c$, $\Phi_c$, and $\Gamma$) have been used to solve for three unknowns ($\rho_L$, $\Phi_L$, $\xi$). Expressions (9)–(15) therefore define a transformation even when a propagation differential phase shift is occurring. Standard circular polarization measurements are equivalent to measuring $\xi$ and $\rho_L$. In addition, the rate of change of $\Phi_L$ with range can be used to estimate $\Delta_{HL}$, the rate range of propagation differential phase shift. This latter quantity may be useful for estimating rainfall (Seliga and Bringi 1978; Jameson 1985b; Sachidananda and Zrnić 1986) particularly since such estimates are independent of the reflectivity factor.

Unfortunately, while all of the parameters in this transformation are affected by particle canting, none provides a measurement which can unambiguously relate to hydrometeor canting. A characteristic of symmetric hydrometeors which is likely to be important for distinguishing among the different classes of precipitation is the canting of their symmetry axes from the vertical (Jameson 1985a, 1986). This canting also affects polarization measurements and is responsible for the linear depolarization ($L$) from Rayleigh–Gans scatterers. It has been suggested, in fact, that $L$ can be used to estimate the variance of particle canting angles (Jameson 1985a, 1986). However, with respect to standard circular polarization measurements, a solution for $L$ introduces one more unknown than measured quantities. An additional observation must be made in order to deduce $L$.

One approach is to measure the cross-correlation function $\nu_{RL}$ between $E_{LL}$ and $E_{RR}$, defined in (5f). It follows that

$$
\text{Re}(\nu_{RL}) = \frac{1 + \xi - 4\xi L - 2\nu_{RL}^0.5 \text{Re}(\nu_L)}{1 + \xi + 4\xi L - 2\nu_{RL}^0.5 \text{Re}(\nu_L)}\quad (16)
$$

This quantity can be measured using rapid pulse-to-pulse polarization switching to cancel the effect of the mean Doppler velocity as discussed earlier. Solving for $L$, (16) becomes

$$
L = \left[1 + \xi - 2\nu_{RL}^0.5 \text{Re}(\nu_L)\right]\gamma\quad (17)
$$

where

$$
\gamma = \frac{1}{4}\left[1 - \text{Re}(\nu_{RL})\right]
$$

is the so-called canting parameter (Jameson 1987). Since $0 \leq \gamma \leq 0.25$, the factor in (17) in front of $\gamma$ represents four times the maximum possible linear depolarization for a particular combination of $\xi$ and Re$(\nu_L)$.

With the measurement of $\nu_{RL}$, the transformation between circular and linear polarization parameters is reasonably complete. It is also independent of propa-
gation differential phase shift. Consequently, there are few, if any, theoretical grounds for selecting linear polarization over circular polarization. (In some situations, there may still be technical and logistical reasons for a particular preference.) Nevertheless, it can be argued that physical interpretations of linear polarization parameters are often easier to grasp intuitively because the horizontally and vertically polarized waves are uncoupled. Therefore, in section 3, circular polarization measurements will be interpreted only in terms of linear polarization parameters.

The first example will be taken from data collected using the 10 cm wavelength circular polarization radar of the Alberta Research Council. Although measurements of four circular polarization parameters would have been desirable, the Alberta data only provide the parameters \( \nu_c \) and \( \Gamma \). Therefore, in section 3 it is only possible to transform \((\rho_c, \Phi_c, \Gamma) \rightarrow (\rho_L, \Phi_L, \zeta)\); nevertheless, as we shall see, this transformation may be quite illuminating. In particular, the behavior of \( \rho_L \) as a function of height through a melting region will be considered in some detail.

