MEASUREMENTS OF CLOUD DROP-SIZE DISTRIBUTIONS

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ABSTRACT

Through the analysis of infrared transmittance of a natural cloud, its drop-size distribution can be inferred. This technique has been used to measure and analyze ten cloud situations on Mount Washington. These clouds show drop-size distributions that are bimodal in character. In all the distributions, a large number of small droplets is inferred. To test the synthesized distributions in another region of the spectrum, the visual range was computed. This determination of the visual range is in agreement with the observed visibilities.

1. Introduction

The techniques employed to measure cloud drop-size distributions may be grouped into two major classifications: first, those techniques in which the measured cloud droplets make physical contact in some manner with a collector, and second, the experimental methods which do not use a collector. Examples of the first group are coated slides, multicylinders, the cascade impactor, *etc.* In each of these instruments, the measured droplet is collected on a surface with the inevitable result that the droplet and its environment are disturbed to some extent. In each case, the collection-efficiency concept results in a minimum measurable drop size.

The second general method, which includes instruments like the transmissometers, the Tyndallometer, *etc.*, may be described as being a passive technique. Because the measured droplets do not impinge on a surface of these instruments, the droplets are not disturbed in their environment. The main disadvantage to this class of instruments is the complexity of the computations necessary to determine the results. However, this objection is becoming less of a limiting factor with the increase in availability of computational aids for data reduction. The drop-size distributions presented below were inferred from transmission measurements made with an infrared spectrophotometer, used as a variable frequency transmissometer.

2. Theory

The optical scattering properties of spheres have been treated by Mie [1], Houghton and Chalker [2], and others. In these papers, the scattering cross-section has been computed chiefly for the non-absorbing case, that in which only the real index of refraction enters. Van de Hulst [3] considered the absorption of small particles, but only recently have the scattering cross-section computations been extended to the absorbing case. In this case, the complex index of refraction must be used to determine the relative amount of incident energy scattered and absorbed by a scattering medium. Johnson and Terrell [4; 5] have computed the transmission cross-sections for water droplets illuminated by infrared radiation. The transmission cross-sections reported give the energy lost from a beam of radiation relative to the energy intercepted by the scatterer's geometrical area. In the absence of absorption, depletion of energy in the beam is caused by scattering only. In the mathematical formulation, the imaginary part of the index of refraction becomes identically zero, and the transmission cross-section is identical to the scattering cross-section.

The use of this theory permits the computation of the transmittance as a function of wavelength for any particle size, and hence for any arbitrary particle-size distribution. In this way, comparison of various synthetic transmission functions with the experimental transmission can be used to infer a particle-size distribution which could have produced it. Though this analytical technique does not yield theoretically unique solutions, it can be shown that if the matching distribution is significantly altered, the experimental transmission can not be matched. Thus, from the point of view of cloud physics, the method gives "unique" drop-size distributions.

3. Instrumentation and measurements

The infrared variable-frequency transmissometer consists of a Perkin-Elmer Model 12-C infrared monochromator, an automatic high-speed scanning mechanism, and associated optical components. The assembly of the major components was made on an optical bench, which was enclosed for protection from the weather [5; 6; 7]. The sampling path length is

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Table I

Drop Size Distribution

Drop diameter (μ)	A Number drops/cc	A LWC gm/m³	B Number drops/cc	B LWC gm/m³	C Number drops/cc	C LWC gm/m³	D Number drops/cc	D LWC gm/m³	E Number drops/cc	E LWC gm/m³
1	34,600	0.018	47,900	0.025	15,100	0.008	1,910	0.001	34,400	0.018
3	2,400	0.035	830	0.012	3,790	0.056	380	0.006	1,050	0.015
5	350	0.023	—	—	770	0.050	130	0.009	—	—
7	—	—	—	—	80	0.014	640	0.115	—	—
9	—	—	—	—	230	0.088	130	0.050	—	—
11	140	0.098	30	0.021	150	0.105	—	—	—	—
13	230	0.264	110	0.127	80	0.092	—	—	200	0.230
15	80	0.142	220	0.389	—	—	—	—	480	0.850
17	—	—	80	0.207	—	—	—	—	70	0.180
19	—	—	30	0.108	—	—	—	—	—	—
21	—	—	—	—	—	—	—	—	—	—
23	—	—	—	—	—	—	—	—	—	—
25	—	—	—	—	—	—	—	—	—	—
Total	37,800	0.580	49,200	0.889	20,200	0.413	3,190	0.181	36,200	1.293

