STUDIES OF SMALL-SCALE TURBULENT DIFFUSION IN THE ATMOSPHERE

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(Manuscript received 3 January 1956)

ABSTRACT

The dispersion of smoke puffs has been used as a method of measuring turbulent diffusion in the atmosphere. Some of the basic equations of turbulent diffusion as applied to the dispersion of a small-scale smoke puff are recalled. The intensity of turbulence in the lower atmosphere is then determined from the experimental measurements. The experimental measurements are analyzed to find the characteristics of turbulence as observed on a small scale. Comparison is made with other smoke-puff and soap-bubble diffusion studies.

1. Introduction

Theoretical and experimental studies of turbulent diffusion are becoming increasingly important from many points of view. An increase in the understanding of turbulent diffusion (often called "eddy diffusion") is of great interest to many meteorological problems, and more particularly to such micrometeorological studies as air pollution, evaporation, and heat transfer.

One of the most common illustrations of turbulent diffusion, and one that is directly related to the air-pollution problem, is diffusion from a smoke stack. The dispersion of the smoke plume is caused by two major factors: (a) the general air motion which carries the smoke downstream, and (b) the turbulent velocity fluctuations which disperse the smoke particles in all directions. Consider a smoke puff leaving the stack. After an interval of time, the center of the puff will be at a distance from the stack depending on the mean wind velocity, and the puff will have grown to a size and shape dependent on the turbulent diffusion.

Fig. 1 illustrates diagrammatically the possible variations of the shapes and trajectories of a dispersing puff, even under the same atmospheric conditions. The eight examples of smoke puffs starting as point sources and observed at successive time intervals are

[1] This work was supported by the Bureau of Ordnance, U. S. Navy, under Contract NOrd-7386.

