THE MEASUREMENT OF CLOUD LIQUID-WATER CONTENT BY RADAR

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ABSTRACT

The depth to which a liquid-water cloud may be detected has been computed as a function of liquid-water content for radars at wavelengths of 0.86, 1.25 and 3.2 cm, with use of Atlas' relationship for Z versus liquid-water content, and with account being taken of attenuation. An estimation is made of the effect of attenuation on calibration error; the resultant error in liquid-water content places a limit on the depth to which the liquid-water concentration in a cloud may be measured to a given accuracy by radar means. For the two shorter wavelengths and for relatively high liquid-water concentrations, error considerations, and not detectability, limit the measurement. The 0.86-cm radar is superior to the other two in cloud detectability for nearly all commonly encountered values of liquid-water content. For cases where an attenuation correction is desirable, a relatively simple method of making this correction is developed.

1. Introduction

The reasonably good correlation of the radar reflectivity of a water cloud with its liquid concentration (Atlas, 1954) has suggested the use of radar for estimating cloud water-content. The inherent difficulties of measuring this important cloud parameter by other means (Bemis, 1951) enhance the attractiveness of the proposed radar technique.

There are, however, several serious limitations to the proposed radar measurement, including failure to detect many water clouds[1] (Plank *et al*, 1955), ambiguities arising from the coexistence of water and ice particles and of large raindrops and smaller cloud droplets, and the limitations imposed by attenuation in the cloud itself. This article will discuss primarily the problems associated with attenuation, problems which increase in severity with decreasing wavelength. More specifically, an attempt will be made to outline a practical method of correcting for attenuation in determining liquid-water content, and to indicate the range of usefulness of such corrections. Particular consideration will be given to several wavelengths presently in use for meteorological purposes, *i.e.* 3.2, 1.25 and 0.86 cm.

The analogous problem of correcting for attenuation in rainfall for the purpose of rainfall measurements by radar was treated comprehensively by Hitschfeld and Bordan (1954). The latter paper has been of great assistance to the writer in the present work.

[1] A rather serious objection has often been raised against ascribing radar reflections from clouds to cloud particles, on the grounds that it is the drizzle and rain *in* the clouds, and not the cloud particles, that are detectable. This objection may be partially met by citing the radar-fog observations of the writer and his associates (1953), in which fog was detected by 1.25-cm radar at a range of 3000 ft. The thinnest detectable fog had a liquid-water content of 0.02 g/m³ and median volume diameter of 24 μ.

2. Development of a method

We must first establish an empirical relationship between W, the liquid-water concentration per unit volume of the cloud, and Z, the summation per unit volume of the sixth power of the droplet diameters. A general form for this relationship is

$$Z = \alpha W^{\beta}, \quad (1)$$

where α is a constant coefficient and β a constant exponent. Atlas (1954) found that $\beta = 2$ and $\alpha = 0.048$ fit Diem's (1948) cloud data quite well when Z is expressed in mm^6/m^3 and W in g/m^3, for W up to 1.3 g/m^3 and median volume diameters up to 29 μ. Equation (1) then becomes

$$Z = 0.048\, W^2. \quad (2)$$

The basic radar equation, attenuation being neglected, may be written

$$P_r/P_t = aZ/r^2, \quad (3)$$

where P_r = the power that would be returned if there were no attenuation in the path to the target and the radar beam were filled, P_t = power transmitted, r = range, and a incorporates all other quantities,[2] which remain more or less constant for a given radar and for a given dielectric constant of the particles in the irradiated volume.

When the radar energy must penetrate liquid cloud particles, the power received is reduced in proportion to the liquid-water concentration. Attenuation by atmospheric gases, principally oxygen and water vapor, also decreases the returned power. For two-way transmission through an element of range dr, the reduction by cloud particles and atmospheric gases takes the

[2] For the quantities involved in a, see Austin (1947).

form

$$(dP_r)/P_r = -2(KW + A)\,dr. \quad (4)$$

Integrating, we have

$$\ln (P_{ra}/P_r) = -2\int_{r_0}^{r} KW\,dr - 2\int_{0}^{r} A\,dr; \quad (5)$$

here P_{ra} is the received power at any range $r \geqslant r_0$, reduced by attenuation; K is the attenuation coefficient per unit concentration of cloud liquid-water; A is the attenuation coefficient of the atmospheric gases; and r_0 and r are the near and far termini, respectively, of the attenuation path in liquid cloud.