Drop diameter (μ)	F Number drops/cc	F LWC gm/m³	G Number drops/cc	G LWC gm/m³	H Number drops/cc	H LWC gm/m³	I Number drops/cc	I LWC gm/m³	J Number drops/cc	J LWC gm/m³
1	61,100	0.032	22,100	0.012	25,600	0.013	45,600	0.024	21,300	0.011
3	1,500	0.022	4,410	0.065	6,170	0.091	4,560	0.067	4,680	0.069
5	—	—	370	0.024	1,550	0.102	370	0.024	850	0.056
7	—	—	550	0.099	—	—	—	—	—	—
9	—	—	370	0.141	510	0.195	—	—	420	0.161
11	—	—	70	0.049	1,060	0.740	370	0.259	1,700	1.188
13	220	0.253	—	—	510	0.586	730	0.841	850	0.977
15	1,140	2.020	40	0.071	—	—	—	—	—	—
17	140	0.360	40	0.103	—	—	—	—	—	—
19	—	—	—	—	—	—	—	—	—	—
21	—	—	—	—	—	—	—	—	—	—
23	—	—	40	0.225	—	—	—	—	—	—
25	—	—	110	0.900	—	—	180	1.472	—	—
Total	64,100	2.687	28,100	1.719	35,400	1.727	51,800	2.687	29,800	2.462

1 m. The sampling time of 22 sec is determined by the time necessary for the automatic scanning mechanism to complete a scan of the transmitted radiation through the spectral range of 1- to 14-μ wavelength. The slit-width program is adjusted so that the resolution of the spectrometer is $\Delta\lambda \approx \lambda/10$, sufficient for analysis of the drop-size distributions in classes of 2-μ width.

The infrared transmissometer was mounted on the west side of the summit of Mount Washington, New Hampshire. The drop-size distributions presented below result from measurements made during the month of October 1953, at which time the winds were generally westerly with velocities between 20 and 40 mi/hr, and the temperature ranged from 20 to 30F. In all cases, the clouds could be described as being stratocumulus in form. However, because the air was being advected over the summit, the clouds were probably in a growing stage.

4. Drop-size distribution

The drop-size distributions are given as the number of droplets per cubic centimeter, per 2-μ drop-diameter class width. The class designator is the median drop diameter in microns. The liquid-water content (LWC) is the product of the median droplet volume and the number of droplets in its class, expressed in units of grams per cubic meter. The ten drop-size distributions in table 1 are representative samples of the data. The data have been presented in the form of individual samples of the drop-size distribution, and not as the average of many samples, in an effort to present the basic characteristics of each distribution.

5. Discussion of results

The two important departures of the above data from previous cloud drop-size distributions are (a) the large number of droplets per cubic centimeter, and (b) the bimodal character of the distribution. These data show that the 1- and 3-μ diameter droplet classes contribute the largest number of drops. In table 2, the 1-μ diameter drop class and the 1- and 3-μ diameter drop classes are progressively subtracted from the total distribution, to emphasize the similarities that exist between these distributions and previous measured drop-size distributions [8; 9; 10; 11]. When the 1- and 3-μ diameter drop classes are removed from the distribution, the total number of droplets becomes consistent with previous slide techniques, because the small droplets are not collected by slides and thus not

TABLE 2. Summary of drop-size distributions.

