Biasing of the Covariance-Based Spectral Mean Estimator in the Presence of Band-Limited Noise

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ABSTRACT

Estimation of spectral mean frequency (spectral first moment) by the covariance technique is considered for a signal process contaminated by band limited, additive noise. It is shown that the covariance-based spectral mean estimator is biased for low signal-to-noise ratios if the noise bandwidth is not large compared to the signal bandwidth. The bias is towards the mean frequency of the noise spectrum, typically equivalent to the center of the frequency band passed by the receiver. This noise-biasing is potentially important in the processing of Doppler data from radars, sodars and sonars operating in a pulse-to-pulse incoherent mode. Biasing of the mean frequency estimator can be easily corrected if measurements of the noise covariance are available. In the absence of noise measurements, correction for biasing can still be accomplished by estimating the signal and noise bandwidths and introducing simple models for the signal and noise covariance functions. This technique allows estimation of noise covariance from measurements of signal-plus-noise covariance at more than one time lag. In addition, the models provide a means of predicting potential biasing problems in a generalized Doppler system.

1. Introduction

The estimation of spectral mean frequency (spectral first moment) by covariance-based processing techniques for a Gaussian shaped signal spectrum contaminated by additive noise has been considered by many investigators including Miller and Rochwarger (1972), Sirmans and Bumgarner (1975b), and Underwood (1981). These authors have concluded that covariance processing (henceforth CP, also known as "pulse-pair" processing) provides an unbiased estimate of the spectral mean frequency for all signal-to-noise ratios if the additive noise is "white." The hypothetical case of true "white" noise would require that the autocovariance of the noise be equal to zero for all time lags not equal to zero or, equivalently, that the noise spectrum have constant energy density and an infinite bandwidth. In practice, band-limited noise can be considered approximately "white" if the noise spectrum has nearly constant energy density over a range of frequencies that is large compared to the bandwidth of the signal of interest.

In a pulsed-Doppler system, the receiver bandwidth must be at least as large as the inverse pulse duration to maintain acceptable fidelity of the received echo (Doviak and Zrnić 1984). For pulse-to-pulse incoherent Doppler systems[1] that do not employ "tracking" circuitry (Rowe and Young 1979) to center the receiver filter with respect to the Doppler spectrum, it is necessary for the receiver bandwidth to be larger than the inverse pulse duration in order to accommodate a range of Doppler shifts. Increasing the receiver bandwidth beyond that required by the above constraints is undesirable since it results in a decrease in the signal-to-noise power ratio (SNR). As a result, many incoherent Doppler systems have a band-limited noise spectrum at the receiver output with a bandwidth that is not significantly larger than the signal bandwidth. This band-limited noise has nonzero autocovariance at small time lags and cannot be considered "white" when evaluating the performance of the CP technique of mean frequency estimation.

This study shows that the CP mean frequency estimator exhibits a significant bias at low SNR in the presence of band-limited noise if the bandwidth of the noise is not large compared to that of the signal. The magnitude of the bias for a given Doppler shift can be expressed in terms of the SNR and the normalized sig-

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[1] Characteristics of pulse-to-pulse coherent and pulse-to-pulse incoherent Doppler systems are discussed by Pinkel (1980, 1987).

nal-to-noise autocovariance amplitude ratio. The nature of the bias is to force the Doppler shift estimated from the signal-plus-noise spectrum towards the mean frequency of the noise spectrum.

In a recent paper Chereskin et al. (1989) showed that noise biasing effects may be seen in the data from commercially available Acoustic Doppler Current Profilers (ADCPs) that use the CP technique. These ADCPs are pulse-to-pulse incoherent systems that employ Doppler frequency tracking and thus have a receiver bandwidth roughly equal to the inverse pulse duration. The theory developed here predicts that a bias towards the mean frequency of the tracking filter will be evident in these systems as the SNR drops below about 10 dB. Using a simulated signal as input to a model of the processing system for an RD Instruments ADCP, Chereskin et al. demonstrate that this is the case. Observational results presented in the same paper show behavior consistent with both the theoretical and model predictions and confirm that noise biasing may exist in field data from ADCPs under certain conditions. It should be noted that properly implemented Doppler frequency tracking will reduce the effect of noise biasing, and the ADCP manufacturer has implemented changes in their tracking algorithm that should minimize problems of the type described by Chereskin et al.

