ON THE ISOTROPY OF LARGE-SCALE DISTURBANCES IN THE UPPER TROPOSPHERE

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ABSTRACT

As a preliminary attempt to investigate the applicability of the turbulence theory to the large-scale atmospheric phenomena, the experimental test is made of the isotropy for the large-scale motion of the atmosphere at the 300-mb level. Uses are made of one-dimensional power- and cross-spectra of wind velocities, presented recently by Benton and Kahn (1957). The requirement of isotropy seems to be satisfied at lat 20N and 70N, for the harmonics whose wave lengths lie between 60 deg and 20 deg of longitude. The latter is the shortest wave length analyzed. In the region between lat 30N and 60N, the amount of kinetic energy in the north-south component of eddy velocities, in comparison with that in east-west component, is found to be more than that expected from the isotropic relations for the same domain of wave number.

1. Introduction

In recent years, extensive studies have been made on the fundamental problems of turbulence, and many important results have been obtained regarding turbulent flows in aerodynamic laboratories. In particular, the theory of turbulence, first put forward by Kolmogoroff (1941) and Obukhoff (1941) and later developed independently by Onsager (1945), Heisenberg (1948), and Weizsäcker (1948), enabled the formulation of specific quantitative predictions and made possible a comparison of theory with observation with respect to the values of many important statistical parameters which characterize a turbulent flow.

With the progress in this field, there appeared some interesting attempts to apply the theory of turbulence to not only small-scale turbulence observed in the natural wind, but also to such large-scale eddies as those associated with the general circulation of the atmosphere. However, the theory of turbulence mentioned above is mainly concerned with homogeneous and locally isotropic turbulence. Therefore, some of the well-defined differences between small- and large-scale turbulent motion must be considered if the utility of this theory is to be evaluated for large-scale phenomena.

We shall be mainly concerned in this paper with the isotropy or lack of isotropy of large-scale turbulence. It is now generally accepted that the zonal atmospheric circulation at higher latitudes is maintained by the horizontal and vertical flux of angular momentum, and that the horizontal flux is primarily due to large-scale eddies. In other words, the kinetic energy of the mean zonal motion is maintained against friction to a large extent by a transfer of kinetic energy from the large-scale horizontal eddies to the mean motions (Starr, 1954). These facts suggest the important role of the cross-product of velocity components, which will be zero in an isotropic velocity field, in the dynamical processes of large-scale atmospheric motion. Therefore, it will be of some interest to see to what degree the requirement of isotropy is satisfied for such large-scale eddies.

In this paper, some comparisons are presented between observed data and theoretical results for disturbances at the 300-mb level, and several features of large-scale turbulence are discussed.

2. Observational results

Before going into the details of the experimental test of the isotropy of the atmospheric turbulence, it seems to be worth while to sketch briefly the present state of the studies on the large-scale atmospheric turbulence. Among many aspects of the studies, we are mainly concerned here with power spectra and correlation functions of fluctuating velocities and pressure.

In 1926, Richardson pointed out, analyzing many observational results, that the eddy diffusivity coefficient (K) in the atmosphere should be regarded as a function of the scale of phenomena (L) under consideration. His conclusion is summarized in his empirical formula

$$K = 0.2\, L^{4/3}, \qquad (1)$$

which seems to fit observational results fairly well up to $L = 10^8$ cm. Later, Stommel (1949) and Inoue (1950) found that the observations of eddy diffusivity in the ocean are also well expressed by the same empirical formula except that a different proportional constant must be used.

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On the other hand, it was pointed out by Obukhoff (1941) and later independently by Weizsäcker (1948) that the empirical formula (1) can be derived reasonably from the theory of isotropic turbulence under a suitable definition of eddy diffusivity. Putting aside some ambiguities involved in (1),[3] the agreement between empirical and theoretical results is quite encouraging to those seeking to apply the theory of isotropic turbulence to large-scale atmospheric phenomena.

Meanwhile, in their researches on the statistical properties of the large-scale disturbances in the free atmosphere, some investigators expanded the fluctuations of the elevations of the 500-mb surface into Fourier series along entire latitudinal circles (Syono and Gambo, 1952; Kubota and Iida, 1954; and Syono, Kasahara, and Sekiguchi, 1955). They found that the distribution of mean-square amplitudes of harmonic components against wave numbers is interpolated approximately by $k^{-7/3}$ for wave numbers larger than about 4 (corresponding to a wavelength 90 deg in longitude). On the other hand, it is well known that the theory of isotropic turbulence gives $k^{-7/3}$ within the inertial range for the spectrum of pressure fluctuations in an isotropic turbulent flow (see for example, Ogura and Miyakoda, 1954).