Distribution	All classes drops/cc	All classes g/m³	Minus 1-μ class drops/cc	Minus 1-μ class g/m³	Minus 1- and 3-μ class drops/cc	Minus 1- and 3-μ class g/m³
A	37,800	0.58	3,200	0.56	800	0.53
B	49,200	0.89	1,300	0.86	470	0.85
C	20,200	0.41	5,100	0.40	1,310	0.35
D	3,190	0.18	1,280	0.18	900	0.17
E	36,200	1.29	1,800	1.28	750	1.26
F	64,100	2.69	3,000	2.66	1,500	2.63
G	28,100	1.72	6,000	1.71	1,590	1.64
H	35,400	1.73	10,000	1.71	3,830	1.62
I	51,800	2.69	6,200	2.67	1,640	2.60
J	29,800	2.46	8,500	2.45	3,820	2.38

measured. Johnson [12] gives a comprehensive discussion of the aerodynamics of collection efficiencies, which indicates how collectors discriminate against the smaller droplets. The omission of a large number of small droplets also shifts the median drop diameter toward the larger droplets in the distribution.

Inspection of table 2 yields another conclusion, that is, the smaller droplets contribute only a negligible amount of liquid water. The validity of the drop-size distributions can be estimated by computation of the meteorological visual range by the following two formulae. The first computation is made with Trabert's formula [13], which is

$$V_m = 2.6\,\bar{d}/w, \tag{1}$$

where V_m is the meteorological visual range (meters), $\bar{d}$ the volumetric median drop diameter (μ), and w is the liquid-water content (grams per cubic meter). The visual range determined by this empirical formula is more dependent upon the liquid-water content than the drop-size distribution. It assumed that a volumetric median drop diameter describes the distribution adequately. A more critical test of the drop-size distribution would be to compute the visual range in terms of scattering-theory parameters. A common form of this equation is

$$V_m = \frac{1}{k_s} \ln \frac{1}{\epsilon} = \frac{3.912}{k_s}, \tag{2}$$

where the standard value of 0.02 is used for ϵ, the threshold of brightness contrast. In (2), V_m is the meteorological visual range (meters). The scattering coefficient, k_s, is

$$k_s = \frac{n\pi}{4} \sum_i \left(\frac{n_i}{n} \right) K_{si} d_i^2 \times 10^{-8}, \tag{3}$$

where n is the total number of droplets (per cubic centimeter) in the distribution. The ratio n_i/n is the fraction of the number of droplets in class i related to the total number of droplets in all classes. The scattering-area coefficient, K_{si}, is determined from Mie scattering theory for those droplets in class i. The quantity d_i (μ) is the median drop diameter of class i. An approximate scattering area cross-section, k_s, cannot be used with any validity because of the predominance of the small droplets. However, to determine K_{si} for each class, it was assumed that $\lambda = 0.55\ \mu$ in the expression $\alpha = \pi d/\lambda$. This wavelength was selected because the normal human eye has its maximum visual response to this color. The use of $\lambda = 0.55$ is quite reasonable, because at $\lambda = 0.45$ or $\lambda = 0.65$ the visual sensation is reduced by a factor of ten or more [14], and thus has a narrow band response to radiation.

TABLE 3. Meteorological visual range.

Distribution	V_m = estimate (meters)	$V_m = \frac{3.912}{k_s}$ (meters)	$V_m = \frac{2.6\,\bar{d}}{w}$ (meters)
A	100 to 200	110	59
B		120	47
C		170	70
D		470	115
E	50 to 100	110	32
F		52	15
G		89	37
H	25 to 50	60	18
I		57	24
J		49	13

The meteorological visual ranges as computed by the above two formulae, and the estimated visual ranges, are presented for each drop-size distribution in table 3. The estimated visual ranges were made by the writer while operating the infrared transmissometer. Due to the nature of the markers, which are the buildings on the summit of Mount Washington, the estimated visual ranges are at best only approximate, and are almost meaningless beyond much more than 100 m.

The visibilities computed with Trabert's formula, (1), are primarily a function of the liquid-water content and only indirectly dependent upon the drop-size distribution when the breadth of the distribution is narrow. The visual ranges computed from scattering theory, (2), are solely a function of the drop-size distribution and not restricted by the character of the distribution.