The CP technique is briefly reviewed in section 2. The work presented in section 3a represents a generalized development of the noise biasing equation used by Chereskin et al. (1989), while section 3b introduces methods of bias correction.

2. Covariance processing (CP)

The covariance-based estimator for spectral moments has been extensively described by many authors (Miller and Rochwarger 1972; Serafin 1975; Sirmans and Bumgarner 1975a; Passarelli and Siggia 1981; Lhermitte and Serafin 1984). A brief review of the CP technique as used for the estimation of mean Doppler frequency is given here.

A simple Doppler system interrogates a sample volume in the medium with a pulsed transmission of the form

$$X_0(t) = a\cos(2\pi f_c t)\Pi(t/T_p - 1/2) \qquad (1)$$

where f_c is the carrier frequency, T_p is the pulse duration, and $\Pi(x)$ is the "boxcar" function

$$\Pi(x) = \begin{cases} 1, & -\frac{1}{2} \leqslant x \leqslant \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases}$$

The received signal is the sum over the sample volume of contributions to amplitude and phase from many individual scatterers. The signal received at time t after the start of transmission comes from a range increment $\Delta r = cT_p/2$ centered at $r = ct/2 - cT_p/4$ and can be expressed as (Serafin 1975)

$$X(t) = \mathrm{Re}\{\exp[i2\pi f_c t]S(t)\exp[i\phi(t)]\} \qquad (2)$$

where $S(t)$ and $\phi(t)$ are the resultant amplitude and phase, respectively.

Since only the complex envelope, $S(t)\exp[i\phi(t)]$ of (2) is of interest, a homodyning or heterodyning system is often used to remove the carrier frequency and separate the received signal into in-phase (I) and quadrature (Q) components. The I and Q components can be combined to form the complex envelope

$$Z(t) \equiv S(t)\exp[i\phi(t)] = I(t) + iQ(t). \qquad (3)$$

The squared modulus of Z gives the received signal power S^2 and the rate of change of the phase of Z gives the Doppler shift.

The mean Doppler shift introduced by an aggregate of moving scatterers is related to the average velocity within the sampling volume. The received signal does not contain a single Doppler shift because the scatterers within the volume may have varying acoustic cross sections and be moving at slightly different speeds. The result is a distribution of received power with frequency forming a Doppler spectrum. In pulse-to-pulse incoherent Doppler systems the spectrum is broadened further due to the bandwidth of the transmitted pulse. Classical methods for determining the mean Doppler frequency involve computing the power spectrum of the complex signal (3) and estimating the power-weighted mean frequency (Serafin 1975; Sirmans and Bumgarner 1975a). With the CP technique, the Doppler spectrum is not computed; instead the spectral moment is determined from the complex auto-covariance function

$$C(\tau) = \langle Z(t)Z^*(t+\tau)\rangle \qquad (4)$$

where the brackets imply ensemble averaging and * denotes conjugation. Henceforth, $C(\tau)$ is referred to as simply "the covariance." Exploiting the fact that the Doppler spectrum is the Fourier transform of the covariance it can be shown (Miller and Rochwarger 1972; Sirmans and Bumgarner 1975a) that the first spectral moment $\bar{f}$ is well approximated by

$$\hat{\bar{f}} = (2\pi\tau)^{-1}\arg[C(\tau)]. \qquad (5)$$

For a noise-free, symmetric Doppler spectrum $\hat{\bar{f}}$ is an unbiased estimator for $\bar{f}$ as τ approaches zero. In practice, the estimator performs well for covariance lags $\tau \ll {\sigma_s}^{-1}$, where σ_s is the bandwidth of the Doppler spectrum.

3. CP in the presence of band-limited noise

a. *Description of noise biasing*

The presence of background noise is unavoidable in an operational Doppler system. Contributions to noise power can be separated into two additive components: 1) system noise, observed at the receiver input, containing contributions from the ambient noise of the medium, transmit–receive switches, and transmission lines and 2) receiver noise due to amplifiers, multipliers, and filters in the receive circuitry. Multiplicative noise, due to distortion or nonlinearity in the system electronics, will not be considered here. The system noise spectrum contains energy over a wide range of frequencies and typically has a bandwidth that is large compared to the bandwidth of the received signal. If, in addition, the system noise spectrum has nearly constant energy density in the frequency range of interest, then the system noise may be considered "white."