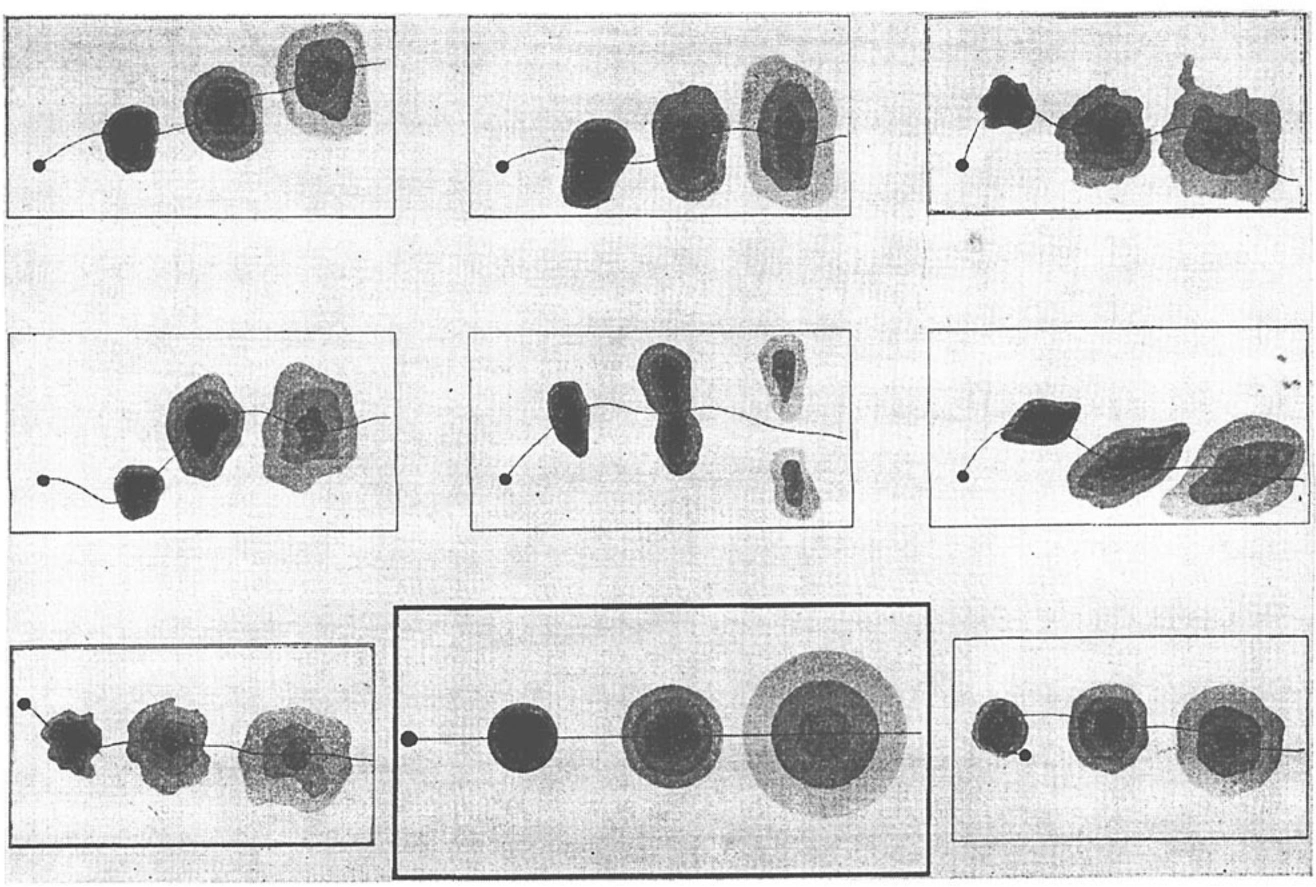

FIG. 1. Possible configurations of smoke puffs under influence of turbulent flow.

schematic. The smoke puffs move in the direction corresponding to the mean wind velocity. It is possible, however, that a sudden gust will momentarily move a puff in an opposite direction. While moving, the puff disperses under the influence of turbulent diffusion. In some cases, a large-scale gust hits the dispersing puff and breaks it into two smaller ones, each continuing to disperse while following its own trajectory. This article will not be concerned with the large-scale gusts, but will be limited to study of the atmospheric turbulence at a smaller scale. Therefore, the case in which a puff is directly affected by large-scale gusts will be eliminated from the analysis. One should keep in mind, however, the fact that the same smoke-puff technique can be used to determine the characteristics of turbulence at a much larger scale by observing the trajectories of the puffs. Since only small-scale turbulence is being studied here, one will not have to be concerned with the trajectories of the puffs but only with their dispersion. Let it be assumed that, during a certain period of time and within a certain space, the statistical characteristics of the small-scale atmospheric turbulence are unchanged. In this case, the average spread and shape of a smoke puff will depend only on the dispersion time of the puff. In fig. 1 is also illustrated an average smoke puff that could be obtained from a large number of individual smoke puffs, each corresponding to the same dispersion time.

Characteristics of turbulent diffusion may be studied experimentally in various ways. A direct measure of turbulent diffusion may be achieved by introducing smoke, soap bubbles, or otherwise identifiable material into the air and recording the resulting spread.

This article deals primarily with smoke-puff measurements and compares the results with those obtained by other methods.

2. A smoke-puff experiment, Tilghman Island

In February 1950, an experiment was performed by the Naval Research Laboratory at Tilghman Island, Maryland, in which smoke puffs were set off and photographed, to obtain data on turbulent diffusion over water at low levels [1]. A tethered balloon was employed to lift a smoke-puff generator to heights up to several hundred feet. The generator was suspended about 50 ft below the balloon, to eliminate the effect of balloon-caused turbulence. The puffs were generated by setting off a small pillbox filled with 4-F gunpowder. Two triangulated motion-picture cameras photographed each puff for its entire visible history. One of the cameras, which was equipped with a telephoto lens, followed the puff to record its apparent areal spread. The second camera, which was equipped with a wide-angle lens, was fixed. Instantaneous positions of the puff were computed from photographs obtained with the second camera and a manual reading of the angle of departure of the puff. Temperature soundings were made to determine the stability conditions present during the experiment.

Data from this type of experiment may be readily analyzed. The developed film from the first camera is projected on a grid, and the visible areas are measured. With a knowledge of the camera focal length and the instantaneous puff positions given by the wide-angle lens, one may derive wind speed, distance of the puff from the first camera, and the true visible puff area. In general, these areas are irregular in shape, since they result from random atmospheric motions. Each area, though irregular, is comparable with a circular area, and it is the rate of growth of the radius of this equivalent circular area which is being studied here.

The data obtained during the Tilghman Island experiment are presented in tables 1 and 2.[2] The values

[2] These data are published here for the first time with the permission of the Naval Research Laboratory.

TABLE 1. Smoke-puff radii (cm), Tilghman Island, 27 February 1950.

