A THEORY OF THE TEMPORAL AND LATITUDINAL DISTRIBUTION OF TEMPERATURE

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(Manuscript received 29 October 1952)

ABSTRACT

An attempt is made to explain the observed distribution of mean temperature near the earth's surface over latitude and through the year in terms of insolation, radiative processes, and large-scale atmospheric eddy conduction of sensible heat. The results are close to the observations and indicate that these three factors explain most of the observed distribution. The larger discrepancies are all explainable in terms of physical factors which were not included in the theory.

1. Introduction

For many years now there have been available moderately reliable observations of the mean temperatures of the latitude circles of the northern hemisphere. These temperatures and their annual course constitute some of the most fundamental climatological data we possess, and the need for an adequate, quantitative explanation of them represents a long-standing challenge to meteorological theory. It is generally conceded that the latitudinal distribution and the annual march of the mean temperatures of the parallels are primarily determined by two factors: the latitudinal and temporal variation of the excess of incoming over outgoing radiation, and the transport of heat by the large-scale circulation of the atmosphere. The first of these factors was dealt with quantitatively by Milankovitch (1930), who calculated the latitudinal distribution of mean annual temperature to be expected on an earth with a uniform surface and a non-circulating atmosphere. He obtained the well-known result that these radiative temperatures are too high equatorward of about latitude 40 deg, and too low poleward of that latitude. It has been accepted that this is because the atmosphere transports heat poleward through its general circulation, thus cooling the equatorial regions somewhat and warming the polar regions. To the best of the writers' knowledge, this second factor has always been introduced qualitatively.[2] Is it possible to deal with the effect of the general circulation on planetary temperatures in a quantitative, analytic fashion? If so, what would be the effect? This paper represents an attempt to answer these questions.

[1] Contribution to a research project in theoretical climatology, being conducted under Contract Nonr-988(01), Project NR-082-071, between the Office of Naval Research and Florida State University.

[2] Defant (1950) has recently attempted to explain the mean meridional temperature-distribution of the troposphere in terms of vertical and horizontal eddy-processes and condensation of water. However, he assumes the temperature distribution close to the earth's surface to be given. It is this assumed distribution which we wish to explain here.

2. Quantitative statement of the problem

Let us consider a strip girdling the earth, parallel to a latitude circle, of width $r\,d\phi$ and height dz (r is the radius of the earth, and ϕ is the latitude). The first contribution to the heat content of the air in this strip will be a radiational one. Let I be the rate at which incoming solar radiation is absorbed, and R the net rate at which outgoing long-wave radiation is emitted, both per unit surface area. The total rate of receipt of radiational energy by the strip will then be given by $(I - R)2\pi r^2 \cos\phi\, d\phi$. To this must be added the influx of heat due to the general circulation. To introduce this factor analytically, we shall assume that the large-scale, lateral transport of heat by the atmosphere may be looked upon as a turbulent phenomenon, wherein the rate of transport of heat across a latitude circle is proportional to the north-south temperature gradient. The accumulation of heat in any latitude strip is, of course, proportional to the divergence of this temperature gradient. The gain of heat in the strip under consideration is, then,

$$\rho c_p K\, \nabla^2 T\, 2\pi r^2 \cos\phi\, d\phi\, dz = \rho c_p K\, 2\pi \cos\phi\, d\phi\, dz \left(-\tan\phi \frac{\partial T}{\partial \phi} + \frac{\partial^2 T}{\partial \phi^2}\right).$$

Here ρ is the density of the air, c_p the specific heat capacity at constant pressure, and K is the "gross-austausch" coefficient which, for simplicity, we shall assume to be constant with latitude and time.

In order that energy shall be conserved, the sum of these contributions, which are the only ones we shall consider, must be equal to the time-rate-of-increase of heat energy inside the strip, which is given by $\rho c_p(\partial T/\partial t)2\pi r^2 \cos\phi\, d\phi\, dz$, where t is time. We thus obtain

$$\frac{\partial T}{\partial t} = \frac{I - R}{\rho c_p\, dz} + \frac{K}{r^2}\left(-\tan\phi \frac{\partial T}{\partial \phi} + \frac{\partial^2 T}{\partial \phi^2}\right). \quad (1)$$

Before (1) can be solved, we must express I and R as functions of either the independent variables ϕ and t, or the dependent variable T.