Also Hutchings (1955) found that auto-correlation functions for the fluctuation of wind velocities observed at a fixed point in the free atmosphere and also those of the pressure at the earth's surface have, respectively, the same functional forms as those predicted by the isotropic turbulence theory up to about 24 hr and 72 hr of lag interval.

We shall now consider the equation for the energy spectrum of turbulence. Recently the present author carried out the mathematical analysis for a homogeneous turbulence associated with mean motion under the following restrictions:

(1) The compressibility of the fluid and the variation of the Coriolis force with latitude are neglected.

(2) The velocity component of the mean motion is assumed to be a linear function of the coordinate y (north-south direction) and constant with respect to time.

(3) The turbulent field is assumed to extend infinitely. In other words, no confining walls are considered.

(4) The motion is purely two-dimensional. This was also assumed by Fjørtoft (1953) and Lorenz (1953).

[3] For example, it is not always clear how we can define the scales of phenomena. The definitions of eddy-diffusivity coefficient, and consequently the procedure employed in estimating their magnitudes, are not same for all the phenomena utilized in deriving (1). In addition, it seems questionable that the concept of eddy diffusivity has actually a physical meaning for phenomena with such a large scale as 10^8 cm, as pointed out by Starr (1953).

The reason for employing the last simplification is that the turbulence considered here covers those whose horizontal scales are of the order of 1000 km, while the vertical scales are of the order of about 10 km or less. Consequently, the requirement of three-dimensional isotropy would not be met.

At the moment, however, it is not obvious to what extent restriction (4) is a good approximation for describing statistically the observed motion of the atmosphere. While large-scale motions are generally regarded as quasi-horizontal, the important role of vertical motion in the evolution of disturbances cannot be denied. However, it is reasonable to suppose that the role of the vertical motion is mainly concerned with the conversion of internal heat, latent heat, and potential energy to kinetic energy, so that the energy exchange between the basic horizontal current and large-scale disturbances, or between various components of disturbances themselves, would be described reasonably well by the two-dimensional equation of motion. This might be more true for the motion in the upper atmosphere, particularly at the "equivalent" barotropic level, than for the atmosphere near the ground.

The details of the mathematical analysis carried out under the above restrictions will be presented elsewhere. Here it is noted that the equation for the power spectrum $E(k)$ is derived as

$$\frac{\partial E}{\partial t} = -2\nu k^2 E + \lambda F - \frac{dS}{dk}. \qquad (2)$$

In this equation, k is the magnitude of the wave number vector, t is time, ν is the viscosity coefficient, λ is the gradient of the mean velocity. S is the transfer function of energy in the wave number space and F is the co-spectrum for the Reynolds stress $\overline{uv}$, that is F is given by

$$\overline{uv} = -\int_0^k F(k)dk.$$

In other words, $\lambda F(k)$ expresses the amount of energy transmitted from the mean flow to the region between k and $k + dk$. In this way, the first term of the right-hand side indicates the loss of turbulent energy through dissipation, the second term is the supply of energy from the mean motion, and the third term is the energy transfer through the spectrum in consequence of the interaction between the various Fourier components of turbulence. It is to be noted that the equation (2), obtained for the strictly two-dimensional turbulence, is exactly the same as that for the three-dimensional turbulence which was discussed by Burgers and Mitchner (1953).

In the steady state, (2) becomes

$$2\nu k^2 E = \lambda F - \frac{dS}{dk}. \tag{3}$$

By integrating (3) between the limits o and k, we have

$$2\nu \int_0^k k^2 E(k)dk = -\lambda\overline{uv} - \lambda \int_k^\infty F(k)dk - S(k). \tag{4}$$

Let us now suppose that the non-isotropy of the turbulence is confined to the rather large eddies. Then $F(k)$ will decrease to zero more rapidly than $k^2E(k)$ with increasing k, and we may use the approximate equation

$$2\nu \int_0^k k^2 E(k)dk = -\lambda\overline{uv} - S(k) \tag{5}$$

for such large values of k, where $\lambda\overline{uv}$ is now considered as a given constant.