The noise at the receiver output is the sum of system noise and receiver noise. This noise is band-limited by the receiver filter(s), and thus has a covariance amplitude $C_n(\tau)$ characterized by a finite decay time or decorrelation time of order σ_n^{-1}, where σ_n is the noise bandwidth taken at the -3 dB points of the receiver frequency response function. Since the noise covariance is nonzero for lags different from zero the concept of true "white" noise at the receiver output, defined by $C_n(\tau) = 0$ for $\tau \neq 0$, is invalid. Even if the system noise at the receiver input is white, the noise at the receiver output can be considered approximately white only if the frequency response of the receiver filter is relatively flat across the pass-band and the noise bandwidth is large compared to that of the signal. In terms of the covariance, the requirement that the signal-to-noise bandwidth ratio be small implies that the noise covariance must have a decay time much shorter than the sampling interval of the signal. If the signal covariance amplitude $C_s(\tau)$ is computed for a lag $\tau = \tau_0$, then the condition for validity of the white noise approximation is

$$C_s(\tau_0)/C_n(\tau_0) \gg 1. \qquad (6)$$

It is the interest of this paper to consider situations where condition (6) does not hold, so that estimation of spectral moments using (5) requires the consideration of additive, band-limited noise in the received signal.

It will be shown that the ratio (6) determines whether biasing of the CP mean frequency estimate is important. If the signal covariance is not large compared to that of the noise at the lag where the CP estimate is computed then biasing will occur. The effect of band-limiting on the signal and noise covariance amplitudes is shown graphically in Fig. 1. The figure is representative of an incoherent Doppler sonar that transmits a rectangular pulse of the form (1) and has a rectangular or "boxcar" receiver filter [see (13)]. In order to minimize the variance of the CP mean frequency estimator, it is desirable to operate the sonar in the region $0 < \tau/T_p \leq 0.2$, where τ/T_p is the normalized covariance lag (Miller and Rochwarger 1972). Inspection of Fig. 1 shows that when the noise is band-limited the noise covariance amplitude is not negligible compared to that of the signal for small values of normalized lag. A more detailed discussion of the effect of band-limited noise on the CP mean frequency estimator is given below.

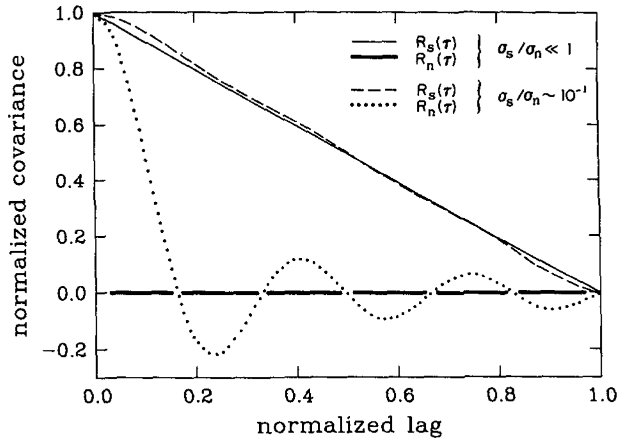

FIG. 1. Normalized amplitude of signal and noise covariance vs. normalized lag τ/T_p for different signal-to-noise bandwidth ratios. For $\sigma_s/\sigma_n \ll 1$ the noise covariance (dark line) is taken to be zero for lags greater than zero, and the signal covariance (light line) is a "triangle" function. For $\sigma_s/\sigma_n \sim 10^{-1}$ the noise covariance (dotted) decays as sinc(τ/T_p), and the signal covariance (dashed) shows some distortion due to band-limiting. In the band-limited case, the amplitude of the normalized noise covariance is not negligible compared to that of the signal for small values of normalized lag. The effect of varying SNR is not included here since the signal and noise covariance amplitudes shown in the figure have been normalized by the signal and noise power, respectively.