Time (sec)	Puff number 4	5	6	7	8	13	16	18	19	20
1	56	58	62	77	64	—	96	63	50	75
2	76	82	86	91	90	111	116	86	72	104
3	101	105	120	99	114	148	138	104	78	119
4	113	119	167	113	137	180	146	125	95	143
5	132	138	187	126	149	184	171	143	118	164
6	145	135	174	187	172	248	173	164	121	183
7	159	158	183	190	175	242	183	178	172	198
8	169	148	172	202	162	281	184	191		198
9	173	190		186	219	291	189	216		237
10	205					266	226	240		250
11	213					266	226	242		260
12	236						233	242		290
13							271	261		262
14							278	262		326
15							272	284		290
16								322		
17								362		
18								376		
19								398		
20								378		

TABLE 2. Smoke-puff radii (cm), Tilghman Island, 28 February 1950.

Time (sec)	Puff number 1(a)	1(b)	2(a)	2(b)	3	5	9	10	11
1	78	72	73	66	58	74	64	70	62
2	102	92	87	88	82	83	77	89	77
3	117	124	105	101	100	107	94	105	101
4	140	155	133	115	114	140	114	120	113
5	160	200	153	126	132	137	140	133	129
6	181	208	150	137	137	158	156	138	143
7	194	254	158	142	149	169	171	155	155
8	199	256	177	149	146	187	177	156	158
9	214	267	164	153	167	198	181	162	160
10	236		175	166	164	216	176	180	168
11	236		201	178	180	236		190	177
12	282		212	179	188	251		202	171
13	296		204	183		282			
14	342		205	185		278			
15	361		239	195		282			
16	421			197					
17	398			206					
18	433			216					
19				231					
20				234					

of average radius *versus* time of each of a total of 19 smoke puffs are tabulated; time is measured from the instant the flash appears. These data were obtained over water under conditions of unstable equilibrium — the air was being heated from below, and active vertical mixing was taking place.

3. Basic equations for dispersion from a point source

Before presentation of an analysis of the data obtained in the Tilghman Island experiment, a few preliminary remarks on the basic equations for dispersion from a point source under turbulent flow conditions may be in order.

In homogeneous and isotropic turbulence, one assumes that the smoke particles in a diffusing puff are distributed normally. This may be written as follows [2]:

$$\frac{\bar{S}_0}{Q} = \frac{1}{(2\pi\overline{y^2})^{\frac{3}{2}}} \exp\left[-\frac{1}{2\overline{y^2}}(x^2 + y^2 + z^2)\right], \qquad (1)$$

where $\bar{S}_0$ is the mean concentration of particles at the point (x, y, z), Q the total number of particles dispersed, and $\overline{x^2} = \overline{y^2} = \overline{z^2}$ are the variances of the mean concentration along each of the coordinate axes. ($\overline{y^2}$ will be used synonymously with $\overline{x^2}$ and $\overline{z^2}$ in this derivation.) In the case of a smoke puff, as viewed from a large distance, it is postulated that one can "see" the smoke puffs provided there is a particle density greater than some threshold value.[3] Because of symmetry, one may integrate with respect to any one of the axes, say z, to obtain a distribution function,

$$\frac{\bar{N}}{Q} = \frac{1}{2\pi\overline{y^2}} \exp\left[-\frac{1}{2\overline{y^2}}(x^2 + y^2)\right], \qquad (2)$$

where $\bar{N}$ is the integrated mean concentration along the line of sight for the entire extent of the puff.

The fundamental expression for the variance of the mean concentration is

$$\overline{y^2} = 2\overline{v^2}\int_0^t (t - \alpha)\, R(\alpha)\, d\alpha, \qquad (3)$$

where $\overline{v^2}$ is the variance of the turbulent velocity, t the dispersion time, and R is the Lagrangian correlation function. (It may be emphasized here that, while the values of $\overline{v^2}$ can generally be assumed to be the same whether the average is Lagrangian or Eulerian, such an assumption would be unjustified for a correlation function.) For small values of t, that is,

$$t \ll \int_0^\infty R(\alpha)\, d\alpha, \qquad (4)$$

$\overline{y^2}$ is found independent of R, and

$$\overline{y^2} = \overline{v^2}\, t^2. \qquad (5)$$

The limits of validity of this relationship will be discussed later.