The role of the small droplets may be further indicated by considering their influence upon the calculated drop-size distribution and the resulting transmittance. As was previously mentioned, the instrument measures the transmissivity as a function of the wavelength. The experimental transmission, τ_E, and the computed transmission, τ_c, (as determined from the synthesized drop-size distribution) are shown in table 4 to illustrate the "fit" between them. Also included in table 4 are the resulting transmissions when the 1-μ class of droplets, τ_c', and the 1- and 3-μ classes of droplets, τ_c'', are omitted from the drop-size distribution. As the smaller droplets are removed from the distribution, the transmissivity increases at the

TABLE 4. Transmissivities.

Wavelength (microns)	τ_E (Experimental)	τ_c (Computed)	τ_c' (Minus 1-μ class)	τ_c'' (Minus 1- and 3-μ class)
2.2	0.72	0.72	0.80	0.86
4.0	0.88	0.86	0.86	0.87
5.0	0.83	0.83	0.84	0.85
6.0	0.83	0.83	0.83	0.83
7.0	0.84	0.84	0.84	0.83
8.0	0.85	0.87	0.87	0.87

shorter wavelengths, but remains practically unchanged at the longer wavelengths. Thus, the smaller drops appear to be necessary to the distribution if the computed transmission is to match the experimental transmission well.

The role of the small droplets in visibility appears to be quite important. This belief has also been mentioned by Fritz [15], who has indicated that an approximate form of scattering theory may be applicable to visibilities. However, the importance of the small droplets can be indicated by computing the visual range with the scattering-theory formula, (2), and successively omitting the smaller droplets. The visual range, V_m, is 110 m when the total distribution A is used to determine k_s. If the 1-μ class of drops is omitted, $V_m = 234$ m; and if the 1- and 3-μ classes are omitted, $V_m = 311$ m. It is quite obvious that any approximate theory to determine the visual range must assess quite carefully the role of the small droplets.

The second feature of the drop-size distributions presented is their bimodal character. The observation of such a feature is possible because a passive measuring technique does not discriminate against the smallest droplets. Table 1 shows most of the distributions to be monomodal when the 1- and 3-μ drop classes are excluded.

The existence of a bimodal distribution is not predicted from the drop-growth equations as they are normally used [16; 17]. The drop-growth equations, however, do not consider the effect of the mixing of parcels of air of different histories. Turbulence existed during the measurements of the drop-size distributions presented, and in most of these situations the growth of new droplets is favored. Consider, for example, a parcel of air with water droplets increasing in size due to the condensation process. If this parcel of air should then subside, or some evaporation take place, and then mix with another parcel of air with growing droplets, a bimodal drop-size distribution could result. Houghton [18] has indicated that mixing and turbulence may be a mechanism causing the broadening of drop-size distributions; an extension of this idea leads to the production of multimodal distributions.

It is improbable that the bimodal character of the natural cloud drop-size distributions can be attributed to the instrumentation. The measuring procedure is the "cell in — cell out" technique, commonly used in spectroscopy, which is capable of yielding reliable data. Prior to the natural cloud measurements on Mount Washington, the instrumentation was tested with artificial clouds produced by pneumatic spray nozzles [6]. These preliminary tests showed the transmissometer to be capable of measuring sizes and distributions of breadths greater than those expected to be found in natural clouds.

6. Conclusions

Many of the laboratory techniques of counting and sizing of small droplets by optical means that have been used in the past have failed for natural cloud drop-size distribution measurements, because they were not capable of measuring broad distributions or were not practical field instruments. The reduction of the data was also limited to wavelengths where the transmission cross-sections computed for the nonabsorbing case could be used. The various collecting techniques that are commonly used are not capable of catching the small droplets in an air stream with sufficient accuracy for measurement. The possibility of, and the requirements for, the existence of the small droplets has been referred to by workers in the fields of solar radiation and cloud albedos, notably Fritz [19]. The essential validity of scattering theory being assumed, the data presented in this article strongly infer the existence of a large number of small droplets in natural clouds. They also indicate the necessity that more natural cloud measurements be made with simultaneous measurement of the visual range and liquid-water content. This point has been well agreed upon in the correspondence [20] among Fritz, Neiburger, and aufm Kampe and Weickmann.

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