The quantity I is determined by astronomical considerations, the albedo of the earth, and the transmission of our atmosphere. The effects of the first and third of these factors have been given analytically by Milankovitch (1930), as functions of latitude and time. The second may be taken into account if we assume that the albedo is constant with latitude and through the year. Unfortunately, the functions given by Milankovitch are entirely too complex to permit an analytic solution of the differential equation. Therefore, we shall replace Milankovitch's functions by more amenable ones which are numerically close to his results. A suitable expression for the northern hemisphere is

$$I = D(\delta - \epsilon \sin^3 \phi - \sin^3 \phi \cos \omega t), \qquad (2)$$

where ω is 2π divided by the length of the year, and D, δ and ϵ are constants. The extent to which such a contrived expression, with suitable values of the constants, numerically matches Milankovitch's values is shown in figs. 1 and 2. Fig. 1 gives the dependence of the annual averages of the two functions on latitude. It is immediately apparent that the two are quite similar in shape; and although there are certain differences in magnitude, these are not large and are certainly acceptable in view of the fact that there are other important influences upon the temperature which we are not including. Fig. 2 gives the annual variation of the two functions at the equator and pole. The fit is satisfactory at the equator, where our contrived function is constant while the actual radiation varies only slightly. The discrepancies are greater at the pole, because of the impossibility of finding a sufficiently simple expression which reflects the complex fact that no insolation at all is received at the pole during winter. Thus, the function we propose to adopt exhibits too much polar heating during the winter. In compensation, it departs during the summer by letting too little solar energy into high latitudes. This sort of discrepancy seems unavoidable under the procedure we are following. It is likely, however, that the deficit of assumed incoming radiation during the summer is largely counteracted by the fact that so much of the actual incoming radiation goes into latent heat of melting of ice and snow instead of into an actual increase in temperature. In winter, however, we should anticipate that the theory will give too high temperatures near the pole.

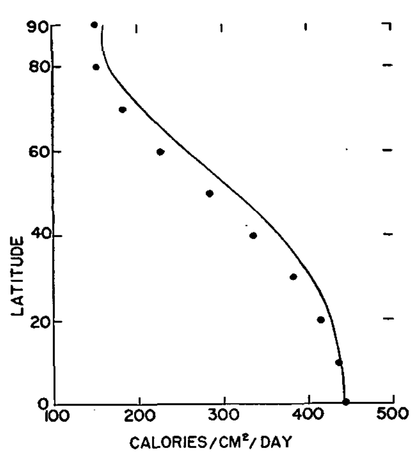

FIG. 1. Curve gives assumed mean annual rate of absorption of solar energy as function of latitude, from (2). Points give rate at which solar energy would actually be absorbed if albedo were 0.39 and atmospheric transmission coefficient were 0.90.

The net outgoing radiation, R, is also a rather complex thing in actuality. It depends primarily upon the temperature of the air, the water-vapor content, and the vertical lapse-rates of these quantities. Furthermore, all these factors may vary with latitude and time. If we were to limit ourselves to this aspect of the problem alone, it would be possible to treat the radiation in a more general way, although even then great simplifications would be required. Since we wish to deal with lateral eddy-transport of heat simultaneously, it will be necessary to simplify the radiative term still further. As a first approximation, we shall assume that we have a layer of air next to the ground, of thickness dz, which is in thermal equilibrium with the earth's surface, and which, therefore, has essentially the same temperature as the surface. The earth radiates like a black body, so the loss of heat is σT^4, where σ is the Stefan-Boltzmann constant. However, because of the greenhouse effect, a certain fraction, e, of this radiation is returned to the surface layer. The net loss is then $R = (1 - e)\sigma T^4$. The quantity e will be assumed to be independent of latitude and time.

The functional form assumed for I is suitable for the northern hemisphere alone, and does not yet express the fact that the radiation curves change phase by 180 deg as we cross the equator. To include this and make the problem applicable to the whole earth, we define a step function, $1(\phi)$, to be zero for $\phi < 0$,

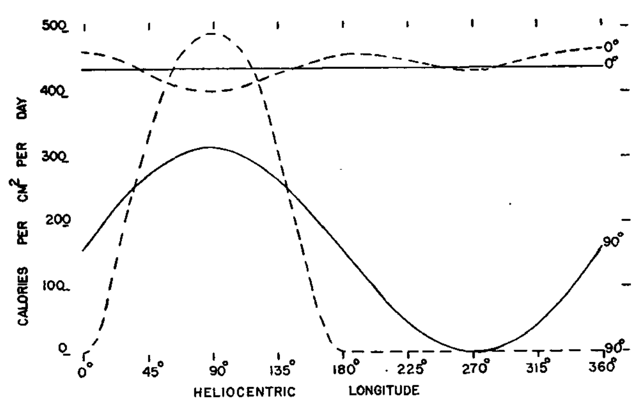

FIG. 2. Solid curves give assumed rate of absorption of solar energy as function of time through year, from (2), at equator and poles. Dashed curves give rate at which solar energy would actually be absorbed if albedo were 0.39 and atmospheric transmission coefficient were 0.90.

one for $\phi > 0$, and one-half for $\phi = 0$. We rewrite (2) as follows:

$$I=D[\delta+\{-1(\phi)+1(-\phi)\}\epsilon \sin^3\phi-\sin^3\phi\cos\omega t]. \quad (3)$$

It can be seen from (3) that inclusion of the step function in this way accomplishes the desired extension to the entire sphere.