For small values of t, then, (2) may be rewritten as follows:

$$\frac{\bar{N}}{Q} = \frac{1}{2\pi\overline{v^2}\, t^2} \exp\left(-\frac{r^2}{2\overline{v^2}\, t^2}\right),$$

where $r^2 = x^2 + y^2$. Setting $r = a$ and $\bar{N} = \bar{N}_0$, and taking the logarithm of both sides, one has

$$\frac{a^2}{t^2} = -(2\overline{v^2} \ln t^2) - \left(2\overline{v^2} \ln 2\pi\overline{v^2}\frac{\bar{N}_0}{Q}\right). \qquad (6)$$

The term $\bar{N}_0$ is defined as the threshold value of $\bar{N}$ corresponding to a, the radius of the outer visible boundary of the smoke puff, which shall be called the radius of the puff.

It may be seen that if a^2/t^2 were plotted against $\ln t^2$, the resulting graph would be a straight line of slope $-2\overline{v^2}$ intercepting the ordinate at

$$(a^2/t^2) = 2\overline{v^2} \ln (2\pi\overline{v^2}\, \bar{N}_0/Q)$$

when $t = 1$. The intercept at the abscissa corresponds to $a = 0$ and leads to

$$\frac{\bar{N}_0}{Q} = \frac{1}{2\pi\overline{v^2}\,(t_{a=0})^2}. \qquad (7)$$

One can therefore apply this procedure to the analysis of experimental measurements and determine the values of $(\overline{v^2})^{\frac{1}{2}}$ and $\bar{N}_0/Q$.

4. Results of Tilghman Island smoke-puff experiment

Fig. 2 is illustrative of the runs made during the Tilghman Island experiment when plotted in the manner previously indicated. It is apparent that the data fit a straight line in the region from about 4 to 9 sec. At the beginning, however, the data tend to diverge from the straight line. Such divergence may be explained by the fact that experimentally one cannot reasonably expect to start with a perfect point source. In fact, 1 sec after the flash, the puff was generally a little under 1 m in radius. Let it be assumed that at that time the smoke is distributed according to a Gaussian law as expressed by (1). One can then evaluate a time interval that will be necessary to

[3] This threshold corresponds to a constant value of the luminous intensity for which it is here postulated that one can "see" the smoke puff. Such a luminous intensity could, for instance, be measured with a microphotometer and properly prepared photographs of smoke puffs.

In the case of visual observation, the human eye is mainly stimulated by the variation of the luminous intensity and not by its absolute value. The visual perception of a contour depends, therefore, on the variation of the luminous intensity over the smoke-puff area. Studies of contour perception seem to show [3] that the value of the second derivative of the luminous intensity is particularly significant in the contour formation. However, higher derivatives may also be important. It may, therefore, be premature to decide about the proper assumptions to be applied in these problems of physiology of vision.

TABLE 3. Characteristics of turbulence, Tilghman Island.

Date	Puff no.	z (10^3 cm)	U (10^2 cm/sec)	$(\overline{v^2})^{1/2}$ (cm/sec)	T (per cent)	$\bar{N}_0/Q$ (10^{-6} cm^{-2})	$(\bar{N}_0/Q)m_p$ (10^{-6} g/cm^2)
27 Feb. 1950	4	6.1	15.9*	8.2	0.5	1.8	12
	5	7.7	15.9	10.0	0.6	3.6	24
	6	5.0	15.9	8.8	0.6	1.0	7
	7	5.0	15.9	12.9	0.8	9.0	60
	8	4.8	15.9	9.4	0.6	1.3	9
	13	0.4	15.9	13.8	0.9	1.1	7
	16	3.9	15.9	15.2	1.0	3.7	25
	18	5.3	15.9	8.8	0.6	1.4	9
	19	4.8	15.9	8.3	0.5	5.4	36
	20	2.5	15.9	11.2	0.7	1.5	10
28 Feb. 1950	1(a)	0.8	5.8	12.0	2.1	2.3	15
	1(b)	0.8	7.7	9.0	1.2	0.2	1
	2(a)	1.5	8.0	11.0	1.4	3.1	21
	2(b)	2.2	4.8	11.7	2.4	5.8	39
	3	3.9	5.2	9.8	1.9	4.0	27
	5	1.9	7.4	8.6	1.2	1.9	13
	9	1.7	5.0	9.9	2.0	3.5	23
	10	1.7	5.0	11.6	2.3	4.5	30
	11	1.7	5.0	10.1	2.0	4.5	30

* Values of U on 27 February 1950 are approximate.

produce a puff of the size measured 1 sec after the flash, and extrapolate to an imaginary point source. Extrapolation to such a point source brings the data closer to a straight-line fit in the first several seconds. Fig. 3 is a recalculation of the puff shown in fig. 2, extrapolated to -1 sec for zero time.