Combining (2), (3) and (5), we obtain

$$\ln (P_{ra}/P_t) = 2\ln b + 2\ln W - 2\ln r - 2\int_{r_0}^{r} KW\,dr - 2\int_{0}^{r} A\,dr, \quad (6)$$

where $b^2 = \alpha a$ in general, and $b^2 = 0.048\,a$ for the particular Z-W relationship of (2).

It will be more convenient to express the returned power in a form in which atmospheric and range attenuation are implicit. Hence, we substitute

$$Y^2 = (P_{ra}/P_t)r^2 \exp\left(2\int_{0}^{r} A\,dr\right) \quad (7)$$

into (6), and obtain

$$\ln Y = \ln b + \ln W - \int_{r_0}^{r} KW\,dr. \quad (8)$$

The quantity Y^2 is, in effect, the radar signal compensated for variations in range and transmitted power and corrected for atmospheric attenuation.

Differentiation of (8) with respect to range frees W from its integral:

$$d(\ln Y)/dr = (1/W)\,dW/dr - KW. \quad (9)$$

Now, freely borrowing an integrating hint from Hitschfeld and Bordan (1954), we substitute $U = 1/W$, collect terms, and integrate to obtain

$$UY = -\int_{r_0}^{r} KY\,dr + \text{const.} \quad (10)$$

The constant of integration may be evaluated easily by setting $K = 0$ and consulting (8); we find that the constant $= b$. We define $Y_1 \equiv Y$ at $r = r_1$. Finally, we obtain a unique relationship for W in terms of normalized signal (Y^2) and other quantities independent of W:

$$W = \frac{Y_1}{\left(b - \int_{r_0}^{r_1} KY\,dr\right)}. \quad (11)$$

An analogous expression, for rain intensity as a function of echo power and range, was found by Hitschfeld and Bordan (1954).

TABLE 1. Two-way attenuation by cloud liquid-water. Values are in (nep/μsec)/(g/m³).

λ (cm)	Temperature 0C	10C	20C
0.86	0.0684	0.0470	0.0447
1.25	0.0367	0.0280	0.0215
3.2	0.00593	0.00435	0.00334

The following outline is a simple, practical method of determining W from (11):

1. Evaluate b, using (2) and (3); the factor a in (3) preferably should be checked by means of an experimental calibration of the radar on a known standard target; (the dimensions of b are range units divided by liquid-water concentration units, since P_{ra}/P_t is dimensionless);
2. Decide what value or values K is to have; the water-cloud attenuation constant K is, in general, a function of temperature T, and radiated wavelength λ; wavelength is known, but the temperature will usually have to be estimated; table 1, which is based on table 5 of Gunn and East (1954), lists some values of K in (nepers/μsec range)/(g/m³ of liquid water) for two-way radar transmission;[3] for calculations involving (11), K must be expressed in nepers; the distance unit chosen here is μsec range; since 1 μsec range = 150 m (for a two-way path), the attenuation values in table 1 may be referred to kilometers by multiplying them by 6.67 and to miles by multiplying them by 10.73;
3. Obtain the echo power over the range in interest, at a suitable number, n, of intervals; echo power is usually most conveniently expressed in decibels above (or below) some standard level;
4. Correct the echo powers for range-variable errors; some of these errors arise entirely within the radar system, including range-attenuation-correcting circuits, TR and anti-TR losses, *etc.;* others originate in the propagation path (in this category we will be concerned here solely with attenuation by atmospheric gases); this quantity is a function of radar wavelength, temperature, pressure and humidity; the range of atmospheric attenuations to be expected may be estimated roughly by examining fig. 2.2 of Atlas *et al* (1952) and is indicated in table 2;
5. Normalize the echo powers with respect to transmitted power; the resulting function should now equal

$$10\log (P_{ra}/P_t) + 8.686\int_0^r A\,dr$$

versus range;

6. Halve the resultant echo powers and add the appropriate $10\log r$ (in the range units used) to each power; the result is now a function of $10\log Y$ versus r; divide by 10 and find the antilog, obtaining Y versus r;
7. Evaluate $\int_{r_0}^{r_1} KY\,dr$ numerically; a trapezoidal approxi-

[3] The natural logarithm (ln) of a ratio is expressed in nepers, and 10 times the common logarithm (log) of a ratio in decibels. Thus, 1 neper = 4.343 decibels.

TABLE 2. Two-way atmospheric attenuation (decibels/mile).