The final differential equation to be solved may now be obtained by a convenient transformation of one of the independent variables. Let $\sin\phi = \mu$; the result is:

$$\frac{\partial T}{\partial t}=\alpha(1-\mu^2)\frac{\partial^2 T}{\partial \mu^2}-2\alpha\mu\frac{\partial T}{\partial \mu} +\beta[\delta+\epsilon\{-1(\mu)+1(-\mu)\}\mu^3-\mu^3\cos\omega t]-\gamma T^4, \quad (4)$$

where the constant coefficients are

$$\alpha=\frac{K}{r^2},\quad \beta=\frac{D}{\rho c_p\,dz},\quad \gamma=\frac{1-e}{\rho c_p\,dz}.$$

3. Solution of the differential equation

We shall seek solutions of (4) of the form

$$T = T_0 + \xi(\mu) + \tau(t,\mu), \quad (5)$$

where T_0 is a constant, ξ a function of latitude only, and τ is a function of latitude and time. Both ξ and τ are assumed to be small compared to T_0. Because of the relative smallness of ξ and τ, we may linearize by substituting (5) into (4) and neglecting terms in ξ and τ which are of higher degree than one. In (5), we are free to choose T_0 and the function $\xi(\mu)$ in any way we please. For any given choice, $\tau(t,\mu)$ will then be specified by substitution in the linearized differential equation arising from (4) and (5). For physical and mathematical reasons, we shall choose T_0 and ξ to satisfy (6) and (7) below; τ must then satisfy (8):

$$0 = \beta\delta - \gamma T_0^4, \quad (6)$$

$$0 = \alpha(1-\mu^2)\frac{\partial^2\xi}{\partial\mu^2} - 2\alpha\mu\frac{\partial\xi}{\partial\mu} + \beta[-1(\mu)+1(-\mu)]\epsilon\mu^3 - 4\gamma T_0^3\xi, \quad (7)$$

$$\frac{\partial\tau}{\partial t} = \alpha(1-\mu^2)\frac{\partial^2\tau}{\partial\mu^2} - 2\alpha\mu\frac{\partial\tau}{\partial\mu} - \beta\mu^3\cos\omega t - 4\gamma T_0^3\tau. \quad (8)$$

From this point of view, (5) is simply a change of dependent variable.

Equation (6) merely says that $T_0 = (\beta\delta/\gamma)^{\frac{1}{4}} = [D\delta/(1-e)\sigma]^{\frac{1}{4}}$. Thus, physically, T_0 is the radiative equilibrium temperature of the surface layer of air which would prevail if there were no eddy transport, and if the solar radiation absorbed at every latitude were constant through the year and equal to the amount actually absorbed at the equator. The value of T_0 is 301K, with a suitable choice of the basic constants of the problem.

It is possible to see, in advance, part of the nature and effects of the functions $\xi(\mu)$ and $\tau(t,\mu)$. Since T_0 is the result of an excessive rate of receipt of solar energy, it is too high a temperature. Thus we would expect ξ, which is in part a correction for the actual diminution of the energy supply poleward, to be negative. Further, its effect should be the same in the two hemispheres, so we expect ξ to be symmetric about the equator.

The quantity τ is in part a correction to T_0 due to the alternate presentation of the two hemispheres to the direct rays of the sun. Therefore, we expect τ to be zero at, and antisymmetric about, the equator. We shall see that these predictions from physical reasoning will be borne out.

Equation (7) may be solved by the following procedure.

Legendre's differential equation is

$$(1-\mu^2)\,d^2P_n(\mu)/d\mu^2 - 2\mu\,dP_n(\mu)/d\mu = -n(n+1)P_n(\mu), \quad (9)$$

where the $P_n(\mu)$ are the Legendre polynomials of degree n. Further, the Legendre polynomials constitute a complete orthogonal set over the range of μ involved in our problem:

$$\int_{-1}^{1} P_n(\mu)\,P_m(\mu)\,d\mu = \begin{cases} 0 & \text{if } n \neq m \\ \dfrac{2}{2m+1} & \text{if } n = m. \end{cases} \quad (10)$$

Therefore, let us seek a solution of (7) in the form of an infinite sum of Legendre polynomials,

$$\xi = \sum_{n=0}^{\infty} a_n P_n(\mu), \quad (11)$$

where the a_n are constants which may be determined in the following way. Substitution of the assumed solution (11) into the differential equation (7) yields

$$0 = \sum_{n=0}^{\infty} \alpha a_n[(1-\mu^2)\,d^2P_n(\mu)/d\mu^2 + 2\mu\,dP_n(\mu)/d\mu] + \beta[-1(\mu)+1(-\mu)]\epsilon\mu^3 - 4\gamma T_0^3\sum_{n=0}^{\infty} a_n P_n(\mu).$$