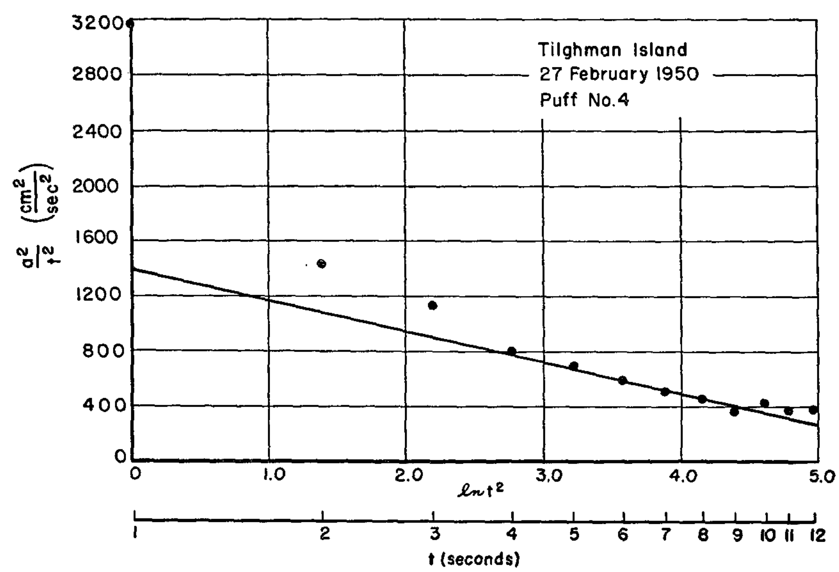

FIG. 2. Sample plot of smoke-puff growth.

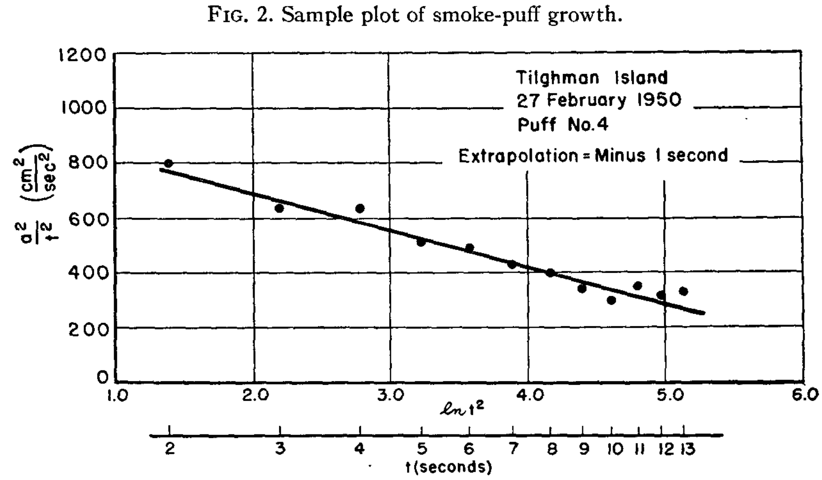

FIG. 3. Smoke puff of fig. 2 extrapolated to -1 sec for zero time.

Characteristics of the turbulence derived from data obtained during 27 and 28 February 1950 are summarized in table 3. Values of the height (z), mean wind (U), root-mean-square value of the turbulent velocity, $(\overline{v^2})^{1/2}$, intensity of turbulence, $[T = (\overline{v^2})^{1/2}/U]$,

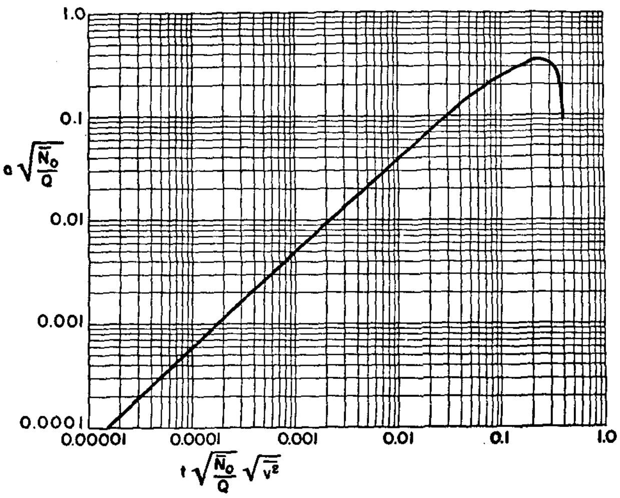

FIG. 4. Universal curve for comparison of smoke-puff data.