Given that the receiver output, composed of in-phase and quadrature signals, is corrupted by additive band-limited noise, the complex envelope of signal-plus-noise can be written

$$Z_{sn}(t) = [I(t) + N_I(t)] + i[Q(t) + N_Q(t)] \qquad (7)$$

where I and Q are as in (3) and the total noise power is $N^2 = \langle N_I^2 \rangle + \langle N_Q^2 \rangle = C_n(0)$. The noise is considered to have zero mean and to be uncorrelated with the signal. If the noise spectrum is symmetric about the center frequency of the receiver pass-band, then N_I and N_Q are statistically independent (Helstrom 1968) and the signal-plus-noise covariance is

$$\begin{aligned} C_{sn}(\tau) &= \langle Z_{sn}(t) Z_{sn}^*(t+\tau) \rangle \\ &= \langle S(t)S(t+\tau) \rangle \exp[i2\pi\bar{f}\tau] \\ &\quad + \langle N_I(t)N_I(t+\tau) \rangle + \langle N_Q(t)N_Q(t+\tau) \rangle \\ &= C_s(\tau)\exp[i2\pi\bar{f}\tau] + C_n(\tau) \qquad (8) \end{aligned}$$

where C_s and C_n are the signal and noise covariance amplitudes, respectively. Note that with these assumptions the noise covariance appears only in the real part of C_{sn}.

Dividing (8) by the signal-plus-noise power, $C_{sn}(0) = S^2 + N^2$, produces the normalized covariance

$$R_{sn}(\tau) \equiv \frac{C_{sn}(\tau)}{C_{sn}(0)} = \frac{\mathrm{SNR}}{\mathrm{SNR}+1} R_s(\tau) \exp[i2\pi\bar{f}\tau] + \frac{1}{\mathrm{SNR}+1} R_n(\tau) \quad (9)$$

where $R_s = C_s(\tau)/C_s(0)$, and $R_n = C_n(\tau)/C_n(0)$. The normalized covariance (9) can be used in place of $C(\tau)$ in (5) to give the CP mean frequency estimator for signal plus band-limited noise

$$\hat{\bar{f}} = (2\pi\tau)^{-1} \arctan\left\{\frac{\sin(2\pi\bar{f}\tau)}{\cos(2\pi\bar{f}\tau) + [\mathrm{SNR}\alpha(\tau)]^{-1}}\right\} \quad (10)$$

where SNR is the signal-to-noise power ratio and $\alpha(\tau) = R_s(\tau)/R_n(\tau)$ is the normalized signal-to-noise covariance amplitude ratio. The expression $[\mathrm{SNR}\alpha(\tau)]^{-1}$ is identified as the biasing factor due to band-limited noise. Using the definitions of R_s and R_n given above and $\mathrm{SNR} = C_s(0)/C_n(0)$ it can be seen that the biasing factor is simply the inverse of the signal-to-noise covariance amplitude ratio introduced in (6). The expression used in (10) separates the part of the bias that is due to the relative magnitude of the noise covariance at nonzero lag from the part due to the noise power (covariance at zero lag). This separation facilitates much of the discussion that follows.

If $\mathrm{SNR} \gg 1$ or $R_s(\tau) \gg R_n(\tau)$ then the biasing factor is small and an accurate mean frequency estimate is recovered from (10). Otherwise there is a bias that depends on SNR and the value of α at lag τ. The covariances R_s and R_n will both be positive for small lags and the effect of the biasing term will be to increase the magnitude of the denominator in (10), thereby biasing $\hat{\bar{f}}$ towards zero, regardless of whether the true mean frequency is positive or negative.

The nature of the bias is to force the CP estimated Doppler shift towards the mean frequency of the noise spectrum. If the noise power is flat across the passband of the receiver and the receiver filter is symmetric, then the bias will be towards the center of the frequency band passed by the receiver. For a system where the frequency of the reference signal used in the demodulation step is held constant and equal to the transmitted frequency the bias will be towards zero velocity. Some Doppler systems incorporate tracking systems that adjust the reference frequency to match the expected Doppler shift in the returned signal. This means that the center frequency of the receiver filter will not represent zero velocity, but a nominal mean velocity that is formed from some combination of depth and time averaging of previous velocities (Rowe and Young 1979; Chereskin et al. 1989). If the center frequency "predicted" by the tracking algorithm is very close to the actual mean Doppler frequency then the noise bias errors will be minimized since the biased mean frequency estimate will be nearly equal to the actual Doppler frequency.