The first sum contains the left-hand side of Legendre's equation, (9); therefore,

$$0 = -\sum_{n=0}^{\infty}[\alpha n(n+1) + 4\gamma T_0^3]a_n P_n(\mu) + \beta[-1(\mu)+1(-\mu)]\epsilon\mu^3. \quad (12)$$

We now multiply by $P_m(\mu)$ and integrate over the range of orthogonality. From (10), we see that the only contribution from the infinite sum comes when $n = m$. The contribution from the second term of (12) is readily obtained by integration; thus a_m may be determined. Substitution of the result into (11) and simplification yields the solution

$$\xi = -\sum_{n=0}^{\infty} \frac{\beta\epsilon}{2^{2n}} \frac{4n+1}{2\alpha n(2n+1) + 4\gamma T_0^3} \times \sum_{j=0}^{n} \frac{(-1)^j (4n-2j)!\, P_{2n}(\mu)}{j!\,(2n-j)!\,(2n-2j)!\,(2n-2j+4)}. \quad (13)$$

Since (13) contains only the even Legendre polynomials, ξ is symmetric about the equator, as expected. The series converges rapidly.

To solve (8), we first replace $\cos \omega t$ by $e^{i\omega t}$ and note that (8) is the real part of this modified equation. But this modified equation is linear, and all operations are real; hence the solution of its real part, (8), is the same as the real part of its solution. If we now assume a solution of the form $\tau = \eta(\mu)\, e^{i\omega t}$, the time dependence vanishes and we obtain

$$(1-\mu^2)\, d^2\eta/d\mu^2 - 2\mu\, d\eta/d\mu - \beta\mu^3/\alpha - 4\gamma T_0^3\eta = i\omega\eta. \quad (14)$$

Again we seek a solution of the form

$$\eta = \sum_{n=0}^{\infty} b_n\, P_n(\mu),$$

and apply the same procedure. The result for the real part of τ is

$$\tau = -\frac{\beta}{5}\Bigg[\frac{3\mu}{(4\gamma T_0^3 + 2\alpha)^2 + \omega^2} \times \{(4\gamma T_0^3 + 2\alpha)\cos\omega t + \omega \sin \omega t\} + \frac{5\mu^3 - 3\mu}{(4\gamma T_0^3 + 12\alpha)^2 + \omega^2} \times \{(4\gamma T_0^3 + 12\alpha)\cos \omega t + \omega \sin \omega t\}\Bigg]. \quad (15)$$

The function τ involves only odd powers of μ and, therefore, is antisymmetric about the equator. Furthermore, it is zero at the equator, thus possessing both of the earlier physically determined properties. It is instructive to note that the solution to (7) resulted in an infinite series, while the solution to (8) by the same method yielded only a finite number of terms. This is because the quantity μ^3, which occurs in (8), is representable as the sum of only two of the Legendre polynomials: $\mu^3 = 2P_3(\mu)/5 + 3P_1(\mu)/5$. Because of the orthogonality of the $P_n(\mu)$, only two terms remain of the infinite sum assumed. In (7), however, we find the term $[-1(\mu) + 1(-\mu)]\mu^3$, and to represent this in a series required an infinite number of the Legendre polynomials. Thus there is no selection due to the orthogonality, and the solution is an infinite sum. This distinction arises from the functional form chosen for the input I. If this expression had been chosen so as to involve a function of μ which was not a finite combination of the $P_n(\mu)$, the solutions for both ξ and τ would have been infinite series. Equation (14), then, contains only a part of the solution that would have been obtained if the intricate Milankovitch function could have been utilized. Presumably, we have found the major part of the more complex solution.

4. Choice of constants

In equations (2) and (3) for I, the rate of input of energy, the following values have been adopted: $D = 160$ cal cm^{-2} day^{-1}, $\delta = 2.75$, $\epsilon = 1.75$, $\omega = 1.99 \times 10^{-7}$ sec^{-1}. The first three of these were chosen in an arbitrary fashion, so as to approximate the results derived from Milankovitch's work. Undoubtedly, somewhat better choices could have been made, but in view of the number of physical factors which have been omitted from this problem it did not seem worthwhile to strive for any closer fit than is shown in figs. 1 and 2.