The equation (5) has the same form as that obtained for three-dimensional isotropic turbulence. Therefore, if it might be true that the cascade process of energy in the wave-number space for the two-dimensional turbulence is actually taking place in the same way as that accepted for the three-dimensional turbulence, then $E(k)$, determined by (5), is easily proved to be proportional to $k^{-5/3}$ also for the inertial range, if it exists, of two-dimensional turbulence. Furthermore, in his kinematical analysis of the two-dimensional isotropic turbulence, the author (1952) showed that the one-dimensional energy spectrum degenerated from the two-dimensional isotropic turbulence has almost the same shape as that degenerated from the three-dimensional isotropic turbulence, if the spectra for two- and three-dimensional turbulence have the same form. This might be considered in turn as a possible explanation of facts summarized in this section. Since there are such distinct differences between two- and three-dimensional motions of a fluid, however, the validity of the statement mentioned above about the cascade process is doubtful. Indeed, for two-dimensional flow, Fjørtoft (1953) showed that only fractions of the initial energy can flow into smaller scales and that a greater fraction simultaneously has to flow into components with larger scales. This process in the energy spectrum would contradict Kolmogoroff's cascade process.

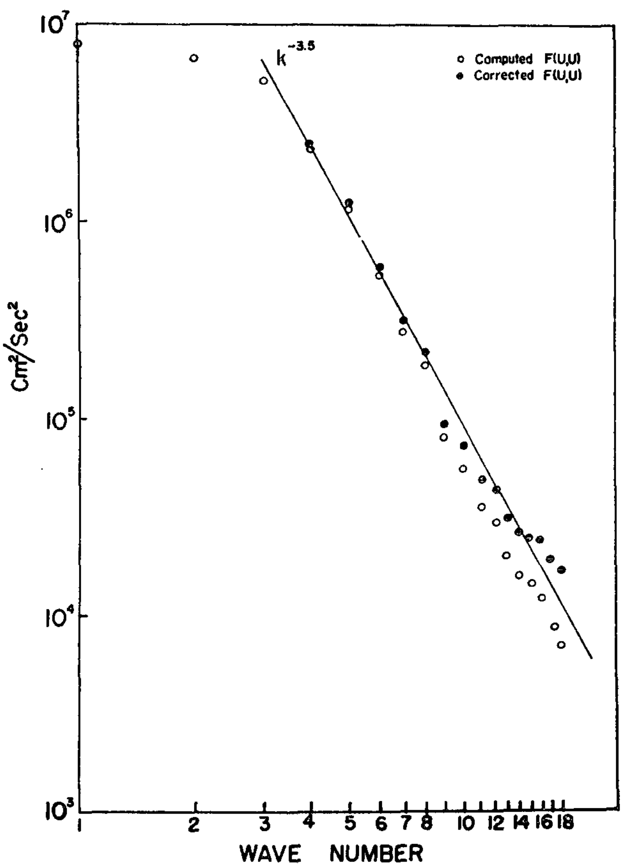

FIG. 1(a). The $\overline{u^2}$-spectrum at 70N, for the 300-mb level, averaged over the period from 1 January 1949 to 28 February 1949. Open dots denote observed values, solid dots corrected ones; the full straight line is fitted to corrected data.