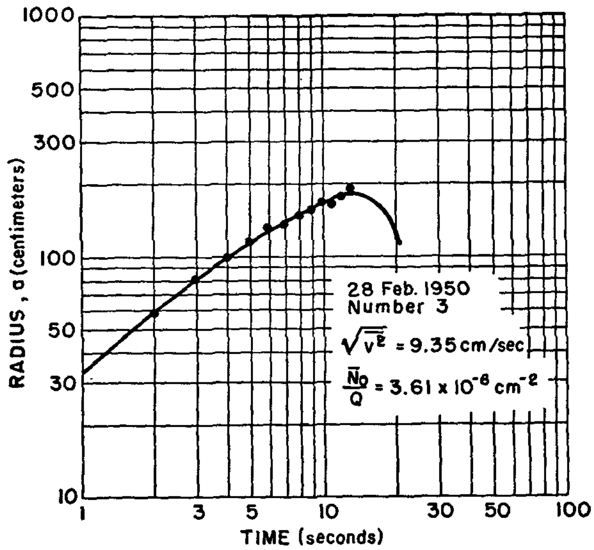

FIG. 5. Sample puff plotted on universal curve.

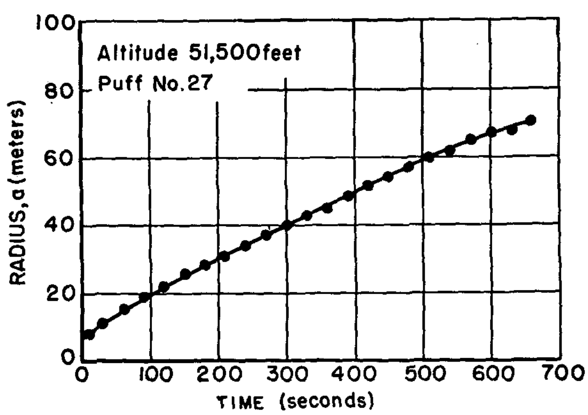

FIG. 6. One curve from Kellogg's smoke-puff data, showing nearly linear growth with time.

and integrated concentration ratio ($\bar{N}_0/Q$) are tabulated. It may be interesting to note the range and order of magnitude of the derived values of intensity of turbulence and integrated concentration ratio. Turbulence intensity varied from 0.5 to 2.4 per cent, and $\bar{N}_0/Q$ varied from 0.2×10^{-6} to 9.0×10^{-6} cm^{-2}. The weight of black powder, m_p, that was burned during the generation of each puff was about 6.7 g. If it were assumed that all the black powder was transformed into smoke particles, one would find that a visible outline was achieved with as little as 1 microgram of smoke per square centimeter of the projected puff area at ranges of about 600 to 800 ft. This value is tentative and is given only to show the order of magnitude. In the last column in table 3 are presented the values of $m_p\bar{N}_0/Q$. Over the small range of heights encountered here, no dependence of turbulence on height was observed.

To make the analysis of smoke-puff measurements more universally adaptable, a plot has been made in somewhat different form with use of nondimensional quantities. If one employs the dispersion factor, defined as

$$\iota^2 = \overline{y^2}/L^2\,\overline{v^2}, \tag{8}$$

where $L = \int_0^\infty R(\alpha)\,d\alpha$ is the "scale of turbulence," puts the puff radius into the form