The term incorporating outgoing radiation is $(1-e)\sigma T^4$, where $(1-e)$ is to be fixed. To do this, let us consider the annual mean temperature of the layer of air near the earth's surface, averaged over the entire earth. For such a steady average over space and time, the outgoing radiation must be equal to the incoming radiation, under the assumptions made here, since the latitudinal transport of heat by the atmosphere can only change the distribution of temperature but cannot affect the earth's mean temperature. Thus, $(1-e)\sigma\bar{T}^4 = \bar{I}$, where $\bar{T}$ is the earth's mean temperature and $\bar{I}$ is the average value of the adopted rule of input of energy. This expression is only approximate, because we really should use the mean of the fourth power of the temperature instead of the fourth power of the mean. Insertion of 287K for $\bar{T}$ and 0.256 cal cm^{-2} min^{-1} for $\bar{I}$, calculated from (3), gives $1 - e = 0.457$.

The combination $\rho c_p\, dz$ appears in our problem. Since we are concerned with representative values, it is reasonable to choose $\rho = 1.20 \times 10^{-3}$ g/cm^3. The value of c_p for air is 0.239 cal g^{-1} deg^{-1}. A choice for dz is more difficult, however. If we are interested in the temperatures at the standard observational level of 2 m, it would seem reasonable to make the standard level the middle of the layer. We would then choose $dz = 4$ m. However, we have so far neglected the effect of vertical transport of heat by turbulence. One of the chief effects of vertical turbulence is to distribute, over a layer a few hundred meters thick, the heat imparted to the very lowest few meters by other processes. In addition to this, it will cause some flux of heat across a horizontal surface bounding these lowest few hundred meters. We shall not incorporate this turbulent flux into our theory directly, but shall include the eddy redistribution of energy through the choice of a value for dz. The idea that vertical eddy transport of properties is essentially limited in height is suggested by such theories and observations as are available. For example, the frictional effect of the earth's surface on the wind is commonly known to decrease with elevation and to be rather small by 500 m above the surface. The amplitude of the diurnal variation of temperature also decreases with elevation, and in theory and observation has decreased to about $1/e$ of the surface value by a height of 500 m (see Brunt, 1939). Sutton (1948) has recently computed from observations that q, the rate of vertical transport of sensible heat on a clear June day in England, decreases upward at a rate given by $q/q_0 = 1 - 1.8 \times 10^{-5} z$ between 10 and 100 m. If we permit ourselves an extrapolation of this formula, we find that q should become negligibly small in about 500 m. From these indications, it appears reasonable to incorporate vertical eddy processes by choosing $dz = 500$ m. At least, this value should be of the right order of magnitude. It will still be possible to compare the results so obtained with observations at 2 m, because the average temperature of the lowest 500 m should not be more than one or two degrees Celsius lower than that at 2 m.

Finally, we must select a value for the large-scale eddy-exchange coefficient K. We shall adopt an approximate value of $K = 6.8 \times 10^{10}$ cm^2/sec, which is appropriate for eddies the size of migratory cyclones and anticyclones (see, for instance, Lettau, 1936, or White and Jung, 1951). With these choices, the constants in (4) become: $\alpha = 1.67 \times 10^{-7}$ sec^{-1}; $\beta = 1.29 \times 10^{-4}$ sec^{-1} deg^{-3}; $\gamma = 4.35 \times 10^{-14}$ sec^{-1} deg^{-3}.

5. Numerical results

The values obtained when the above constants are substituted in (13) and (15) are represented in fig. 3. The equatorial temperature is found to be constant throughout the year, at 300K. This is because the rate of receipt of radiation at the equator is independent of time in our model. At other latitudes, the temperature varies sinusoidally through the year, with increasing amplitude as the latitude increases. Thus one recognizes, qualitatively, that certain important features of the actual temperature field have been included successfully. These will be discussed in more detail shortly.

In the real atmosphere, there is an easily detectable lag of the dates of maximum and minimum temperatures behind the solstices. This lag amounts to several weeks over much of the world (Prescott and Collins, 1951). One finds this phenomenon explained in some textbooks as due to the fact that, for a time after the summer solstice, more energy is gained from the sun than is lost by radiation, even though insolation is decreasing with time. If this is the primary factor at work, the theoretical results in fig. 3 should include lags of the proper size. The scale of fig. 3 is such that no lag is apparent, but calculations from (15) indicate theoretical lags of 2 to 3 days. This is insufficient by