3. Examples and discussion

a. Estimating linear polarization parameters from circular polarization measurements biased by propagation differential phase shift

For the last several years the radar of the Alberta Research Council has been operated in an experimental hail suppression program. This radar operates at a wavelength of 10.41 cm and simultaneously receives and processes the signals at both co- and cross-polarization. Although in principle the radar is capable of transmitting any elliptical polarization, the polarization must be set by hand and therefore rapid interpulse polarization switching is not possible. Usually, the transmitted polarization is at left-hand circular. The signals are processed over 1 km segments in range, and time averages of the powers in the cross- and copolarization channels, \( \text{Re}(\nu_c) \), and \( \text{Im}(\nu_c) \) are recorded on tape. The calculated reflectivity factor is the linear average of values corresponding to vertical and horizontal polarizations. Calibrations of the \( I \) and \( Q \) ports of both the co- and cross-polarized channels are performed frequently. Although the antenna is usually swept quickly to provide a hemispheric scan every few minutes, the special data presented in these examples were collected when the antenna was stationary or descending only very slowly in elevation at a constant azimuth angle.

In Fig. 1, measurements through a small decaying Alberta thunderstorm are presented. Although the reflectivity core lies between 31–36 km, it was enveloped in a blanket of anvil precipitation extending for several kilometers. While \( \rho_c \) remains essentially constant between 25–41 km, CDR increases monotonically to substantial values, suggesting that propagation differential phase shift is present (Humphries 1974). This is confirmed by computing \( \Phi_L \) from \( \Phi_c \) using (11b) and (15). A striking increase in \( \Phi_L \) is exhibited after only 1 km of penetration into the storm core. The values of \( \Phi_L \) become much too large to be attributed to phase shifts from backscatter. Between 33–36 km there is an average one-way increase in \( \Phi_L \) of about \( \Delta_{HF} = 1.35^\circ \) km\(^{-1}\). Although this is a rather modest value considering the large reflectivity factors, its effects on the polarization parameters are significant. In particular, if \( \zeta \) is calculated ignoring propagation differential phase shift [i.e., setting \( \Phi_c = 0 \) in (14) and solving for \( \zeta \)], this biased estimate increases monotonically with increasing range up to the substantial value of 1.62 (10 log\( \zeta \) = 2.1 dB) at 41 km.

Curves corresponding to CDR (i.e., \( \Gamma \) in dB), \( \zeta \), and \( \rho_c \) in Fig. 2 illustrate the parameters corrected for propagation differential phase shift (lower lines) and the original, uncorrected values (upper lines). While the unbiased \( \zeta \) is derived from (14), the estimates of the corrected CDR and \( \rho_c \) are computed using (9) by setting \( \text{Re}(\nu_c) = \rho_L \) and \( \text{Im}(\nu_c) = 0 \). In essence these substitutions cancel the bias due to propagation differential phase shift. This can be seen by noting from (7c) that \( \text{Re}(\nu_c) = \rho_L \cos(\Phi_L) \) and \( \text{Im}(\nu_c) = \rho_L \sin(\Phi_L) \). Using (3), Jameson and Mueller (1985) show that \( \Phi_L = (\phi_H - \phi_V) + (\delta_H - \delta_V) \) where \( \phi_H - \phi_V \) is the net mean propagation differential phase shift and \( \delta_H - \delta_V \) is the mean phase shift due to backscatter. Now assume that \( \phi_H - \phi_V \) is known and is subtracted from \( \Phi_L \);
then $\Phi_L = \delta_H - \delta_V$. At a long radar wavelength like 10 cm, this backscatter phase shift is small for Rayleigh–Gans scatterers (Jameson and Mueller 1985) and is probably small even for most large hail. Hence, $\text{Re}(\nu_L) \approx \rho_L$ and $\text{Im}(\nu_L) \approx 0$.

Figure 2 shows that after only a short penetration, CDR, $\rho_c$, and the estimate of $\xi$ are significantly affected by propagation differential phase shift. In particular, while the uncorrected $\xi$ is about 1.61 at 41 km, the correct value is much closer to 1.24 (a difference in $Z_{DR} = 10 \log(\xi)$ of 1.15 dB). Similarly, by 41 km, propagation enhanced CDR by 2.3 dB and $\rho_C$ by 0.3. These substantial effects were generated from only 10° (two-way) of propagation differential phase shift. Clearly circular polarization measurements are of marginal value unless propagation phase effects are eliminated.

