# ON THE MEASUREMENT OF PRECIPITATION INTENSITY BY RADAR

*By S. Twomey*

Commonwealth Scientific and Industrial Research Organization, Australia

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The power of the radar echo received from a group of water drops is such that the received power $P_r = (C\Sigma ND^6)/S^2$, where $N$ is the number of drops of diameter $D$ in a unit volume, $S$ the range, and $C$ is a constant for any one radar. [See, for example, Ryde (1947) or Wexler (1948).] The precipitation rate is given by $R = \frac{1}{6}\pi\Sigma ND^3\ V(D)$, where $V(D)$ is the terminal velocity of drops of diameter $D$. It is obvious that the precipitation rate will be determined uniquely by radar-echo power only if the drop-size distribution is determined unambiguously by precipitation rate. Observations of drop-size distributions in rain made in Sydney, Australia, during the period November 1950 to June 1951 (to be reported elsewhere) gave distribution curves over short periods of time, which often varied considerably in shape; no one-to-one correspondence existed between drop-size distribution and precipitation rate. Some definite relation [*e.g.*, that of Laws and Parsons (1943), giving average drop-size distribution for various precipitation rates] must be assumed, however, in deducing precipitation rate from radar-echo power. The approximation involved in this procedure introduces an uncertainty into the measurements, which fundamentally limits the possible accuracy achievable by radar measurement of precipitation rate. With the distribution curves which have been obtained experimentally as a starting point, computations were made to assess the magnitude of the above-mentioned error caused by variations in shape of the drop-size spectrum.

The following quantities were calculated for each of a large number of either 5- or 10-min periods:

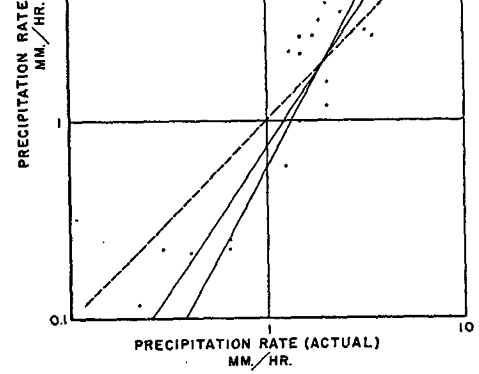

FIG. 1. Plot of "deduced" rainfall *versus* "actual" rainfall.

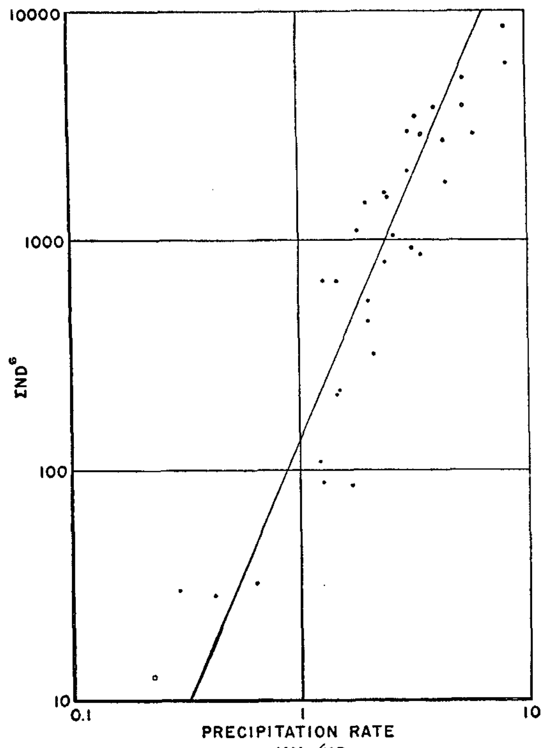

FIG. 2. Plot of $\Sigma ND^6$ *versus* rainfall intensity.

(a) $\sum ND^6$, calculated from the experimental curves of number per unit area *versus* drop diameter, with use of the values of terminal velocity published by Gunn and Kinzer (1949); (b) the mean precipitation rate $R$; (c) the precipitation rate which would correspond to the calculated value of $\sum ND^6$, if a Laws and Parsons distribution existed.

The "deduced" precipitation of (c) was plotted against the actual precipitation rate (fig. 1). The 45-deg line, for which "deduced" precipitation rate equals actual precipitation rate, represents a Laws and Parsons distribution; it is obvious that the scatter of the points about this line is quite appreciable (root-mean-square deviation 0.52). The scatter is reduced somewhat if one of the regression lines is used. This result shows that, apart from all other sources of error, fluctuations in the shape of drop-size distribution curves introduce an uncertainty in the value of precipitation rate deduced from radar observation. This uncertainty seriously militates against accurate determination of precipitation intensity by radar methods.

Use of the regression lines of fig. 1 is equivalent to deducing a regression equation connecting the calculated value of $\sum ND^6$ with actual precipitation rate, independent of the Laws and Parsons distribution. This procedure is represented by fig. 2. The scatter of the points about the regression line of the latter figure is still quite significant; the root mean square of the relative deviations in precipitation rate from this line was found to be 0.33. The regression equation for the data of fig. 2 was, in non-dimensional form,

$$\sum ND^6 = 127\ R^{2.287}$$

($D$ = diameter in mm; $R$ = precipitation rate in mm/hr). The correlation coefficient between the variables was 0.89. For comparison, the following equations from the literature are included:

$\sum ND^6 = 1600\ R^{1.4}$ (Smith, 1944);
$\sum ND^6 = 228\ R^{1.43}$ (calculated from Laws and Parsons, 1943);
$\sum ND^6 = 630\ R^{1.45}$ (Best, 1950, Shoeburyness);
$\sum ND^6 = 224\ R^{1.54}$ (Best, 1950, Ynyslas);
$\sum ND^6 = 220\ R^{1.6}$ (Marshall and Palmer, 1947);
$\sum ND^6 = 295\ R^{1.612}$ (Hood, 1950);
$\sum ND^6 = 190\ R^{1.72}$ (Marshall *et al.*, 1947);
$\sum ND^6 = 23.5\ R^{2.028}$ (Humphreys, quoted by Hood, 1950).

The exponent 2.287 found for the Sydney measurements may represent a tendency for the relative number of large drops in Sydney to be greater for higher rain intensities. However, the main feature of the above equations is the considerable disagreement between them. This emphasizes the uncertainty inherent in the deduction of precipitation rate from the quantity $\sum ND^6$. It is significant that the scatter of points representing different periods during continuous rain, or between different showers on the same day, was found not to be appreciably less than that of the total. It may safely be concluded therefore, that radar methods can give only an approximate measure of precipitation rate; the value deduced from the radar echo may be in error by a factor of 2:1 either way, and this randomly distributed error is independent of instrumentation or procedure adopted.

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