Wavelength of radar (cm)	mT atmosphere, sea level, T=28C, water-vapor density =10 g/m³	mP atmosphere, altitude 2 mi, T=−5C, water-vapor density =3 g/m³
0.86	0.3	0.11
1.25	1.1	0.2
3.2	0.05	0.03

mation to the integrand may serve well enough, in which case the integral equals

$$\Delta r\left(\frac{K_0Y_0}{2}+K_{\frac{1}{n}}Y_{\frac{1}{n}}+\cdots+K_{n-\frac{1}{n}}Y_{n-\frac{1}{n}}+\frac{K_1Y_1}{2}\right);$$

8. Use (11) to find W.

3. Limitations by detectability and error

What are the limitations to this method? Obviously, it cannot be applied in any situation where ice crystals contribute significantly to the echo. Air temperatures above the freezing point of water should eliminate the possibility of ice crystals. However, 0C is not a mutually exclusive dividing line between water droplets and ice crystals; supercooled water droplets often exist, with or without accompanying ice crystals, at temperatures below 0C.

The method also depends on the validity of a linear relationship between attenuation and liquid-water content [see (4)]. Haddock's (1948) fig. 2 indicates the departure of the attenuation from a Rayleigh approximation as a function of the quotient of drop diameter by wavelength. The true attenuation does not exceed twice the Rayleigh approximation for drop diameters up to about 0.6 mm for the two lower wavelengths, and up to 1.3 mm for 3.2-cm wavelength. If greater accuracy is necessary, we may place an upper limit of 1.2 on the factor by which the true attenuation may exceed its Rayleigh approximation. Even here, the maximum permissible drop diameters are 0.3 mm for 1-cm waves, and 0.5 mm for 3.2-cm radiation. Thus, within fairly small errors, this method may be applied not only to cloud particles but also to drizzle drops.

It should be noted, however, that Atlas' Z-W relationship (2) was derived for cloud particles of maximum median volume diameter of 29 μ. A preliminary examination of some data taken by Mr. Duncan C. Blanchard in Hawaiian rains, and privately communicated to Mr. Dave Atlas, indicates that drizzle and

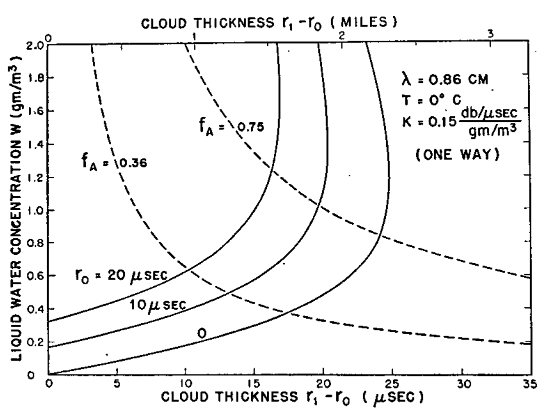

FIG. 1. Thickness to which liquid-water cloud may be detected by means of 0.86-cm radar, for various ranges (r_0) to near side of cloud. Dashed lines are loci of fractional attenuation (f_A). Example: for $W = 1$ g/m³ and $r_0 = 10$ μsec, cloud may be detected through thickness of 19.7 μsec; $f_A = 0.75$ occurs at 20.3 μsec.

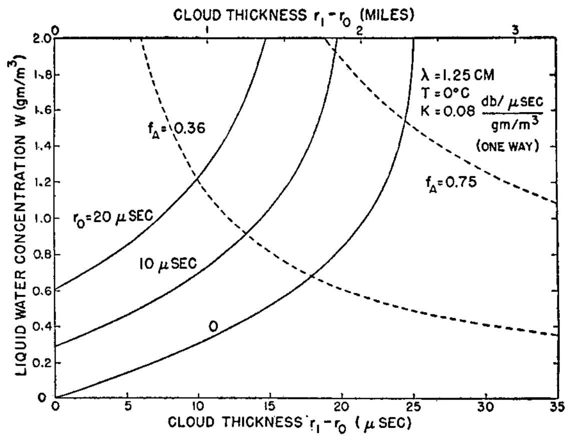

FIG. 2. Thickness to which a liquid-water cloud may be detected by means of 1.25-cm radar, for various ranges (r_0) to near side of cloud. Dashed lines are loci of fractional attenuation (f_A).

small raindrops with median volume diameters in the 0.1- to 0.5-mm range may also be fairly represented by means of a $Z = \alpha W^2$ relationship. However, the coefficient α in this case (drizzle and small raindrops) is about 200. This is some 4000 times the coefficient found by Atlas for cloud droplets. Hence, great care must be exercised in choosing an appropriate coefficient for (1).