The rather low value of corrected $\rho_C$ (0.41) at 41 km can probably be attributed to the penetration of the radar beam into the melting region. Since ice processes play the predominate role of precipitation formation in Alberta thunderstorms (Krauss and Marwitz 1984), melting precipitation will almost always be found beneath Alberta storms. The large $Z$ associated with the precipitation core, if due solely to rain, would imply very large liquid water contents ($W$) on the order of 8 g m$^{-3}$ (Sekhon and Srivastava 1971). If the equilibrium size–shape relation (Pruppacher and Pitter 1971) is assumed, $Z$, in combination with $\xi = 1.38$ (Seliga and Brigni 1976), implies $W$ on the order of 16 g m$^{-3}$ (Jameson 1983b). On the other hand, an estimate derived from $\Delta_{HV}$ and $\xi$ independently from $Z$ (Jameson 1985b) suggests that $W$ is only 1–2 g m$^{-3}$. Because of the unrealistically large values deduced when $Z$ is included in the estimate, the reflectivity factor has likely been enhanced, presumably by the wetting of ice particles still in the process of melting even though they are only 1.2 km above ground. The evidence for melting will become even more apparent in subsection 3b. However, this example illustrates the importance of combining as many independent methods as possible for estimating rainfall. Inconsistent results can serve to flag suspicious values.

b. On the significance of $\rho_L$

In Fig. 3, circular and linear polarization parameters are displayed as functions of height at a range of 25 km from the radar. These data, while not simultaneous, were collected within a few minutes of the measurements in Fig. 2 near the same location. Not atypical of summertime in Alberta, the melting level appears to be near 2 km above ground level (AGL) although the exact height is not known.

Since these data were near the front of the precipitation canopy, propagation differential phase shift was slight as the small values of $\Phi_L$ indicate. An inspection of data at 24 km indicated some propagation differential phase shift was present at 25 km. Since even a small amount of propagation may have important effects, $\Phi_L$ has been incorporated into the estimates of $\xi$ and $\rho_L$. Measurements were collected about every 40 m in height, although, once again, they represent averages $\sim 1$ km in range.

![Diagram](img1)

**FIG. 2.** Plot of measured and computed quantities both corrected and uncorrected for propagation differential phase shift. The upper lines for CDR, $\xi$, and $\rho_c$ correspond to the uncorrected values in Fig. 1. The lines beneath each of these represent estimates of the parameters after propagation differential phase shift has been removed. Note the impact after only a few degrees of phase shift.

![Diagram](img2)

**FIG. 3.** Measured (CDR, $Z$, $\rho_c$, $\Phi_L$) and derived ($\Phi_L$, $\rho_L$, $\xi$) parameters as functions of height above ground at a range of 25 km. $\rho_L$ is the magnitude of the linear cross-correlation function. The melting level is thought to be near 2 km. The structures of $\rho_L$ and $\xi$ are discussed in the text. The $\Phi_L$ plotted here is the measured $\Phi_L - 180°$.
The circular polarization parameters are plotted on the left-hand side of Fig. 3. The most noticeable feature is the substantial increase in $\Phi_c$ above the melting level. Large $\Phi_c$ are often found aloft in Alberta hailstorms. Since in these storms the precipitation aloft is mostly ice (Krauss and Marwitz 1984), $\xi$ is usually very close to unity (Hall et al. 1984). The denominator in (11a), therefore, can approach zero. For scatterers with even a small but finite $\text{Im}(\nu_L)$, $\tan \Phi_c$ and hence $\Phi_c$ can become very large. A substantial $\Phi_c$ does not necessarily imply that very large hydrometeors are within the sampling volume.