$$\rho = a/UL, \tag{9}$$

and lets

$$\sigma = U^2L^2\,\bar{N}_0/Q, \tag{10}$$

(6) can be rewritten in the form

$$(\rho/T)^2 = -2L^2 \ln 2\pi L^2T^2\sigma. \tag{11}$$

A curve of $\sqrt{\sigma}\rho$ plotted against $\sqrt{\sigma}Tt/L$ (which is the equivalent of plotting $(\bar{N}_0/Q)^{1/2}$ *versus* $t(\bar{N}_0/Q)^{1/2}(\overline{v^2})^{1/2}$ on log-log scales) results in a universal curve to which smoke-puff data may be compared. Fig. 4 is a graph of such a curve, calculated for dispersion time much less than L. The points plotted in fig. 5 are the experimental values of the puff radius for puff 3 on 28 February 1950. The line in this figure is that of the universal curve displaced upward by $(\bar{N}_0/Q)^{1/2}$ and to the right by $(\overline{v^2})^{1/2}(\bar{N}_0/Q)^{1/2}$. The values of $(\bar{N}_0/Q)$ and $(\overline{v^2})^{1/2}$ were obtained from the best fit of the data to the curve; these compare favorably with the values given in table 3 for this puff.[4]

[4] In the preceding footnote, reference was made to the visual contour perception which indicated that the appearance of a contour may be dependent on the value of $d^2\bar{N}_0/da^2$ rather than on $\bar{N}_0$. A similar curve to the one represented in fig. 4 can be obtained by using this second derivative as a characteristic value for smoke-puff outlines. We are not elaborating on this problem at the present time, since the relationship between the contour perception and the luminous intensity is not sufficiently well known. We may, however, indicate that, by using $d^2\bar{N}_0/da^2$ as a contour characteristic, one finds smaller values for $(\overline{v^2})^{1/2}$. The resulting values of $(\overline{v^2})^{1/2}$ obtained by such a method would be equal to those listed in table 3 multiplied by the same factor of about 0.5.

5. Smoke-puff measurements in the stratosphere

An experiment using smoke puffs for finding diffusion data for the stratosphere was reported by Kellogg [3]. He exploded vials of titanium tetrachloride and water at heights of 23,600 to 63,300 ft and obtained useful information in 18 runs. These puffs were tracked much longer than those in the Tilghman Island experiment, nine of them lasting for 660 sec. The initial puff radii were about 7 m. Recording of the smoke puffs was accomplished by a network of three phototheodolites. Fig. 6 shows an example of these data, indicating that the diameter of the puff first increases nearly linearly with time and then tends to level off slowly. From these data, Kellogg obtained for $(\overline{v^2})^{\frac{1}{2}}$ values from 4 to 10 cm/sec for the 18 puffs.

From Kellogg's wind-speed data, values of the intensity of turbulence have been computed. These were as low as 0.2 and as high as 1.9 per cent — only slightly lower than those computed from the Tilghman Island puffs.

The relationship given by Kellogg for the radial spread of a smoke puff is of the same analytic form as (6), indicating that similar laws hold for the diffusion of a 1-m puff for 10 to 20 sec near the ground and of a 7-m puff for many minutes in the stratosphere. Kellogg gives values of $(\overline{v^2})^{\frac{1}{2}}$ as 4 to 10 cm/sec, whereas the Tilghman Island experiment yielded values of 8 to 15 cm/sec. There is one difference, however. Whereas, in the stratosphere, the Lagrangian correlation coefficient remains approximately unity for a long time, it is highly unlikely that such is the case near the ground.

A question may be raised as to which law of growth is valid for the data — that given in (6) or a simple linear relationship between radius and time, as suggested by the nearly straight line in fig. 6. Equation (6) may be written simply as $a^2/t^2 = b_1 \ln t^2 + c_1$. Results of the smoke-puff experiments given by Kellogg and the Tilghman Island data may be seen to fit $a = b_2t + c_2$, at least during the early portions of the curves. Equation (6) may be rewritten as $a = t(b_1 \ln t^2 + c_1)^{\frac{1}{2}}$, and it may be pointed out that the expression appearing in the parentheses is indeed a slowly varying function of t. One finds that, when a is plotted against t, there results an approximately straight line. There is clearly, then, no disagreement between the analytical form of (6) and the data obtained from smoke-puff experiments.

6. Smoke puffs to measure wind-speed fluctuations

A valuable comparison may be made between the methods described above and one given by Durst [5]. In an investigation of the "fine" structure of the wind, Durst used smoke puffs at heights of 5000 to 20,000 ft. He used a camera obscura to follow puffs released from aircraft flying directly above the camera. Puff trajectories were recorded manually by marking the puff positions every 10 sec. During a 1-hr period, it was possible to release 15 to 25 puffs at intervals of about 2 to 5 min. Downwind and crosswind velocity fluctuations over about a 1-hr period were obtainable from the velocities of each puff.