The speed of fall of a radar echo is a rough indicator of the size of water droplets, although vertical air currents may be misleading. According to Gunn and Kinzer (1949), the terminal velocity under normal sea-level conditions of 0.3-mm diameter water droplets is 1.2 m/sec; of 0.5-mm drops, 2.1 m/sec. In general, one may suspect that a non-falling echo at a temperature above 0C represents cloud-size liquid-water particles; but, in any specific case, a knowledge of the synoptic situation and the observer's judgment are important factors.

Aside from the decision whether a given radar signal represents only cloud liquid water, there are two other important considerations. One is detectability. This depends on the characteristics of the radar system and on the amount of attenuating liquid water traversed.

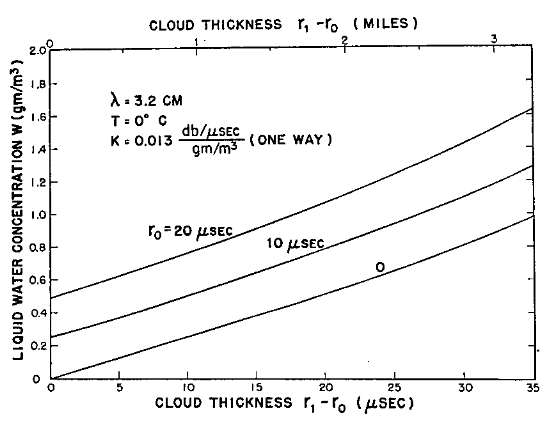

FIG. 3. Thickness to which liquid-water cloud may be detected by means of 3.2-cm radar, for various ranges (r_0) to near side of cloud.

Figs. 1, 2 and 3 illustrate the detectability of cloud liquid water for three wavelengths of radar: 0.86 cm (the AN/TPQ-6), 1.25 cm (our modified version of the APS-34), and 3.2 cm (AN/CPS-9), respectively. On each diagram, the lines labeled $r_0 = 0$, $r_0 = 10$ μsec and $r_0 = 20$ μsec, delineate the range of detectability. These lines give the maximum thickness $(r_1 - r_0)$ to which a cloud of given liquid-water content W (assumed constant) may be detected for three ranges to the near side of the cloud (r_0). If the radar is pointing vertically, r_0 is the distance to cloud base. All distances are expressed in μsec range. The curves labeled f_A will be discussed later.

These detectability lines were computed from two relationships: (i) minimum reflectivity, $\eta_{\min}$, *versus* range r, and (ii) the dependence of reflectivity on liquid-water content. The former equation is discussed by Plank *et al* (1955) and has the general form

$$10 \log \eta_{\min} = C + S_{\min} + 20 \log r + \text{attenuation in db.} \quad (12)$$

The sensitivity of the radar, including such parameters as transmitted power, antenna gain, wavelength, beam width, and pulse length, is included in the constant C; $S_{\min} = 10 \log P_{r\,\min}$ is the minimum detectable signal that can be handled by the receiving and recording circuits. ($P_{r\,\min}$ is measured in milliwatts.)

The other equation takes the form

$$10 \log \eta = E + 20 \log W; \quad (13)$$

it is derived from the wavelength and temperature dependence of reflectivity, η, on the summation per unit volume of the sixth power of the droplet diameters, Z, and from the empirical relationship of W and Z [see Gunn and East (1954)]. The term E is simply a convenient manner of expressing the empirical proportionality between W^2 and η.

Note that the factors of $b^2 = \alpha a$ are involved in the terms C and E of (12) and (13). The reciprocal of the radar calibration-constant a is included in C, and the Z-W^2 empirical coefficient α is included in E. Equations (12) and (13) may be combined to express the range of detectability directly:

$$\begin{aligned} 20 \log r &= 20 \log W + E - C - S_{\min} - \text{attenuation} \\ &= 20 \log W + 10 \log (\alpha a) + \text{const.} - S_{\min} - \text{attenuation.} \end{aligned} \quad (14)$$

A "practically attainable" value of $S_{\min}$, 3 db higher than the rated minimum detectable power, has been adopted in the calculation of maximum detectable range. This allows for some unforeseen losses in overall system sensitivity and takes account of the difficulty in detecting a threshold signal. The values of C used here imply approximately the same antenna efficiency estimated by Plank *et al* (1955). These constants, together with radar parameters and cloud and atmospheric attenuation coefficients, are listed in table 3. Some of the quantities listed there are estimates of varying degrees of accuracy, but it is hoped that they may be fairly representative of most equipment and meteorological situations.