The linear polarization parameters are shown on the right-hand side of Fig. 3. The unique feature here is the display of $\rho_L$ through the melting level. An understanding of the physical meaning of $\rho_L$ will aid in interpretation of these data. Consider an ensemble of spheres which are Rayleigh–Gans scatterers. Suppose that the size distribution is a decreasing exponential. Since the radar cross section increases with drop diameter while the number concentration decreases, there will be some drop size which will contribute most to the average power of the backscattered signal. For spherical particles, the contributions as a function of drop size to the average power will be identical for both horizontal and vertical polarizations. Therefore $\xi = 1$ and $\rho_L = 1$. Suppose, next, that a few large horizontally oblate spheroids are added to the ensemble. Not only will the average power corresponding to vertical polarization decrease with respect to that for horizontal polarization, but the distributions of the contributions as a function of drop size to the power will shift more toward larger sizes at horizontal polarization than at vertical polarization. Consequently, the horizontal and vertical polarization signals decorrelate ($\rho_L$ decreases) while $\xi$ increases. Conversely, if the oblates are once again removed, $\rho_L$ increases while $\xi$ decreases. If prolate with the long axis oriented vertically had been added instead of oblate spheroids, then $\rho_L$ would again decrease, but this time $\xi$ would decrease as well. Conversely, the removal of these prolate would have produced increases in both $\rho_L$ and $\xi$. In summary, when $\xi$ and $\rho_L$ change temporally or spatially in opposite senses, observations can be discussed in terms of the removal or addition of particles with a larger radar cross section at horizontal than at vertical polarization.

When $\xi$ and $\rho_L$ behave similarly, discussion can be in terms of the addition or removal of particles with larger radar cross sections at vertical than at horizontal polarization. From these two quantities alone, however, it is not possible to tell whether these changes are the result of alteration in the canting of the particles or of modification in the actual shapes of the hydrometeors.

At 3.3 km (top of Fig. 3), $\xi$ is only 1.02 while $\rho_L$ is 0.997. The measurements using horizontal and vertical polarizations are highly correlated. Both of these measurements are what would be expected if a scattering medium were nearly isotropic. While $\xi$ suggests that the hydrometeors may be nearly spherical, it is not possible to estimate the potential reduction of $\xi$ due to particle canting. This cannot be resolved without an independent measurement directly related to particle canting. Nevertheless, the low $\xi$ does indicate that the hydrometeors are ice (Hall et al. 1984). Although both $\xi$ and $\rho_L$ are consistent with measurements in snow (e.g., Hendry et al. 1987), the reflectivity factor is much too large. It is likely that the precipitation is predominately graupel or small hail, very common forms of hydrometeors even in small Alberta storms (Krauss and Marwitz 1984).

From 3 to 2 km, there is a monotonic increase in $\xi$ but a monotonic decrease in $\rho_L$. As discussed above, these trends are consistent with more oblate particles becoming more prevalent with decreasing height. Again, it is not possible to tell whether this is largely because the actual shapes of the particles are more oblate or whether some particles are simply becoming more horizontally oriented. From 1.8 to 1.6 km, $\xi$ and $\rho_L$ are both increasing, indicating that some of the hydrometeors contributing significantly to $|E_{VV}|$ are being removed either because they are becoming horizontally oriented or because their shapes are changing as they melt. Finally, from 1.6 to the bottom, $\rho_L$ increases while $\xi$ decreases. This suggests that particles contributing strongly to $|E_{HH}|$ are disappearing, perhaps as a result of completion of the melting of ice and a collapse to more spherical shapes.

Although these data are from a melting region in a thunderstorm, an analysis of observations by Hendry et al. (1987) through a classic snow-to-rain melting band shows that $\rho_L$ and $\xi$ behave identically with decreasing height. However, in part because of the slower fall speed of the hydrometeors, the vertical scale is compressed, producing much more dramatic features. While Hendry et al. measured $\xi$, CDR, and $\rho_c$, Re($\nu_L$) was estimated using (15) assuming $\Phi_c = 0^\circ$. This latter assumption was found to be appropriate after comparing the measured $\xi$ to that computed using (14) with $\Phi_c = 0^\circ$. In this examination of their data it will be assumed that the hydrometeors are Rayleigh–Gans scatterers so that $\rho_L$ can be equated to Re($\nu_L$) as discussed earlier.