The behavior of the fractional bias $(\bar{f} - \hat{\bar{f}})/\bar{f}$ versus α for various values of SNR is shown in Fig. 2. The magnitude of the bias is largest for $\alpha = 1.0$, decreasing with increasing α and increasing SNR. The bias effect is negligible for α greater than 100 and SNR greater than 0 dB. For $\alpha = 10$, the fractional bias ranges from 0.01 to 0.07 as SNR falls from 10 to 0 dB. For $\alpha = 1.0$ the bias ranges from 0.07 to 0.5 for the same values of SNR. Hence, for an incoherent Doppler sonar with a signal-to-noise bandwidth ratio near unity and a correspondingly small covariance amplitude ratio ($1 \leqslant \alpha \leqslant 10$) operating in a low SNR environment ($0 \leqslant \mathrm{SNR} \leqslant 10$ dB) biasing due to band-limited noise may cause errors as large as 7 to 50 percent in the CP mean frequency estimate.

Hansen (1985) presents an expression for a noise biased CP mean frequency estimator similar to (10). Hansen's expression does not include the parameter α and hence is not valid for arbitrary signal and noise covariance except in the limit as $\tau \to 0$ where $\alpha(\tau)$ approaches 1.0. Since the covariance for a given Doppler system must be computed for some finite lag $\tau = \tau_0$, considering only the limiting case obscures the role of the signal-to-noise covariance amplitude ratio as well as differences in signal and noise covariance characteristics for different systems. The importance of this effect can be seen from Fig. 2: Hansen's expression may significantly overestimate the biasing effect if the value of α is greater than one for $\tau = \tau_0$.

The nature of noise biasing for a wide range of Doppler systems can be understood by the consider-

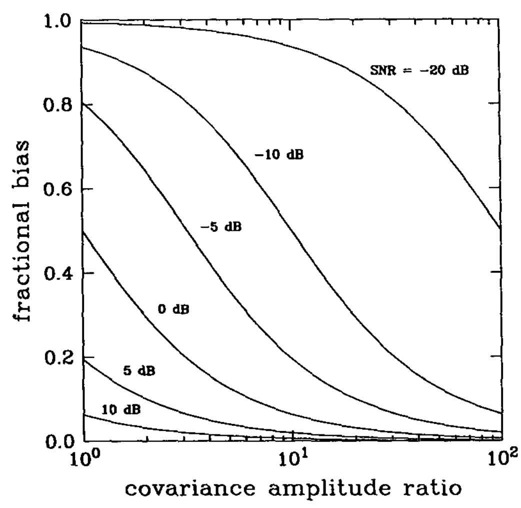

FIG. 2. Fractional bias $(\bar{f} - \hat{\bar{f}})/\bar{f}$ vs normalized covariance amplitude ratio $\alpha = R_s/R_n$, and $\mathrm{SNR} = S^2/N^2$. The curves are computed from (10) after normalization by the Nyquist frequency $f_{\mathrm{nyq}} = (2\tau_0)^{-1}$, where τ_0 is the sampling interval. The value of the true Doppler frequency is fixed at $\bar{f}/f_{\mathrm{nyq}} = 1/2$ so that the curves represent the maximum fractional bias for SNR > 0 dB (see Fig. 3). The covariance lag is assumed to be the first available lag for a given sampling interval ($\tau = \tau_0$).

ation of the noise biasing equation (10). Commercially available Doppler profilers like those discussed by Chereskin et al. (1989) use a Doppler frequency tracking technique so that the receiver filter need be no wider than the inverse pulse duration. The result is that the noise bandwidth is nearly equal to the signal bandwidth and $\alpha(\tau)$ is very close to one for small values of τ (cf. Fig. 1). In this case (10) reduces to the approximate expression presented by Hansen (1985) and used by Chereskin et al. (1989) in their evaluation of model and field data. However, for other Doppler systems where the signal and noise bandwidths are not equal, the effect of varying α is important. For an incoherent Doppler system that does not employ pulse-to-pulse frequency tracking, and thus uses a receiver filter somewhat wider than the signal bandwidth, the appropriate value of α may be between 1.5 and 2.0 (Pluedemann 1987). In this case the approximate expression (which assumes $\alpha = 1$) would overestimate the bias by as much as a factor of two for SNR between 5 and 10 dB (Fig. 2). For incoherent Doppler systems using a broadband pulse (e.g., Brumley et al. 1990) the signal covariance can be enhanced by coding at a relatively large lag where the noise covariance is small. Appropriate values of α for these systems may be significantly greater than 1 and noise biasing effects will be much less than those predicted by the approximate expression.