The other consideration limiting the radar measurement of liquid-water content is the presence of errors in W generated by errors in the terms of the defining equation (11). Numerical integration techniques and the resolution limits of the radar system will introduce some error in the attenuation integral, and a mistaken estimate of air temperature may result in an error in the cloud-attenuation coefficient by a factor as large as two. Careless reading of the returned signal may also deviate the values of Y. However, these are all controllable errors: they may be reduced by a careful reading and integration of Y, and a sufficiently accurate knowledge of temperature along the attenuating path.

A faulty correction for atmospheric attenuation may also contribute a measure of confusion. In an extreme case (*e.g.*, very moist maritime tropical air) this problem may be serious for 1.25-cm observations, where (for a temperature of 20C and water-vapor concentration of 18 g/m^3) the atmosphere may be as effective an attenuator as 1.3 g/m^3 of cloud! It is of much less importance for 0.86 cm and 3.2 cm, where atmospheric losses under the same conditions are equivalent to 0.3 and 0.4 g/m^3, respectively, of cloud liquid-water.

The uncontrollable errors are located in the factors α and a, which determine the constant b in (11). There is, first of all, an inevitable error in a. If the radar has never been calibrated against a standard

TABLE 3. Detectability parameters for three radars.

	TPQ-6	Modified APS-34	CPS-9 (short pulse)
Wavelength (cm)	0.86	1.25	3.2
Peak transmitted power at antenna (kw)	12.5	25	250
Pulse length (μsec)	1.0	0.4	0.5
Beam width (deg)	0.29	0.37	0.98
Antenna gain	2.7×10^5	1.7×10^5	2.4×10^4
Antenna diameter (ft)	7	8	8
C (db)	−55	−55	−66
$S_{\min}$ (db)	−87	−89	−95
*Atmospheric attenuation (db/μsec range)	0.02	0.05	0.004
*Cloud attenuation (db/μsec range)/(g/m^3)	0.3	0.16	0.026
η/Z (cm^{-1})/(mm^6/m^3)	5.0×10^{-10}	1.13×10^{-10}	2.7×10^{-12}
E (db)	−106.2	−112.7	−128.8

* Attenuation values refer to a two-way path.

target, there may be a fixed error in a of a few decibels. In addition, the performance of a radar can and does vary between calibrations against a known power source. Our experience with a 1.25-cm system indicates that ± 1 db would be a judicious estimate of the error in the radar constant from day to day, although others quote figures as high as ± 3 db, and where no calibrations are made, one might expect discrepancies as high as ± 6 db.

The most intractable error, however, is found in α, the proportionality constant in the Z-W empirical relationship. Atlas (1954) gives a standard error in W of ± 53 per cent for $Z = 0.048\, W^2$. This means that the standard error has limits of $W = Z^{\frac{1}{2}}(0.048)^{-\frac{1}{2}} \pm 53$ per cent. In terms of decibels, the standard error is 2 db; that is, the standard error is represented by $10 \log W = 5 \log Z - 5 \log 0.048 \pm 2$.

However, the error in Diem's data may not be representative of the error in *radar* measurements of liquid-water content. Mr. Dave Atlas, in a private conversation, has pointed out that the variability in the Z-W relationship represented by Diem's observations may arise partially from variability between regions of cloud of a size comparable to liquid-water content sampling volumes but much smaller than radar-pulse volumes. Insofar as this represents the true state of affairs, a radar sample would tend to average out some of this variability, and the variability in the Z-W coefficient among radar-pulse volumes would be somewhat less than is indicated by Diem's data. Unfortunately, there is no adequate information available on the fine structure of the drop-size distribution within clouds. However, Mr. Atlas' proposal must be kept in mind as an attractive possibility for improving the predictability of cloud water-concentration from measurements of radar reflectivity.