Results of Durst's study showed values of $(\overline{v^2})^{\frac{1}{2}}$ ranging between 15 and 130 cm/sec, and values of the intensity of turbulence between 1 and 21 per cent. It should be made clear at this point that there is one important difference between this method and those described earlier. Durst measured the fluctuations in wind as seen by the puff as a whole, a larger-scale phenomenon over a distance comparable to tens of miles, whereas the radial spread of the puff yields information on the smaller-scale diffusion taking place within and at the edges of the smoke puff. The effect of larger eddies was purposely avoided in reducing the data in both preceding smoke-puff experiments. It appears, then, that the larger-scale eddies caused turbulent velocity fluctuations up to tenfold greater than those eddies contributing to the spread of puffs of the size described previously.

7. Diffusion measurements by soap bubbles

Soap-bubble measurements to study diffusion were carried out in a wind tunnel at Lille, France, in the 1930's [6; 7]. The distribution of bubbles downstream from a point source was found to be approximately Gaussian. In an experiment carried on more recently, Edinger [8] studied atmospheric diffusion by photographing the spatial distribution of bubbles released sequentially at an altitude of 1000 ft. Although these experiments fall in the class of diffusion from a continuous point source, they are mentioned here because they help verify certain assumptions made in the previous analysis. The Lille experiments verify the assumption of a Gaussian distribution, the basis for (1). One may characterize Edinger's results, on the standard deviation of bubble positions as a function of time, by the expression $(\overline{y^2})^{\frac{1}{2}} = 0.107\, t$, which represents his data with an accuracy within $\pm$ 2 per cent. This result confirms the validity of (6) for small values of t and is especially convincing in that it stems from a somewhat different experimental procedure. Not only is the analytic form supported, but it may be seen that the constant indicates a value of $(\overline{v^2})^{\frac{1}{2}}$ of 10.7 cm/sec. This value lies near the upper end of Kellogg's values and falls within the range of values obtained at Tilghman Island.

8. Conclusions

There are at least two basically different approaches to the problem of turbulent diffusion. One is usually referred to as the "Fickian" treatment, wherein the differential equations are solved, it being assumed that K, the "coefficient of eddy diffusion" is independent of the dispersion time. If the value of $\overline{y^2}$ were to be replaced by $2Kt$ in (1), one would have a solution to the Fickian diffusion equation. This treatment is correct for large dispersion times, where $t \gg L$. The other method, the broad outline of which is mentioned here, is based on the statistical theory of turbulent diffusion. In this method, one utilizes the correlation function R in seeking the functional form of the diffusion process.

The experimental evidence presented in this article points toward the relationship $\overline{y^2} = \overline{v^2}\, t^2$, indicating that these processes are taking place for $t \ll L$. When $t \gg L$, the variance is given by $\overline{y^2} = 2\overline{v^2}\, Lt$, the variance being directly proportional to t. The variation of $\overline{y^2}$ as a function of the dispersion time can be described by (3). This equation can be approximated by a relation of the form $\overline{y^2} \doteq t^n$, with $n = 2$ when $t \ll L$, and $n = 1$ when $t \gg L$. Sutton [9] computes, under certain assumptions, a value $n = 1.75$, which describes diffusion in the atmosphere for distances varying from meters to hundreds of kilometers. This value lies inside the bounds of n and closer to the value corresponding to small dispersion times. In the present article, only the smaller-scale turbulence is studied, corresponding to $t \ll L$, which leads to a value $n = 2$.

Smoke puffs offer a versatile tool in the study of turbulent diffusion. If values of $\overline{v^2}$ and U were to be obtained from the study of smoke puffs under varying stability conditions and at various heights, a clear-cut advance would be made in the knowledge of turbulent diffusion. The experiment does not involve complex equipment, and data analysis is relatively simple. In fact, it has been demonstrated recently that smoke puffs may be set off by remote control, obviating the need for a tethered balloon or an airplane. Randall and Clark [10] have developed a simple puff generator that may be sent up on a small balloon and fired from the ground when desired. The study of smoke puffs will contribute to the accurate forecasts of concentrations of smoke, radioactive materials and other pollutants, as well as to various micrometerological applications.

Acknowledgments.—The writers wish to thank Miss Doris Rubenfeld for preparing the figures and Mrs. Betty Grisamore for her assistance in the numerical computations.

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