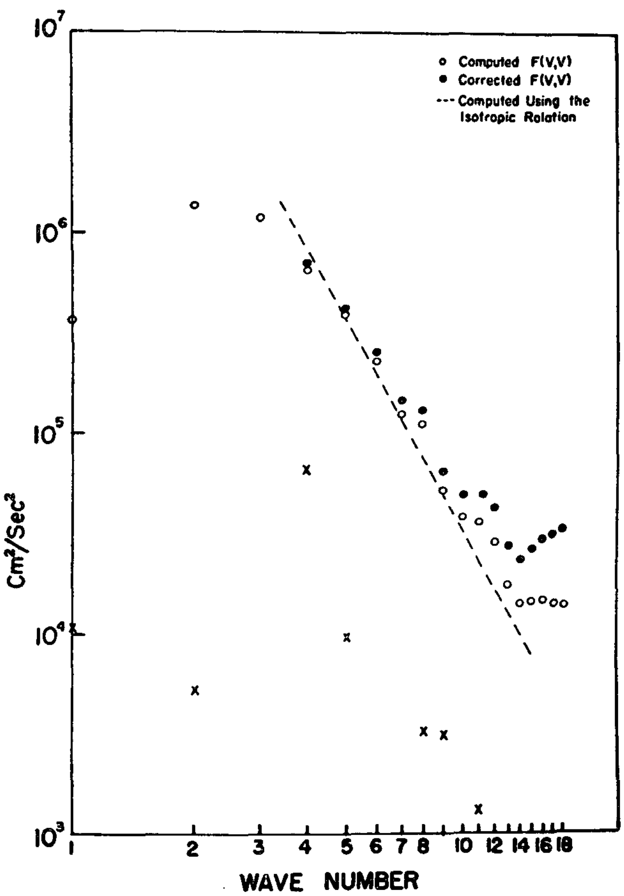

FIG. 1(b). The $\overline{uv}$- and $\overline{v^2}$-spectra at 70N, for the 300-mb level, averaged over the period from 1 January 1949 to 28 February 1949. Crosses denote observed values of $\overline{uv}$-spectrum. Open dots are observed values of $\overline{v^2}$-spectrum, solid dots corrected ones. Dotted line is computed from observed $\overline{u^2}$-spectrum with the aid of the isotropic relation.

The full investigations of the dynamical behavior of the atmospheric turbulence on large scales are beyond the scope of this paper and we shall examine here to what extent the large-scale horizontal motion of the atmosphere can be regarded as two-dimensionally isotropic.

3. Experimental test of horizontal isotropy

Needless to say, it is a rather difficult task to confirm experimentally whether the local isotropy exists or not in a turbulent flow. One method which would be practical for this purpose is to compare the spectrum of $\overline{u^2}$ or $\overline{v^2}$ with that of $\overline{uv}$. If the latter decreases to zero more rapidly than the former does for large wave numbers, as was mentioned above, then the local isotropy for such small eddies would be expected to exist. This is the method used by Laufer (1951) in his studies on the turbulent flow in a two-dimensional channel.

Then another method is to check experimentally the relationship which connects the one-dimensional spectra of $\overline{u^2}$ with that of $\overline{v^2}$ for an isotropic turbulence (Klebanoff, 1954). For two-dimensional isotropic turbulence, this relation can be easily derived. Let the longitudinal and lateral correlation functions be given by $f(r)$ and $g(r)$. Then $g(r)$ is related with $f(r)$ by the continuity equation as follows:

$$g(r) = f(r) + r\frac{df}{dr}. \tag{6}$$

The one-dimensional spectrum of $\overline{u^2}$ and $\overline{v^2}$ is given by

$$\left.\begin{aligned} F_u(k_1) &= \overline{u^2}\int_0^\infty f(r)\cos k_1 r dr, \\ F_v(k_1) &= \overline{u^2}\int_0^\infty g(r)\cos k_1 r dr, \end{aligned}\right\} \tag{7}$$

where k_1 is the wave number in the direction of x-axis. Substituting (7) into (8), we have

$$F_v(k_1) = -k_1\frac{dF_u(k_1)}{dk_1}. \tag{8}$$

Consequently we can compute the $\overline{v^2}$-spectrum from the observed $\overline{u^2}$-spectrum, if the turbulence is isotropic.

Benton and Kahn (1957) have recently prepared

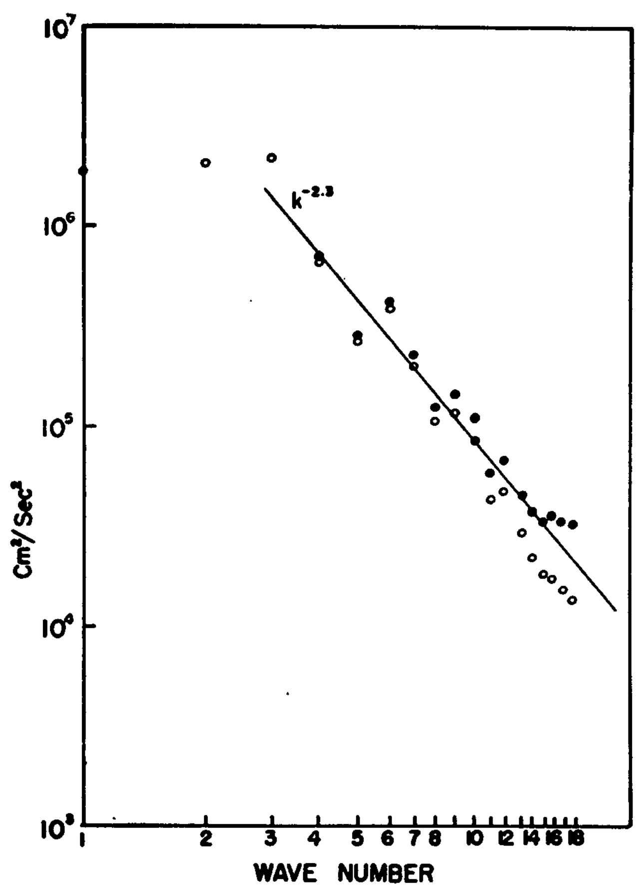

FIG. 2(a). Same as fig. 1(a), but for 40N.