Let us now examine the effect on W of an error in $b = (a\alpha)^{\frac{1}{2}}$. It is useful, first of all, to simplify the denominator of (11) somewhat. From (8),

$$Y = bW \exp\left(-\int_{r_0}^{r} KW\, dr\right). \qquad (15)$$

The attenuation integral in (11) may be written, therefore,

$$\int_{r_0}^{r_1} KY\, dr = b\int_{r_0}^{r_1} KW \exp\left(-\int_{r_0}^{r} KW\, dr\right) dr$$
$$= b\left[-\exp\left(-\int_{r_0}^{r} KW\, dr\right)\right]_{r_0}^{r_1}$$
$$= b\left[1 - \exp\left(-\int_{r_0}^{r_1} KW\, dr\right)\right]$$
$$= b[1 - (1 - f_A)^{\frac{1}{2}}]. \qquad (16)$$

The final two forms of (16) define the fractional attenuation, f_A, or the proportion by which the signal has been reduced by attenuation. (The fraction of signal that survives attenuation $= 1 - f_A$.) Finally, (11) simplifies to

$$W = Y_1/[b(1 - f_A)^{\frac{1}{2}}]. \qquad (17)$$

Now suppose the *true* value of b deviates by a factor ϵ from the assumed value, b', on which the calculations of liquid-water content are based, *i.e.*, $b = \epsilon b'$. Use of the erroneous value will effect a corresponding error in W, as follows:

$$W' = Y_1/[b'(1 - f_A')^{\frac{1}{2}}]. \qquad (18)$$

Here the primes refer to the assumed, but erroneous values. From (16),

$$(1 - f_A')^{\frac{1}{2}} = 1 - \epsilon + \epsilon(1 - f_A)^{\frac{1}{2}}. \qquad (19)$$

Dividing (17) by (18), we have

$$\frac{W}{W'} = \frac{(1 - f_A')^{\frac{1}{2}}}{\epsilon(1 - f_A)^{\frac{1}{2}}}; \qquad (20)$$

in terms of the true fractional attenuation,

$$\frac{W}{W'} = \frac{(1/\epsilon) - 1 + (1 - f_A)^{\frac{1}{2}}}{(1 - f_A)^{\frac{1}{2}}}; \qquad (21)$$

and in terms of the apparent but erroneous fractional attenuation,

$$\frac{W}{W'} = \frac{(1 - f_A')^{\frac{1}{2}}}{\epsilon - 1 + (1 - f_A')^{\frac{1}{2}}}. \qquad (22)$$

Equations (21) and (22) are completely analogous but somewhat simplified forms of the error equations found by Hitschfeld and Bordan (1954).

Fig. 4 is a plot of (21) for several values of ϵ. The

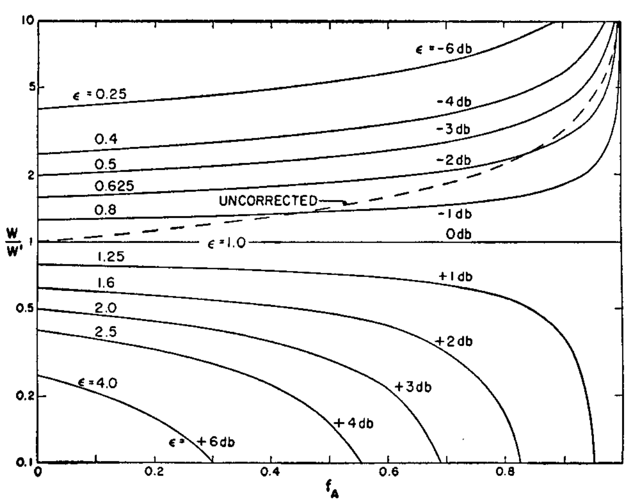

FIG. 4. Effect of error ϵ in calibration constant b on ratio W/W' of actual to computed liquid-water content, as function of fractional attenuation f_A. Dashed line represents error in making no corrections for attenuation. Example: for $f_A = 0.75$, actual W is twice uncorrected W, and for $\epsilon = 0.5$, actual W is three times W corrected for attenuation but computed on assumption that $\epsilon = 1$.

values of ϵ chosen represent errors (+ and −) of 1, 2, 3, 4 and 6 db. The dashed line represents the error incurred in making no corrections at all. It is the ratio of the true W to the W that would be computed if attenuation were ignored, perfect calibration being assumed ($\epsilon = 1$).

Equation (21) and fig. 4 may also be used to find the limit to the useful range of the basic equation (11). In terms of fractional attenuation, we distinguish three regions:

(a): $0 \leqslant f_A \leqslant x$: not economical to correct;
(b): $x \leqslant f_A \leqslant y$: both economical and safe to correct;
(c): $y \leqslant f_A \leqslant 1$: not safe to correct.