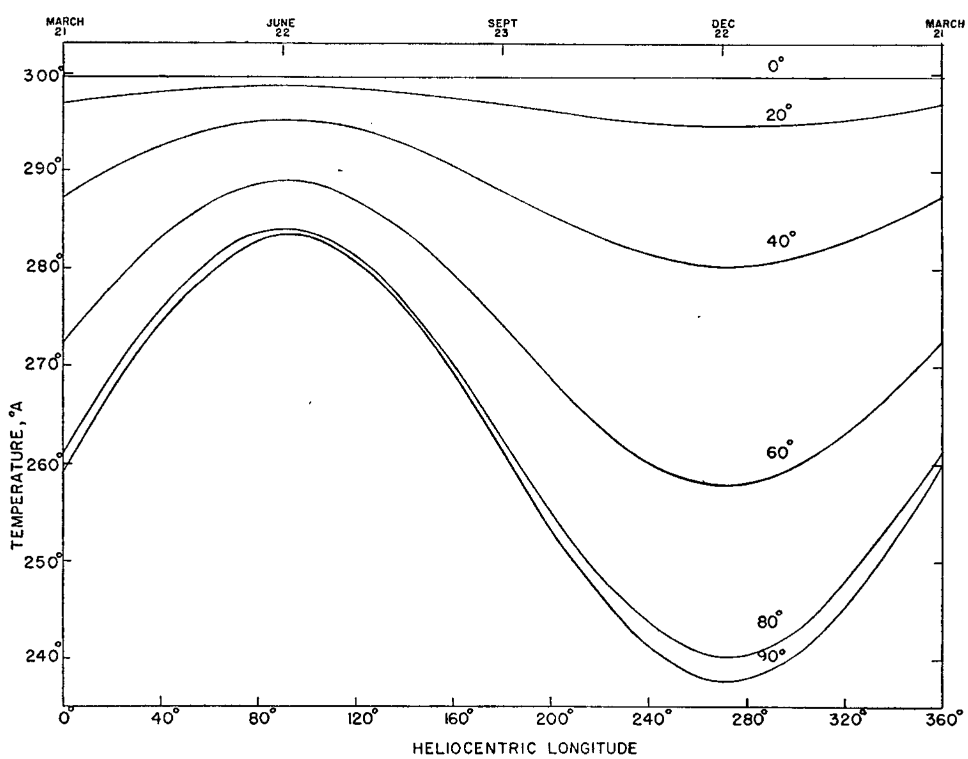

Fig. 3. Theoretical annual variation of temperature at various latitudes, calculated from (6), (13) and (15).

an order of magnitude or more. Therefore, the explanation offered above must be rejected and one must be sought in terms of physical factors which have not been included in the present theory.

An explanation of the observed lags may lie in the following concepts. Consider three regions into which the solar energy absorbed at the earth's surface may go: the underlying surface, the layer of air next to the ground, and the overlying atmosphere. The total amount of solar energy is, of course, a function of time, but the apportionment of the available energy may also be a function of time. This last time-dependence may introduce a lag in the temperature of the lowest layer of air, since it is quite possible, for example, that the portion of the available energy which goes into the lowest layer may increase with time after the summer solstice at a rapid enough rate to produce a maximum temperature after the solstice. Some evidence in favor of an explanation along these lines may be found in the fact that the lags are considerably greater over the oceans than over the land (Prestcott and Collins, 1951), and it is for the oceans at certain seasons of the year that a much greater fraction of the absorbed radiation goes into the underlying surface than for the land. At other seasons, of course, this energy is released again to the atmosphere. Thus, the observed lags are greatest where this apportionment of energy seems to be most significant. Other factors which may play important roles are evaporation and the seasonal variation of atmospheric water-content. A quantitative theory of the relative significance of these and other factors in producing the observed lags is clearly needed and is not supplied here. Gleeson (1950) has made an approach to this problem, but more work is needed to evaluate the relative importance of the many factors he considers.

When one deals with a heat-conduction problem, as we are doing here in principle, it is to be expected that time lags will appear. In our problem the sun's rays alternately warm the polar regions and permit them to cool, while the equator is kept at a constant temperature. The heating and cooling near the poles should then be "conducted" equatorward, with the time of maximum and minimum temperature occurring later and later as the equator is approached. This is found in the theoretical results, to a slight extent. Thus, the maximum or minimum temperature occurs one day later at latitude 20 deg than at the pole. It appears that the radiative part of the problem is dominant in this respect, so that the effect of conduction on the lag is distinctly minimized. Nevertheless, some reserve must be maintained towards this conclusion, because the extent to which the radiative terms dominate the conductive terms in their effects on lags may depend markedly on the precise functional form chosen to represent the rate of input of energy. For example, the input function utilized here, (3), involves μ^3, which is a linear combination of two of the Legendre polynomials. This produced the small dependence of lag on latitude discussed above. If μ^3 is replaced by μ in (3), one has a poorer approximation to the correct input function and, because μ involves only one Legendre polynomial, no dependence of lag on latitude appears at all. A closer approach to the Milankovitch input function, involving more than two polynomials, might result in a greater dependence of lag on latitude.

To compare these results more closely with observations, the theoretical curves were averaged over January and July. The resulting means are compared with the average January and July temperatures of the latitude circles of the northern hemisphere (Haurwitz and Austin, 1944) in fig. 4. The match between theory and observation is quite satisfactory in January, at all latitudes except the pole itself. There is some question as to the reliability of the reported polar temperatures; nevertheless, we have already anticipated just such a discrepancy because the adopted input function supplies too much energy near the pole during winter.