An analysis of the melting band data of Hendry et al. (1987) yields $\xi = 1.04$ and $\rho_L = 0.994$ at 400 m above the melting level. At the bright band, however, $\xi = 1.29$ while $\rho_L$ is only 0.805. This value is especially low since $E_{HH}$ and $E_{VV}$ are usually observed to be very highly correlated, i.e., $\rho_L$ is very close to unity. This value may even be as low as 0.64 if an $LDR$ of $-12$ dB, previously reported in different melting level data (Hendry and Antar 1984), is included in the solution of (11) for Re($\nu_L$). At 200 m below the melting band, $\xi$ and $\rho_L$ return again to 1.05 and 0.997, respectively. The unusually low value of $\rho_L$ at the melting level indicates that much of the contribution to $|E_{HH}|$ is derived from a set of particles distinctly different from
those contributing most to $|E_{VV}|$. Thus, there is probably no incompatibility between the presence at the same location of both large $\xi$ and substantial LDR (an indication of significant apparent canting). The horizontally oriented hydrometers producing the large $\xi$ are not the same as those responsible for the significant LDR. The coincident location of both large $\xi$ and LDR appears to be an inconsistency in melting layer measurements (see the discussion in Atlas 1984) only when the horizontal and vertical signals are assumed nearly perfectly correlated.

This example illustrates the potential importance of $\rho_L$ for qualitative interpretations. This parameter may also be useful in more quantitative applications. In the above example, for instance, if it is assumed that the hydrometers at the melting layer are approximately oblate spheroids and that $\xi$ is due to essentially wet particles, then a reflectivity weighted mean axis ratio $R$ can be estimated to be 0.896 from the relation $R = \xi^{-3/7}$ (Jameson 1983a). Because of the low correlation, this $\xi$ may not be the appropriate value since a significant fraction of the backscattered powers at vertical and horizontal polarizations are coming from different particles. Jameson (1987) shows that an estimate which includes $\rho_L$ can be calculated from $R = \xi^{-3/7} \rho_L^{1/7}$. Using $\rho_L = 0.805$ and $\xi = 1.11$ measured by Hendry et al. (1987), we compute $R$ to be 0.744 which is considerably smaller than the original estimate of 0.896. The particles responsible for $\xi$ in the melting layer are therefore actually much flatter than the measured $\xi$ implies.

Although the difference between these two estimates is extreme in the melting layer, it may also be important to include $\rho_L$ when attempting to use $\xi$ to estimate rainfall rate. In most rain, $\rho_L$ is always slightly less than unity. Consequently, if $\xi$ is measured without accounting for $\rho_L$, it will be too small which, in turn, leads to an underestimate of the mean drop size. In order to reproduce the observed reflectivity factor, too large a drop concentration is then required. The net result is an inflated estimate of the radar rainfall rate.

Rainfall rates deduced using $\xi$ unadjusted by $\rho_L$ should therefore always be biased toward values higher than actually measured. In addition, the magnitude of this bias should increase at larger rainfall rates since more big drops are then in the size distribution. Consequently, $\rho_L$ will decrease (see the discussion above and in Jameson 1987). This decorrelation between measurements at horizontal and vertical polarizations may explain some of the bias toward radar overestimation actually observed in intercomparisons between measured rainfall rates and those calculated using $\xi$ and $Z$ (Goddard and Cherry 1984). For example, a 2% underestimate of $\xi$ will lead to a 4% overestimate of the reflectivity weighted axis ratio $R$ (Jameson 1983a). If the drops are assumed to be equilibrium shaped, this overestimate of $R$ corresponds to a 4% underestimate of the reflectivity weighted mean drop diameter $D$. From the relationships between $D$ and rainfall rate (Jameson 1983b), the net result is a bias of 8%–12% overestimation of the rainfall rate. This is the same magnitude as the bias in the measurements of Goddard and Cherry (1984).

4. Concluding remarks

In this work it has been shown that earlier fears were well founded of the effect of propagation differential phase shift on circular polarization measurements. It has also been shown, however, that differential propagation phase shift can be removed and need no longer restrict the use of circular polarization for meteorological research. Circular polarization measurements can be transformed into estimates of linear polarization parameters unbiased by differential propagation phase shift. These quantities can then be used to describe the precipitation. Although the general applicability of these transformations need further substantiation, this technique provides greater flexibility in interpreting circular polarization measurements.