The limits to these three regions are quite arbitrary and depend to a great extent on the preference of the observer; however, there are some guides which may be helpful in making a decision.

The limit x, below which it is not economical to correct for attenuation, is a measure of the fractional error one can tolerate in W. Thus, the uncorrected $W = Y_1/b$ will be too small by a factor $1/(1 - f_A)^{\frac{1}{2}}$. In view of the size of other errors likely to be present, a fair value for x would be 0.36. At this limit, the uncorrected W would be just 0.8 of the correct W, it being assumed that $\epsilon = 1$ (*i.e.*, perfect calibration). This represents an error in W of −1 db.

The limit y, above which it is not safe to correct, depends primarily on ϵ and on the level of confidence that the observer is willing to accept. From (21), we see that f_A as large as $(2/\epsilon) - (1/\epsilon^2)$ results in $W/W' = 0$ (for $\epsilon > 1$). For a finite W, this implies that the calculated W' becomes infinitely large. Clearly such high attenuations must be avoided.[4] Quite arbitrarily, we have chosen an upper limit of $y = 0.75$.

These two limits to the useful range of attenuation corrections for liquid-water content determinations by radar have been plotted on figs. 1 and 2. (Both limits are outside the range of fig. 3 for 3.2 cm.) This was done by finding $\int_{r_0}^{r_1} KW\,dr$ from f_A [see (16)] and dividing by K (assumed constant, and in nepers, not decibels) corresponding to the temperature and radar wavelength to obtain $\int_{r_0}^{r_1} W\,dr$. For a constant W, this integral reduces to $W(r_1 - r_0)$. Each value is, therefore, the locus of a hyperbola on figs. 1 and 2.

While the reader's attention is directed to figs. 1 to 3, it must be pointed out that detectability itself is affected by an error in b; this is evident in (14). As an example, for 0.86 cm (fig. 1), $W = 1$ g/m³ and $r_0 = 0$, $\epsilon = 0.63$ reduces the range of detectability, $r_1 - r_0$, from 24.7 to 19.2 μsec, and $\epsilon = 1.6$ increases this range to 31.1 μsec.

[4] Where the attenuation is high enough to generate intolerably large errors in the measurement of W, and two radar systems are available, the method proposed by Atlas (1954) may be of compelling interest. This technique requires the use of two different wavelength radars, and results in a determination of *average* W over a path of sufficient length to give a measurable difference in attenuation. It is relatively simple and neat, and is independent of radar-calibration constants and the Z-W correlation.

4. Summary

An examination of figs. 1 to 3 shows that detectability is the limiting factor in the radar measurement of cloud liquid water for all smaller, more commonly observed, values of W. For example, at 1.25 cm a cloud having a uniform liquid-water concentration of 0.5 g/m³ can be detected through a thickness of 14.7 μsec for an initial range $r_0 = 0$, but only through 5.9 μsec at an initial range of 10 μsec, and is not detectable at all at a distance of 20 μsec. Error considerations limit the measurement only at fairly high values of liquid-water content. Table 4 indicates, for 0.86 cm, the value of W above which a quantitative radar measurement (not detection) would be limited by a fractional attenuation of 0.75, and the percentage of cases in Diem's cloud data (Atlas, 1954) and a weighted average of Lewis' compilation (1951) in this category. This is done for the three initial ranges $r_0 = 0$, 10 and 20 μsec.

A comparison of the useful range of the three radars is interesting.[5] Such a comparison of utility for measuring cloud liquid-water is portrayed in fig. 5 for $r_0 = 0$, and fig. 6, for $r_0 = 10$ μsec. The curve for each radar is the maximum range for cloud liquid-water *measurement* (not necessarily *detection*) imposed both by expected detectability and by an error limitation corresponding to a fractional attenuation of 0.75.

Both diagrams emphasize the superiority of the 0.86-cm equipment for all but the very highest values of W. For these extremely high values, the 3.2-cm radar is much more suitable than the other two equipments. The 3.2-cm system, because of its lower attenuation coefficient for all manner of particles, is also a superior device for detecting cloud liquid-water through paths containing some intervening large water droplets, such as rain, drizzle or melting snow, although measurement of W would be very difficult in this case,

[5] If the CPS-9 radar is used with its "long pulse" of 5 μsec and the peak transmitted power can be maintained constant, the maximum detectable range of clouds is increased by a factor of ten, neglecting attenuation. This is effected both by a ten-fold increase in the energy transmitted in each pulse, and by a 10-db reduction in minimum detectable power. However, the longer pulse width reduces the range resolution by a factor of ten.