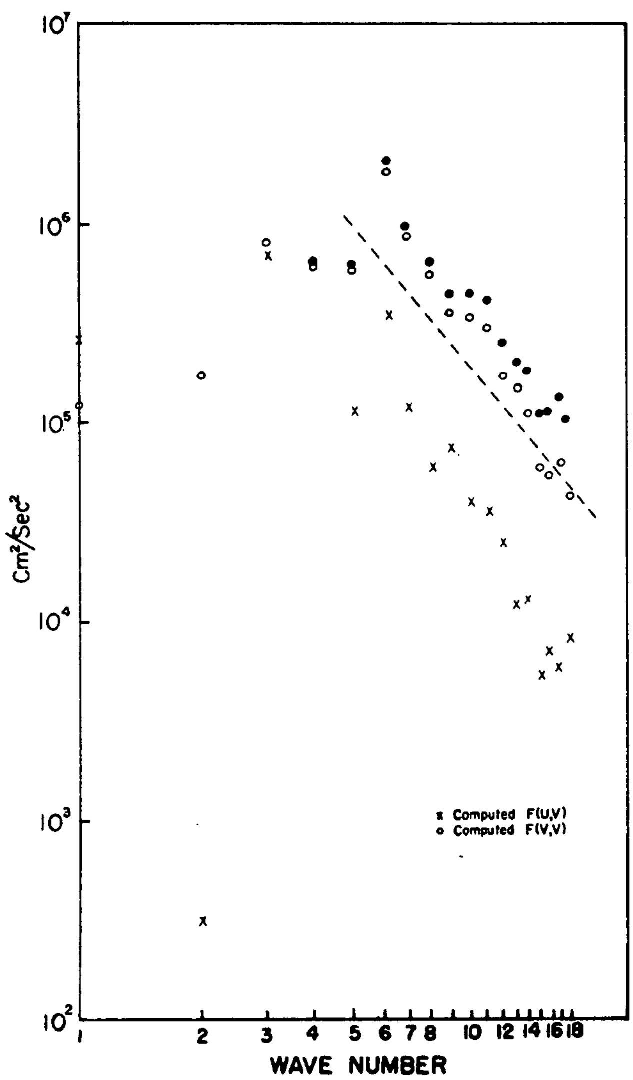

FIG. 2(b). Same as fig. 1(b), but for 40N.

extensive spectral analyses of various physical quantities, including $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$, along the entire latitude circles at every 10 deg of latitude between 20N and 70N. Their analyses cover the period from 1 January to 28 February 1949, and from 1 June to 31 July 1949, and are made at the 300-mb surface where the horizontal transfer of the east-west momentum is considered to be most active throughout the troposphere. Since the more violent horizontal mixing of the atmosphere occurs during the winter, we shall confine our considerations mainly to the winter months. Some examples of results of their analyses, averaged over the winter period, are shown in fig. 1(a) to fig. 3(b). The values of $F_{uv}(k_1)$ are omitted in the figures when they are negative or less than 10^3.

In making use of these data for checking the isotropy of the large-scale horizontal motion, we meet some difficulties in practice. The first of these is that the original data of the wind velocities, used for spectral analysis, are computed from observed values of the elevation of the 300-mb surface at every 5 degrees of latitude and longitude using the geostrophic approximation. Therefore the computed spectra of velocities are subject to slight errors, as was pointed out by the author (1957). For isotropic turbulence, we can estimate the error introduced by the use of finite difference technique as follows:

When the stream function ϕ is assumed to be a random, steady function of space, it will be represented as a stochastic Fourier integral:

$$\phi = \int e^{i(k_1x_1+k_2x_2)}dh(\boldsymbol{k}), \tag{9}$$

where $h(\boldsymbol{k})$ is a random function of k_1 and k_2. From (9), we have

$$\overline{\phi^2} = \int E(\boldsymbol{k})d\boldsymbol{k},$$

where

$$E(\boldsymbol{k})d\boldsymbol{k} = \overline{dh^*(\boldsymbol{k})dh(\boldsymbol{k})}$$

and the asterisk indicates a complex conjugate. From (9), we have also

$$\overline{v^2} = \overline{\left(\frac{\partial\phi}{\partial x_1}\right)^2} = \int k_1^2E(\boldsymbol{k})d\boldsymbol{k}.$$