In July there are two regions of appreciable error: north of latitude 60 deg, where the theoretical temperatures are too high; and in the subtropics, where the theoretical temperatures are too low. Both of these deviations are easily understood in terms of factors not included in the theory. Near the pole, in summer, much of the sun's energy goes to melting snow and ice instead of raising the temperature. Since such phase changes have not been taken account of here, it is quite understandable that the predicted polar temperatures are too high in July. The discrepancy in the subtropics may be attributed to two neglected factors. First, the true radiation received at the outside of the atmosphere during summer has a maximum in the subtropics which is not contained in our input function. Second, there is a pronounced minimum in cloudiness and albedo in July from 20°N

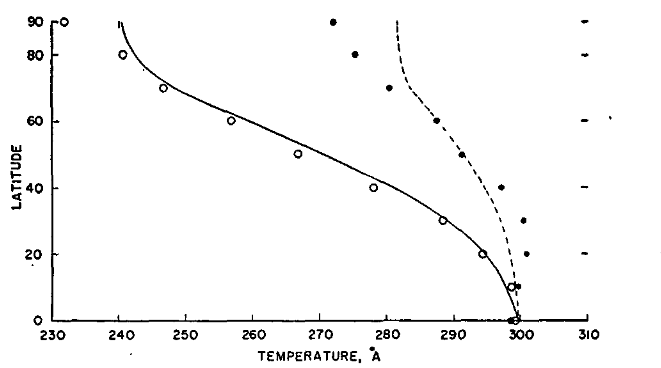

FIG. 4. Theoretical mean January temperature as function of latitude (solid line) compared to observed mean January temperatures (open circles), and theoretical mean July temperature as function of latitude (dashed line) compared to observed mean July temperatures (dots).

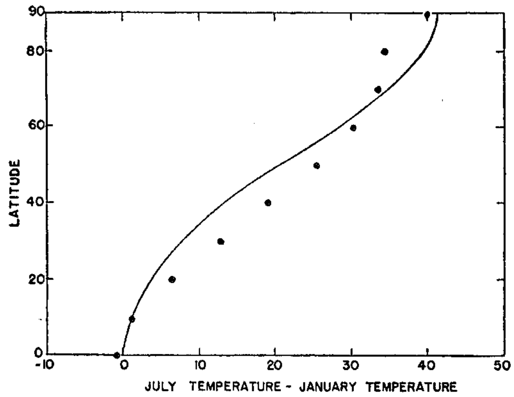

Fig. 5. Annual range of temperature as function of latitude. Curve is the theoretical result, and dots are observed values.

to 40°N, but we have neglected meridional variations in albedo.

Another comparison of the theory with observation is contained in fig. 5, where we have plotted the theoretical annual range, determined from monthly means, together with the observed annual range. It will be seen that the actual increase in annual range with latitude is fairly well represented by the theory. The discrepancies are all explainable in terms of the neglected factors discussed above.

Part of the effect of including eddy exchange-processes may be seen in fig. 6, where the theoretical mean annual temperature is plotted as a function of latitude for the case of purely radiative equilibrium ($K = 0$) and the case of equilibrium due to radiation and eddy conduction ($K \neq 0$). The inclusion of eddy processes lowers the mean equatorial temperature and raises the mean polar temperature, compared with the radiative equilibrium case, as was expected. The magnitude of the effect is greater near the pole than near the equator, because of the smaller surface area between successive parallels of latitude near the pole. The most important influence of the introduction of the eddy processes cannot be seen in the annual mean, but occurs during winter. If one considers radiative effects only, the winter polar temperatures must fall to extremely low values. Eddy transport in that season constitutes the predominate source of heat for the arctic and antarctic, and keeps the temperatures up to the levels indicated in fig. 3.

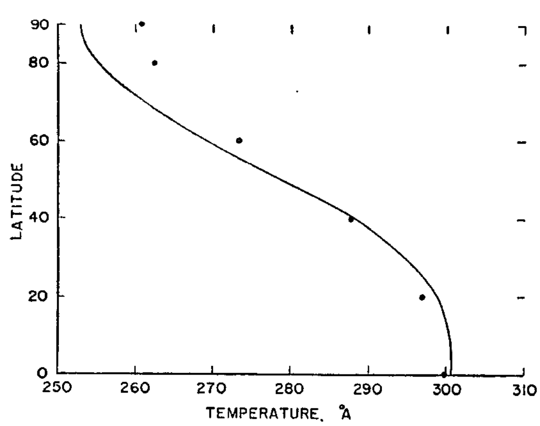

Fig. 6. Theoretical annual mean temperature as function of latitude if no lateral atmospheric heat-transport takes place (solid curve), and if heat transport takes place as assumed in this paper (points).

