CORRESPONDENCE

Comments on "A general survey of factors influencing development at sea level"

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In a recent paper,[1] Dr. Petterssen reviewed and enlarged upon the theory of development due to R. C. Sutcliffe. Dr. Petterssen's discussion and illustrations of the terms in the development equation are very illuminating. I think it should be emphasized, however, that the theory can be derived from statical considerations alone without the introduction of dynamics.

To illustrate the statical character of the development equation, it will be derived from the hypsometric identity,

$$h_0 \equiv h_1 - h', \tag{1}$$

where h_0 and h_1 denote the geopotential heights of a low-level and a higher-level isobaric surface, respectively, and h' is the geopotential thickness of the layer. We apply the isobaric Laplacian operator to (1) and obtain

$$\nabla^2 h_0 \equiv \nabla^2 h_1 - \nabla^2 h', \tag{2}$$

which can also be written as

$$q_0 = q_1 - (g/f)\,\nabla^2 h', \tag{3}$$

where

$$q_0 = \frac{g}{f}\nabla^2 h_0, \quad \text{and} \quad q_1 = \frac{g}{f}\nabla^2 h_1.$$

q_1 and q_2 are defined as the geostrophic relative vorticities, and g and f are standard gravity and the Coriolis parameter, respectively. If we add f to both sides of (3), we obtain

$$Q_0 = Q_1 - (g/f)\,\nabla^2 h', \tag{4}$$

where

$$Q_0 = q_0 + f, \quad \text{and} \quad Q_1 = q_1 + f.$$

Q_0 and Q_1 are defined as the geostrophic absolute vorticities.

Now, differentiating (4) partially with respect to time, we have

$$\frac{\partial Q_0}{\partial t} = \frac{\partial Q_1}{\partial t} - \frac{g}{f}\nabla^2\left(\frac{\partial h'}{\partial t}\right). \tag{5}$$

From the hydrostatic equation,

$$h' = \left(\frac{R}{g}\ln\frac{p_0}{p_1}\right)\bar{T}, \tag{6}$$

where R is the gas constant, and $\bar{T}$ is the mean virtual temperature of the layer. Differentiation of (6) and expansion of the individual temperature derivative yields the Sutcliffe-Forsdyke equation,

$$\frac{\partial h'}{\partial t} = \left(\frac{R}{g}\ln\frac{p_0}{p_1}\right)\left[\bar{A}_T + \overline{\omega(\Gamma_a - \Gamma)} + \frac{1}{c_p}\overline{\frac{dW}{dt}}\right], \tag{7}$$

quoted by Petterssen, wherein the three terms in brackets represent, respectively, the horizontal thermal advection, the vertical advective-adiabatic temperature change, and the non-adiabatic temperature change. The symbols are Dr. Petterssen's.

The individual derivatives of Q_0 and Q_1 can be expanded in the form

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + V\cdot\nabla Q + \omega\frac{\partial Q}{\partial p}, \tag{8}$$

where V is the horizontal wind vector, and ω is the individual pressure derivative.

Substituting (7) and (8) in (5), we obtain the following equation for the development of geostrophic absolute vorticity at the lower level:

$$\frac{dQ_0}{dt} = V_0\cdot\nabla Q_0 - V_1\cdot\nabla Q_1 + \omega_0\frac{\partial Q_0}{\partial p} - \omega_1\frac{\partial Q_1}{\partial p} + \frac{dQ_1}{dt} - \left(\frac{R}{f}\ln\frac{p_0}{p_1}\right)\nabla^2\left[\bar{A} + \overline{\omega(\Gamma_a - \Gamma)} + \frac{1}{c_p}\overline{\frac{dW}{dt}}\right]. \tag{9}$$

If there is no vorticity development (dQ_1/dt) at the upper level (in Dr. Petterssen's discussion this is the level of non-divergence), and if we ignore the vertical advection of vorticity, (9) reduces to the Sutcliffe-Forsdyke-Petterssen development equation. The first two terms on the right in (9) can, of course, also be written in terms of the wind shear,

$$V_s \equiv V_1 - V_0,$$

and the geostrophic absolute vorticity difference,

$$Q_s \equiv Q_1 - Q_0.$$

Thus,

$$V_0\cdot\nabla Q_0 - V_1\cdot\nabla Q_1 \equiv -V_1\cdot\nabla Q_s - V_s\cdot\nabla Q_0. \tag{10}$$

The right-hand side of (10) is the form of the advection term preferred by Sutcliffe and Petterssen, although its advantages over the left-hand side of the identity are not very clear.

It appears that the replacement of (1) by (9) merely substitutes the problem of predicting the Laplacian

[1] S. Petterssen, "A general survey of factors influencing development at sea level," *J. Meteor.*, 12, 36–42, 1955.

of h_0 for the problem of predicting h_0 itself. The problem of forecasting the Laplacian of the temperature (actually, virtual temperature) replaces the problem of forecasting the thickness. And, as Petterssen has pointed out, the problem of forecasting the configuration of the upper isobaric surface is replaced by the problem of forecasting the level of non-divergence.

Since (9) is merely a reformulation of identity (1), it should be considered only as another statement of the condition of hydrostatic equilibrium which the atmosphere must satisfy. Statics is, of course, a valuable tool in analysis and forecasting. But we should expect no more from statics than internal hydrostatic consistency.

On the other hand, Dr. Petterssen's empirical approach to the development problem, which employs the so-called development equation for the purpose of formulating hypotheses, has already proved to be quite fruitful as shown by his successful subjective forecasts of cyclogenesis.[2]

Spar seem to indicate that some success may be expected. It appears doubtful whether Dr. Spar's identity (1) would serve equally well.

[2] S. Petterssen, G. E. Dunn, and L. L. Means, "Report of an experiment in forecasting of cyclone development," *J. Meteor.*, 12, 58–67, 1955.

[1] N. E. LaSeur, "On the asymmetry of the middle-latitude circumpolar current," *J. Meteor.*, 11, 43–57, 1954.

[2] R. R. Long, "The flow of a liquid past a barrier in a rotating spherical shell," *J. Meteor.*, 9, 187–199, 1952.