These transformations also demonstrate that circular polarization measurements have much to offer, such as in estimating rainfall rate using differential reflectivity and the reflectivity factor. Although differential reflectivity can be measured using linear polarization, standard circular polarization measurements provide $\xi$, $\rho_L$, and the rate of change with distance of propagation differential phase shift ($\Delta_{HV}$). In rain this latter quantity is directly related to the liquid water content and the mass weighted mean axis ratio (Jameson 1985b). Estimates of rainfall using $\Delta_{HV}$ are independent of the reflectivity factor ($Z$). All of these measurements can then be used to produce a set of radar rainfall measurements as consistent as possible. In the one example presented in this work, these measurements revealed an enhancement of $Z$ probably due to melting. In such a situation, the use of $Z$ alone or in combination with $\xi$ to estimate rainfall would have produced significant errors.

Ironically, circular polarizations might even provide a better method for estimating the linear polarization parameters $\Delta_{HV}$ and $\rho_L$. These quantities can be estimated from simultaneous measurements of the two orthogonal circularly polarized backscatter signals. If alternating linear polarizations are used, $\Delta_{HV}$ and $\rho_L$ must be calculated from the cross correlation between vertically copolarized and horizontally copolarized signals separated in time by one or more interpulse periods (Jameson and Mueller 1985; Sachidananda and Zrnić 1986). During this time the precipitation moves, thereby producing a change in the phases of the signals. Although Jameson and Mueller (1985) and Sachidananda and Zrnić (1986) argue that this phase shift can probably be eliminated after sufficient averaging, for realistic finite sampling times there is likely still to be some slight residual phase noise. Using circular polarization, this source of noise can be eliminated.
Since these transformations show that linear and circular polarizations provide equivalent information, the question worthy of future research should not be which polarization is better (i.e., provides more information), but rather which can provide the best measurements of which parameters.

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APPENDIX
Definitions

a. Linear polarization

$E_{HH}$ complex amplitude of the horizontally polarized backscattered wave from the transmission of a horizontally polarized wave

$E_{VV}$ complex amplitude of the vertically polarized backscattered wave from the transmission of a vertically polarized wave

$E_{HV}$ complex amplitude of the vertically polarized backscattered wave from the transmission of a horizontally polarized wave

$E_{VH}$ complex amplitude of the horizontally polarized backscattered wave from the transmission of a vertically polarized wave

$\xi = |E_{HH}|^2/|E_{VV}|^2$, differential reflectivity

$L = |E_{HV}|^2/|E_{HH}|^2$, linear depolarization ratio

$\nu_L = \rho_L \exp(j\Phi_L) = \langle E_{HH}E_{VV} \rangle / (|E_{HH}|^2 \times |E_{VV}|^2)^{0.5}$, linear copolarization cross-correlation function in which $\langle \cdots \rangle$ denotes a time average, $j = \sqrt{-1}$, and $\rho$ denotes complex conjugation.

b. Circular polarization

$E_{RR}$ complex amplitude of the right circularly polarized backscattered wave from the transmission of a right circularly polarized wave

$E_{LL}$ complex amplitude of the left circularly polarized backscattered wave from the transmission of a left circularly polarized wave

$E_{RL}$ complex amplitude of the left circularly polarized backscattered wave from the transmission of right circularly polarized wave

$E_{LR}$ complex amplitude of the right circularly polarized backscattered wave from the transmission of left circularly polarized wave

$\Gamma = |\langle E_{RR}E_{RL} \rangle / (|E_{RR}|^2|E_{RL}|^2)^{0.5}\rangle$, circular depolarization ratio

$\nu_{RC} = \langle E_{RR}E_{RL} \rangle / (|E_{RR}|^2|E_{RL}|^2)^{0.5}$, cross-correlation function between $E_{RR}$ and $E_{RL}$

$\nu_{LC} = \langle E_{LL}E_{LR} \rangle / (|E_{LL}|^2|E_{LR}|^2)^{0.5}$, cross-correlation function between $E_{LL}$ and $E_{LR}$

$\nu_c = \rho_c \exp(j\Phi_c) = \nu_{RC} = \nu_{LC}$, circular cross-correlation function

$\nu_{RL} = \langle E_{RR}E_{LL} \rangle / (|E_{RR}|^2|E_{LL}|^2)^{0.5}$, circular copolarization cross-correlation function.

REFERENCES