Therefore, the one-dimensional spectrum of $\overline{v^2}$ is given by

$$F_v(k_1) = \int_0^\infty k_1^2E(\boldsymbol{k})dk_2.$$

When the stream function ϕ is assumed to be isotropic, $E(\boldsymbol{k})$ is a function of k only, where $k^2 = k_1^2 + k_2^2$, and we have

$$F_v(k_1) = 2k_1^2\int_{k_1}^\infty \frac{kE(k)}{\sqrt{k^2 - k_1^2}}dk. \tag{10}$$

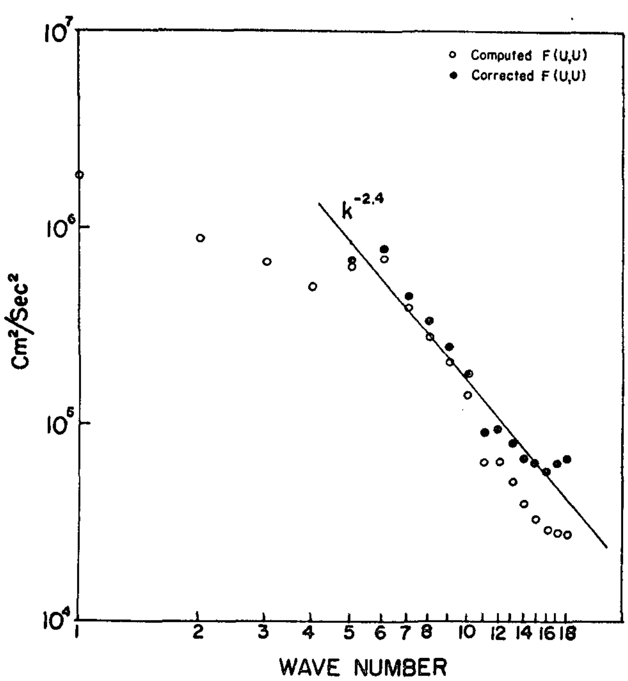

FIG. 3(a). Same as fig. 1(a), but for 20N.

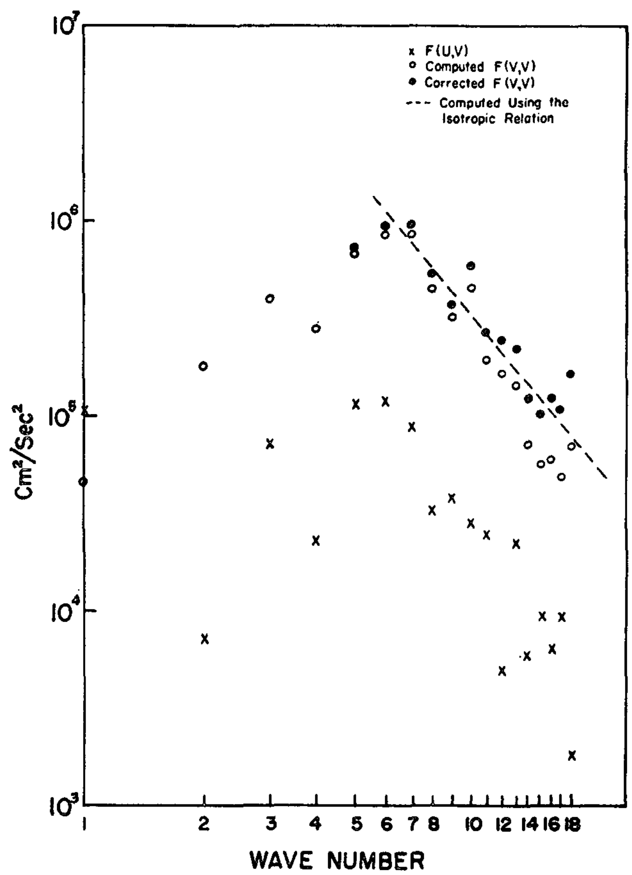

FIG. 3(b). Same as fig. 1(b), but for 20N.