In pulse-to-pulse coherent Doppler systems the receiver bandwidth must be broadened to accommodate the signal spectrum from a transmitted pulse that is short compared to that of an incoherent system. This results in a small noise covariance at the lag where the CP mean frequency estimate is made. The signal covariance remains high because the returns from successive pulses are processed in a phase-coherent manner. The appropriate values of α for these systems are of order 10^4 and noise biasing is not important because the condition (6) is true and the noise can be considered approximately white. As an extreme example, it has been shown by previous authors (Miller and Rochwarger 1972; Sirmans and Bumgarner 1975b) that the CP mean frequency estimator is unbiased for all SNR if the noise covariance is zero for lags greater than zero (true white noise). This result is recovered from (10), but not from Hansen's expression, which predicts a bias dependent only on SNR.

The expression (10) can be generalized by normalizing the Doppler shift by the Nyquist frequency $f_{nyq} = (2\tau_0)^{-1}$, where τ_0 is the sampling interval. The dependence of the normalized bias on the normalized Doppler frequency is illustrated in Fig. 3 for various values of SNR. The normalized bias increases with increasing Doppler frequency for all SNR if $\bar{f}/f_{nyq}$ is less than 1/2. For SNR less than 0 dB the bias continues to increase with increasing frequency. For SNR greater than 0 dB, the bias increases to a maximum value and then decreases with further increase in frequency. The fractional bias is equal to the ratio of ordinate to abscissa from the curves in Fig. 3, and is nearly constant for $\bar{f}/f_{nyq} < 1/2$. Significant values of fractional bias (≥ 0.1) occur for all but the highest Doppler frequencies when SNR is less than 10 dB.

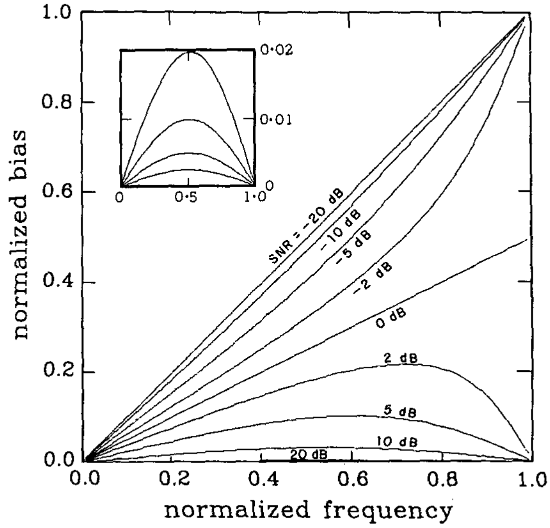

FIG. 3. Normalized bias $(\bar{f} - \hat{\bar{f}})/f_{nyq}$ vs normalized Doppler frequency $\bar{f}/f_{nyq}$, and SNR. Values are computed from (10) with the normalized covariance amplitude ratio fixed at $\alpha = 1.0$, so these curves represent the maximum expected biases. Inset shows normalized bias vs. normalized Doppler frequency for SNR of 16 (upper line), 32, 64, and 128. Note that the noise biasing term has the same effect as a DC offset in the in-phase channel, as described by Sirmans and Bumgarner (1975b), if SNR is considered as the signal-to-DC power ratio (compare their Fig. 9 to inset).

b. Correction of noise biasing

The bias predicted by (10) can be corrected if the magnitude of the noise covariance can be determined for the lag at which the signal-plus-noise covariance is measured. Most Doppler systems employing the CP technique to measure the echo intensity (covariance at $\tau = 0$) as well as the complex covariance at one or more nonzero lags. The available covariance lags are determined by the sampling interval of the system. If the complex envelope is sampled at intervals τ_0, then the covariance has the form $C(k\tau_0)$, where $k = 0, 1, 2, \cdots$ is the lag index. The discretely sampled signal-plus-noise covariance for the first three lags ($k = 0, 1, 2$) can be written

$$C_{sn}(0) = S^2 + N^2 \tag{11a}$$

$$C_{sn}(\tau_0) = S^2 R_s(\tau_0) \exp[i2\pi\bar{f}\tau_0] + N^2 R_n(\tau_0) \tag{11b}$$

$$C_{sn}(2\tau_0) = S^2 R_s(2\tau_0) \exp[i2\pi\bar{f}2\tau_0] + N^2 R_n(2\tau_0). \tag{11c}$$

If the noise covariance (equivalently the noise spectrum) for the system is known and considered invariant from pulse to pulse, then $C_n(\tau) = N^2 R_n(\tau)$ can be subtracted from $C_{sn}(\tau)$ to produce an unbiased estimate of $\bar{f}$ from any nonzero lag covariance (Miller and Rochwarger 1970, 1972). In practice, $C_n(\tau)$ is rarely known a priori, but must be estimated from a time interval where there is no transmission, or from a small fraction of the return at far range where the echo intensity has decayed significantly.