6. Discussion of the basic assumptions

Let us now summarize and make explicit the basic assumptions underlying this work, so as to understand fully the results and their limitations.

First, we have assumed an artificial form of the rate of receipt of solar energy. This expression, (3), was arbitrarily constructed so as to approximate the rate of receipt of energy as a function of time at various latitudes if the albedo is 0.39 and the atmospheric transmission coefficient is 0.9. The albedo of the earth has been assumed to be independent of both latitude and time.

Second, we have assumed a rather elementary form for the radiative loss of heat from the surface layer under consideration. This does not include the influence of vertical gradients of temperature and water vapor. The numerical results are not greatly dependent upon the value of $1 - e$.

Third, we have assumed a meridional transport of heat by large-scale eddy processes which is proportional to the mean meridional temperature-gradient. This is an assumption for or against which little published quantitative evidence exists. Aside from the analogy with smaller scale phenomena, in which such a proportionality does exist, the chief reason for believing this lies in the character of the observed latitudinal temperature-gradients in summer and winter. In summer, when the north-south temperature gradient is small, one would expect a minimum in the eddy heat-transport and, indeed, radiative calculations indicate that there must be only a small heat transport then. In winter, on the other hand, the latitudinal temperature gradient is large, and the large excess of incoming radiation at the equator over that at the pole indicates that a large transport of heat across the latitude circles must exist. Thus, for a long-term, seasonal problem such as this one, it seems reasonable to assume heat transport approximately proportional to the temperature gradient. Recently White and Jung (1951) compared the observed geostrophic meridional transport of sensible heat to the observed meridional temperature-gradients, averaged over periods up to 12 days long. They found a poor correlation between these quantities and concluded that the eddy flux of sensible heat over short periods of time is not controlled primarily by the temperature gradient. This result is not applicable to a long-term seasonal average over perhaps one to two months. Regardless of the results of White and Jung, it seems appropriate to make the assumption of a linear relationship between these two variables in a long-term seasonal problem such as the present one.

In addition, the factor of proportionality, involving the "grossaustausch" coefficient, has been assumed to be a constant independent of time and latitude. The sole justification for such an approximation is the possibility that the most important aspect of the large-scale mixing may be its mere existence, and not its variations in space and time. Thus, the present procedure is to be looked upon as a first approximation, designed to take into account the supposedly primary effects of large-scale mixing but not the secondary effects. Furthermore, the magnitudes of the terms in (13) and (15) indicate that the results do not depend critically on the adopted value of K.

Fourth, the heat received by a layer of air near the ground is assumed to go only into raising the temperature of the air, outgoing radiation to space, or horizontal meridional transport to an adjacent layer of air. No provision has been made for heat storage in the underlying surface, nor, consequently, for release of stored heat from the surface to the overlying atmosphere. Thus, we have not taken account of the character of the underlying surface. Neither have we incorporated loss or gain of sensible heat due to changes of state. The effect of this is appreciable in polar regions during summer, when much melting occurs. Loss and gain of sensible heat may be appreciable over the oceans, also.

Fifth, the eddy transport of latent heat through large-scale diffusion of water vapor has been completely neglected, although recent investigations indicate that it is not a completely negligible part of the total poleward energy-flux. (See, for instance, White, 1951.) This, of course, is related to the neglect of changes of phase discussed above.

Sixth, it was necessary to assign a depth of air above the surface of the earth, dz, into and out of which pass the radiative fluxes of energy. It is in the choice of dz that considerations of vertical turbulent exchange make their only appearance. Since the lowest hundreds of meters are mixed by vertical turbulence, it was thought that a value of $dz = 500$ m would come close to representing the lowest atmospheric layer within which surface observations are made. This choice, however, may be justified in only a rough fashion. The numerical results are not markedly dependent upon dz, but the adopted value of this parameter is more important than those of $1 - e$ or K.

Finally, the terms involving T^4 were linearized by assuming small deviations from a constant value T_0.

7. Conclusion

This paper represents an attempt to explain theoretically the observed temporal and meridional variations of the low-level temperatures of the latitude circles in terms of only three physical factors: absorbed solar radiation, emitted long-wave radiation, and meridional transport of heat by the general circulation of the atmosphere. The fit between the results of the analysis and the observed data seems sufficiently good to permit the conclusion that most of the major features of the observed distributions are indeed explainable in terms of these three factors, as has been qualitatively supposed for many years. All the significant differences between this theory and the observations are qualitatively explainable as due to physical factors not included in the theory. The chief of these neglected factors appear to be exchange of heat with the underlying surface and the effect of changes of phase of water substance.

Acknowledgment.—The writers wish to express their gratitude to Mr. John Joern, who performed most of the calculations for this paper.

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