In a finite difference form, v is given by

$$v = \frac{\phi(x_1 + a) - \phi(x_1 - a)}{2a} = \frac{i}{a}\int e^{i(k_1x_1+k_2x_2)} \sin k_1 a dh(\mathbf{k}).$$

By the same procedure as was used above, we have in this case

$$F_v'(k_1) = 2k_1^2\psi(k_1a)\int_{k_1}^{\infty} \frac{kE(k)}{\sqrt{k^2 - k_1^2}} dk, \quad (11)$$

where

$$\psi(\alpha) = \sin^2 \alpha/\alpha^2. \quad (12)$$

The comparison of (11) with (10) shows that the influence function for the spectrum, calculated by the finite difference technique, is expressed by $\psi(k_1a)$.

Unlike the isotropic case, we cannot find the influence function for the non-isotropic turbulence. As we will be shown later, however, the large-scale turbulence can be regarded, to some extent, as isotropic and the spectrum modification due to the use of finite differences seems to be slight for the wave numbers considered here. So we shall employ the influence function (12) as a correction factor for our computed one-dimensional spectra of $\overline{v^2}$ and also of $\overline{u^2}$.

The other difficulty, which we meet in checking the relation (8) experimentally for our case, arises from the fact that the values of computed spectra fluctuate considerably with wave number, and consequently the reasonable value of $dF_u(k_1)/dk_1$ is hard to evaluate. To avoid this difficulty, we shall proceed in the following way.

Since we can hardly expect isotropy for the low-wave-number region, and since observed points of the $\overline{u^2}$-spectrum lie more or less on a straight line for the relatively high wave numbers, we shall assume that this $\overline{u^2}$-spectrum can be interpolated, for the interval between k_a and k_b and for each latitude, by the equation

$$F_u(k_1) = Ak_1^{-n} + B,$$

where

$$A = [F_u(k_a) - F_u(k_b)][k_a^{-n} - k_b^{-n}]^{-1},$$
$$B = [F_u(k_a)k_b^{-n} - F_u(k_b)k_a^{-n}][k_a^{-n} - k_b^{-n}]^{-1},$$

and $F_u(k_a)$ and $F_u(k_b)$ are values of $F_u(k_1)$ at k_a and k_b, respectively. The values of three parameters A, B and n are determined by drawing a straight line so that it fits the observed points as well as possible.

Then, from (8), $F_v(k_1)$ is given by

$$F_v(k_1) = nAk_1^{-n}$$

for the same interval of k_1. The dotted lines on the figures are drawn in this way.

Despite the somewhat arbitrary operation in drawing the interpolation lines for $\overline{u^2}$-spectra, the figures show several interesting features:

(1) For all latitudes and for almost all wave numbers higher than 3 or 4, the magnitude of $F_{uv}(k_1)$ is of one order smaller than that of $F_v(k_1)$. This suggests that the non-isotropy of the turbulence is more predominate in the region of relatively small wave number than in the high-wave-number region.

(2) The agreements between the observed $\overline{v^2}$-spectra and those calculated from $\overline{u^2}$-spectra using the isotropic relation is fairly good for lat 20N and 70N but not for other latitudes. In the region between lat 30N and 60N, inclusive, the values of $F_v(k_1)$ are always higher than those predicted from the isotropic relation. This means that more energy is supplied into the component of the velocity that can be expected in isotropic motion. At the moment, it is not clear how much of this discrepancy could be attributed to the non-zero value of the $\overline{uv}$-spectrum and how much to the baroclinicity of the atmosphere because we have considered here only the one-dimensional spectrum of $\overline{uv}$.

(3) At the same time, it is also to be noted that the values of n for the $\overline{u^2}$-spectrum at each latitude are almost the same as those for the $\overline{v^2}$-spectrum, as is true for the isotropic turbulence.

(4) The value of n increases monotonically with latitude in the region between 30N and 70N.

(5) For all latitudes under consideration, the values of the exponent n are considerably higher than that predicted from the isotropic turbulence theory (*i.e.*, 5/3). As mentioned before, the power spectrum in the inertial range for the pressure fluctuations in an isotropic turbulence is given by $k^{-7/3}$, whereas that for the velocity is expressed by $k^{-5/3}$. Now suppose the fluctuations of the elevation of the 300-mb surface is expanded into elementary waves along the entire latitude circle, and the spectrum for the fluctuating elevation is interpolated by $H(k) \sim k^{-m}$ for a certain range of wave numbers. Then we can imagine from the fact mentioned above that the value of m would be higher than 7/3, the predicted value for the isotropic turbulence.