A straightforward way to correct for biasing due to band-limited noise without making the assumption of time-invariant noise covariance is to choose the pulse repetition time (PRT) to be longer than the decay time of the return echo. In this case, a fraction of the return at far range will have virtually no contribution from the signal, and the noise covariance for a given lag can be estimated directly from C_{sn} (i.e., for SNR $\ll 1$, $C_{sn}(\tau) \approx C_n(\tau)$). A time-varying bias correction can then be made by subtracting estimates of $C_n(\tau)$, averaged over the "noise-only" region of a pulse or group of pulses, from the corresponding signal-plus-noise covariance. Unfortunately, extending the PRT by a sufficient amount to obtain accurate noise covariance estimates results in fewer pulses, and hence larger velocity variance, in a given averaging interval.

Without prior knowledge or real-time measurement of the noise covariance, bias correction cannot be done by the methods described above. Instead, a correction method that accounts for the biasing effect of band limited noise without requiring direct measurement of noise covariance is needed. This can be accomplished by introducing models of the signal and noise covariance amplitudes. The covariance modeling approach allows correction for noise biasing from measurements of signal-*plus*-noise covariance at more than one lag. It is sufficient to model the covariance function amplitude as a function of lag; the environmentally dependent parameters, Doppler shift and SNR, are determined from the data.

A composite covariance $R_{sn}(\tau) = R_s(\tau) + R_n(\tau)$ is considered where R_s and R_n are the covariance function amplitudes normalized by the signal and noise power, respectively. If the system noise is considered white, then the noise covariance at the receiver output can be specified in terms of the characteristics of the receiver filter. For a receiver filter with a boxcar frequency response, the noise covariance will have the form of a sinc function with zero crossings determined by the receiver bandwidth σ_n. The signal covariance for a pulsed Doppler system is determined by the combined effects of the receiver bandwidth and the pulse duration T_p (Doviak and Zrnić 1984). For a transmission of the form (1) the signal covariance at the receiver input will have the form of a "triangle" function. The band-limiting of the receiver filter causes a distortion of the signal covariance from the triangle shape at the receiver output (cf. Fig. 1). This distortion increases as the bandwidth-pulse duration product $\sigma_n T_p$ decreases, and is most pronounced for small values of covariance lag. The signal covariance at the receiver output is modelled as the convolution of the "triangle" function and the noise covariance. Specifically, the signal and noise covariance amplitudes are

$$R_s(\tau) = \begin{cases} (1 - b_s\tau) * R_n(\tau), & |\tau| < b_s^{-1} \\ 0, & \text{otherwise} \end{cases} \quad (12a)$$

$$R_n(\tau) = \mathrm{sinc}[b_n\tau] \quad (12b)$$

where $\mathrm{sinc}(x) = \sin(\pi x)/\pi x$, $*$ denotes convolution, and the bandwidth parameters b_s and b_n are related to the -3 dB widths of the signal and noise spectra by $b_s = 1.13\sigma_s$, $b_n = \sigma_n$. The corresponding normalized spectra are

$$P_s(f) = b_s^{-1}\,\mathrm{sinc}^2[f/b_s]P_n(f) \quad (13a)$$

$$P_n(f) = b_n^{-1}\Pi[f/b_n]. \quad (13b)$$

Since this model results from taking the transmitted pulse to be boxcar in time [see (1)] and the noise spectrum to be boxcar in frequency, (12) and (13) will be called the boxcar model.