TABLE 4. Minimum liquid-water content above which range for quantitative radar measurement is limited by 0.75 attenuation for $\lambda = 0.86$ cm and $T = 0$C.

	W (g/m³)	Fraction of clouds in this category: Diem	Lewis
$r_0 = 0$	0.84	6.2%	4 %
$r_0 = 10$ μsec	1.02	3.1%	1.4%
$r_0 = 20$ μsec	1.23	3.1%	0.6%

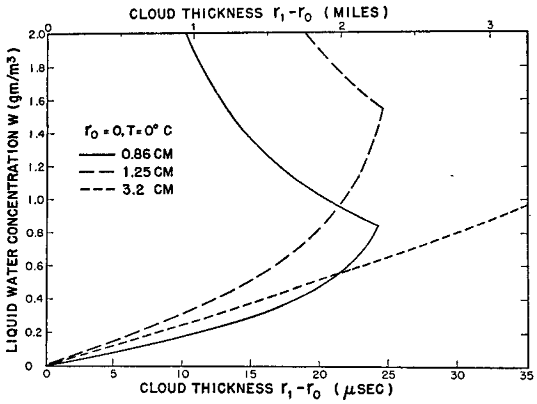

FIG. 5. Comparison on three radars of maximum thickness of cloud amenable to quantitative measurement of cloud liquid-water content, for near side of cloud at range $r_0 = 0$. Limitation is imposed by detectability (lines sloping upward and to right) and by prohibitive errors at fractional attenuation $f_A = 0.75$ (lines sloping downward and to left).

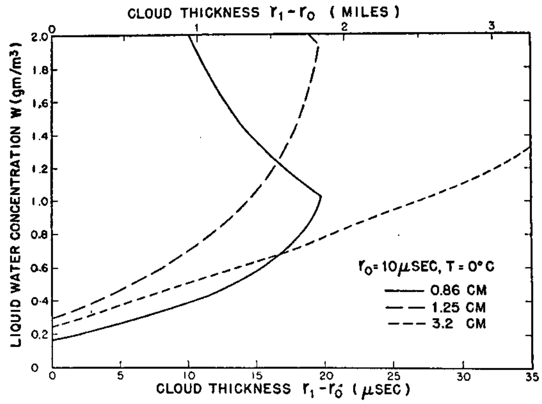

FIG. 6. Same comparison as in fig. 5, except $r_0 = 10$ μsec.

if not impossible, because of a probable lack of sufficient data on the nature of the attenuating medium.

These diagrams illustrate the superiority of high-frequency equipment, such as the TPQ-6, for detecting and measuring, from the ground, liquid water in and near cloud *bases*. The lower frequency CPS-9 finds its usefulness in detecting and measuring liquid water of very high concentrations and through great thicknesses; thus, under many conditions, it is a more reliable cloud *top* indicator. As mentioned previously, the attenuation by atmospheric gases (principally water vapor) at 1.25 cm may be greater than, or at least a very large fraction of, the attenuation caused by the great majority of natural water clouds; hence, unless this is adequately corrected for, somewhat poor accuracy in measuring liquid-water concentration should be expected for 1.25-cm systems.

In conclusion, the writer would recommend a high-power 3.2-cm system for detecting dense liquid-water clouds over long paths or through intervening rain. An 0.86-cm radar would be essential for a high probability of detecting and measuring the liquid water in all but a very small fraction of natural clouds. The 1.25-cm system is not recommended by the writer, because it is inferior in detectability to one or the other radars considered, and because of its high water-vapor attenuation.

Once a liquid-water cloud has been detected, and the observer is satisfied that it *is* a liquid-water cloud, an initial quantitative estimate should be attempted with use of $W = Y_1/b$. If this initial estimate of W and the attenuating path are sufficiently high [that is, if the point $(r_1 - r_0)$, W on fig. 1, 2 or 3 falls near or to the right of the $f_A = 0.36$ line], a more accurate determination of W may be warranted, with use of (11) and the procedure outlined just below that equation. If, however, twice the initial estimate of W and the attenuating path define a point in the vicinity of $f_A = 0.75$ or greater, a quantitative computation of W should be attempted only with the realization that the result may be in serious error.

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