On the other hand, as mentioned at the beginning of this paper, the value of m seems to be approximately 7/3, for the disturbances at the 500-mb level. This suggests that the value of m increases with height in the troposphere or, in other words, the amplitudes of disturbances with relatively short wavelengths will decrease with height more rapidly, or increase more slowly, than those with long wavelengths do. Of course, this conclusion must of necessity be tentative because the experimental evidence for the value of m at the 300-mb level is not available at present and the value of 500 mb is based on a rather limited amount of data. In this connection, however, it is interesting to note that the analytical solution of the vorticity equation for the baroclinic atmosphere, obtained by Kuo (1953) on the basis of perturbation technique, indicates that the perturbation velocity v for the short waves decreases rapidly with increasing height, while that for the very long waves increases with height.

(6) So far the discussions are limited to the velocity spectra in the winter season. As to the spectra in the summer season, which are also presented by Benton and Kahn (1958) for the period from 1 June to 31 July 1949, it appears the spectra are more or less similar to the winter spectra discussed above. In comparing the spectra for the summer season with those for the winter season at the same latitude, however, there seems to be one remarkable feature: namely, there are little differences in the values of these two spectra for the same wave number in the domain of relatively large wave number, while in the domain of relatively small wave number the values of spectra for the winter season in the middle latitudes are considerably larger than those for the summer season. An example is illustrated in fig. 4. Considering the substantial differences in the basic state of the atmospheric motion in the middle latitudes between the winter and summer seasons, this would suggest that statistical properties of the large-scale turbulence in the domain of wave number larger than about 5 seem to be little influenced by the distribution of the wind velocity or temperature in the mean flow. On the other hand, disturbances in the domain of wave number less than 4 would be primarily associated with the fixed topographic and oceanographic features of the earth's surface, as was pointed out previously by Kubota and Iida (1954) and White and Cooley (1956).

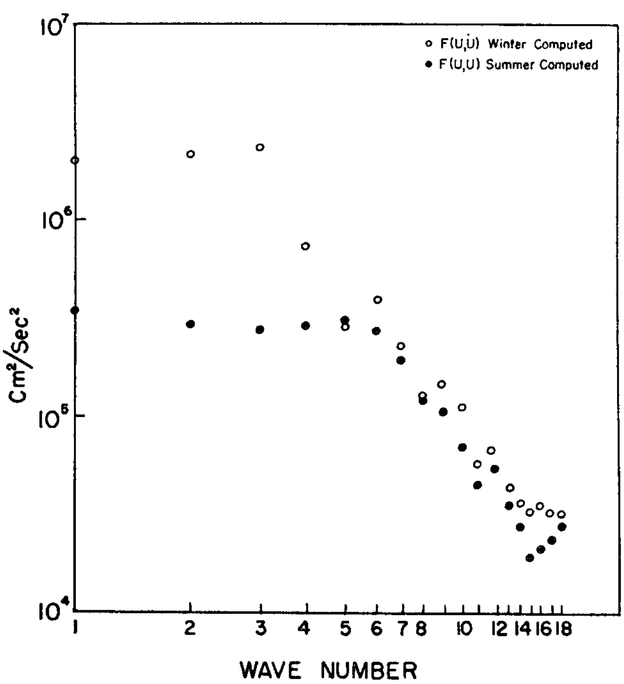

Fig. 4. Open and solid dots denote average values of $\overline{u^2}$-spectra at 40N for the 300-mb level in the winter and summer seasons, respectively.

In connection with this, Benton and Kahn (1958) made another analysis of the velocity spectra. They computed the power spectra for the velocity average over the whole winter and summer season. Then the differences between the average of spectra, computed on the day-by-day bases, and the spectra for the mean motion for each season and for each latitude circle were obtained. These differences represent the mean spectra for disturbances changing with respect to time and space. One of the results thus revealed is that the spectra for the mean motion are rather restricted to the domain of low wave numbers. In other words, the shapes of spectra for the domain of high wave numbers —which are the main concern in this article—are almost similar for time-space-eddy and total motion.

4. Conclusion

One of the main problems in the statistical studies on the large-scale atmospheric turbulence would be to determine experimentally values of many important statistical parameters which characterize the turbulent flow and, at the same time, to predict theoretically those values on the basis of a set of more fundamental equations (such as the equations of motion, the thermodynamical energy equation, and the equation of state).

In this paper, some preliminary studies are made on the interrelations between the power- and co-spectra of velocity. In view of the fact that the turbulent field occurring naturally in the atmosphere is influenced by many more factors than those encountered in aerodynamics laboratories, it is obvious that much further work will be necessary before the exact nature of the atmospheric turbulence can be made clear.

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