The boxcar model yields solutions for $R_s(\tau)$ and $R_n(\tau)$ for any lag if the signal and noise bandwidths are known, and in principle allow a solution for the unknowns $\bar{f}$, S^2 and N^2 from the three equations (11). This general solution is nonlinear and is quite sensitive to small deviations of the data from the model predictions. A much simpler and more robust solution that requires only two autocovariance lags is possible if the phase change of the signal is small over the sample interval. This condition can be expressed in terms of the magnitude of the mean Doppler frequency compared to the Nyquist frequency. Assume that $\bar{f}$ is being estimated from the first nonzero lag covariance (11b). If $|\bar{f}| \ll (4\tau_0)^{-1} = f_{\mathrm{nyq}}/2$ then $|2\pi\bar{f}\tau_0| \ll \pi/2$ and $\cos(2\pi\bar{f}\tau_0) \approx 1$. This implies that the magnitude of $C_{sn}(\tau_0)$ can be well approximated by

$$|C_{sn}(\tau_0)| \approx C_s(\tau_0) + C_n(\tau_0) = S^2R_s(\tau_0) + N^2R_n(\tau_0) \quad (14)$$

so that S^2 and N^2 (equivalently SNR) can be estimated from (11a) and (14). The values of R_s and R_n are obtained from the model covariances (12) using values of signal and noise bandwidth appropriate for the system of interest. Bias correction can be accomplished by subtracting the value of $N^2R_n(\tau_0)$ estimated using (11a) and (14) from (11b), and computing $\bar{f}$ according to (5). The conditions for validity of this approximation can be generalized for any covariance lag by requiring

$$|\bar{f}| \ll (2k)^{-1} f_{\mathrm{nyq}} \quad (15)$$

where k is the lag index. Bias correction can be made using any two lags which satisfy (15).

4. Conclusions

The mean frequency estimator based on the covariance processing (CP) or "pulse-pair" technique is unbiased in the presence of true "white" noise ($C_n(\tau) = 0$ for $\tau \neq 0$). Although the case of true white noise does not arise in practice, the white noise approximation can be made for a given lag τ if the signal-to-noise bandwidth ratio is small or, equivalently, if the signal-to-noise covariance ratio is large. If the signal-to-noise covariance ratio is not large, then the signal must be considered corrupted by additive, band-limited noise, and the CP estimator will show a bias at low SNR. The magnitude of bias for the CP mean frequency estimator in the presence of band-limited noise can be expressed in terms of the signal-to-noise power ratio, SNR, and the normalized signal-to-noise covariance amplitude ratio, $\alpha(\tau)$. Biasing is negligible for $\alpha > 100$ at practical limits of SNR (SNR > 0 dB), but many pulse-to-pulse incoherent Doppler systems are operated in the range $1 \leqslant \alpha \leqslant 10$ where biasing may be significant at low SNR. Low SNR is typically encountered at the limits of the profiling range as signal levels drop due to spreading and absorption of the energy in the transmitted pulse. For SNR between 10 and 0 dB, bias errors as large as 7% to 50% in the CP mean frequency estimate may be encountered. It should be noted that biasing of the CP mean frequency estimator at low SNR is but one of many possible biasing effects in a Doppler system (cf. Hansen 1985; Chereskin et al. 1989).

The nature of the bias is to force the CP estimated Doppler shift towards the mean frequency of the noise spectrum. If the noise power is flat across the passband of the receiver and the receiver filter is symmetric, then the bias will be towards the center of the frequency band passed by the receiver. In situations where tracking of Doppler shift is not used, the bias will be towards zero velocity. If tracking is employed, the bias is towards the velocity corresponding to the frequency at the center of the tracking band. A properly functioning tracking system will minimize the effect of biasing by keeping the center of the frequency band passed by the receiver near the mean frequency of the signal spectrum. However, designing an accurate tracking algorithm is difficult and some biasing problems may still arise in practice (e.g., Chereskin et al. 1989).

Several methods of bias correction are available. The simplest correction methods assume that the noise covariance is known or measurable, and may also assume that noise power is time-invariant. The presence of noise power that varies from pulse to pulse, combined with inadequate or unavailable noise covariance measurements, may make the use of these techniques impossible. For systems where multiple lags of covariance are measured, a method of estimating time-varying noise covariance and SNR has been presented. The method can be used even when no direct measurements of noise covariance are available, but does require models for the signal and noise covariance functions. In addition to providing a generalized bias correction method, the covariance models allow prediction of potential bias effects for the Doppler systems to which they apply. In this paper covariance models based on a simple gated-pulse transmission and a rectangular receiver filter, appropriate for a Doppler sonar system familiar to the authors, are introduced as an illustration of the correction method. Different functional forms for the envelope of the transmitted pulse or the receiver frequency response function may be appropriate for other Doppler systems and could be easily substituted for those used here.

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