A STUDY OF THE LARGE-SCALE SPECTRA OF SOME METEOROLOGICAL PARAMETERS

By ***Robert M. Henry***[1] ***and Seymour L. Hess***

Florida State University

(Original manuscript received 27 November 1957; revised manuscript received 25 April 1958)

ABSTRACT

Methods are developed for determining the contribution by each harmonic wave component to the geostrophic kinetic energy of meridional and zonal motion, and to the geostrophic fluxes of angular momentum and enthalpy. Evaluations are carried out for each day of January 1951 for selected latitudes and pressure surfaces and the resulting spectral distributions discussed. Salient features are large spatial and temporal variations of the energy spectra and unexpectedly large contributions by the lower wave numbers to the fluxes of momentum and heat.

1. Introduction

In recent years, increased emphasis has been placed on the study of wave motions in the atmosphere. One way of rigorously defining atmospheric waves is by means of harmonic analysis of the height of a pressure surface. Such an analysis around a latitude circle yields a set of parameters which defines the component waves at a particular latitude and pressure in a manner which is free from ambiguity.

If one makes the geostrophic assumption, it is possible to compute the contribution of each such harmonic to the kinetic energy of meridional and of zonal motion and the contribution to the poleward flux of angular momentum. If the hydrostatic assumption is also made, the contribution to the poleward flux of sensible heat may be computed.

These values indicate the roles played by various scales of motion in the atmosphere, especially their part in the general circulation. In addition, they may be used in evaluating theoretical studies, such as studies of wave instability.

2. Method of computation

In this section the details of computational procedure are discussed.

Harmonic analysis of height field.—The first step in the computation of the various spectra consists of harmonic analyses of the variation with longitude of the geopotential height of standard pressure surfaces around certain latitude circles.

It can be shown (see, *e.g.*, Kaplan 1953) that any single-valued function having period 2π and having a piecewise continuous first derivative may be represented by a Fourier series. Thus, the observed height of a standard pressure surface around a latitude circle may always be represented precisely by such an infinite series and to any desired degree of accuracy by a sufficient number of terms. The decision as to the number of terms required is made largely on the basis of experience. Syōno, Kasahara, and Sekiguti (1955) evaluated the first eighteen harmonics of 500-mb height for lat 20N to 60N for 20 days in January 1949. In all cases the amplitude decreased rapidly with increasing wave numbers for wave numbers above eight. As a result, it seemed reasonable to assume that disturbances having a wave length of 30 degrees longitude or less do not contribute greatly. Therefore, in the present study only the first eleven harmonics have been considered.

Kinetic energy of northward motion.—Harmonic analysis of geopotential height yields a series of the form

$$\frac{1}{g}\psi(\varphi, \lambda, p) = \tfrac{1}{2}a_0(\varphi, p) + \sum_{k=1}^{n} a_k(\varphi, p)\cos k\lambda + b_k(\varphi, p)\sin k\lambda \quad (1)$$

where ψ is the geopotential, ϕ is latitude, λ is longitude, p is pressure, g is the acceleration of gravity, the a_k and b_k are the Fourier coefficients, and n is number of harmonics to be evaluated. When we introduce the geostrophic assumption for v, the S-N speed, and utilize the orthogonality property of Fourier series we obtain

$$\overline{v^2} = \frac{1}{2(fE\cos\varphi)^2}\sum_{k=1}^{n} k^2[a_k(\varphi, p)]^2 + k^2[b_k(\phi, p)]^2 \quad (2)$$

which is twice the average value of the kinetic energy per unit mass contributed by the northward component of geostrophic motion. The square of the velocity component will be used throughout this report as a measure of the kinetic energy per unit mass. The value of $v^2 = 1\ m^2\ \text{sec}^{-2}$ corresponds to 5×10^3 erg gm^{-1}.

Kinetic energy of eastward motion.—The eastward

---

[1] Present affiliation: National Advisory Committee for Aeronautics.

component of geostrophic motion may be approximated by a finite difference expression,

$$u = -\frac{1}{f}\frac{\partial\psi}{\partial y} \approx \frac{1}{fE}\,\frac{\psi_1 - \psi_2}{\varphi_2 - \varphi_1}, \tag{3}$$

where f is the Coriolis parameter, y is distance northward, and E is the radius of the earth. Substitution of the series expression for ψ given by equation (1) into the above equation and utilization of the orthogonality property gives

$$\overline{u^2} = \frac{1}{2(fE\Delta\varphi)^2}\Big\{\tfrac{1}{4}[a_0(\varphi_1, p) - a_0(\varphi_2, p)]^2 + \sum_{k=1}^{n}[a_k(\varphi_1, p) - a_k(\varphi_2, p)]^2 + [b_k(\varphi_1, p) - b_k(\varphi_2, p)]^2\Big\}. \tag{4}$$

This is twice the average value of the kinetic energy per unit mass contributed by the eastward component of geostrophic motion. The values of ϕ_1 and ϕ_2 may be chosen equidistant north and south of the latitude for which the evaluation is desired to form a centered difference, or one of them may be chosen equal to it for a forward difference approximation which will generally be less accurate but simpler to compute.

Momentum flux.—Lorenz (1951) has shown that the geostrophic poleward transport per unit time (flux) of absolute angular momentum per unit pressure across a circle of latitude is

$$J_m = -\tfrac{1}{2}\pi g^{-1}\omega^{-2}\cos\varphi\,\sin^{-2}\varphi\,\overline{\frac{\partial\psi}{\partial\lambda}\frac{\partial\psi}{\partial\varphi}}, \tag{5}$$

where

$$\overline{\frac{\partial\psi}{\partial\lambda}\frac{\partial\psi}{\partial\varphi}} = \frac{1}{2\pi}\int_0^{2\pi}\frac{\partial\psi}{\partial\lambda}\frac{\partial\psi}{\partial\varphi}\,d\lambda. \tag{6}$$

When we differentiate with respect to λ the Fourier series for ψ at ϕ_1 and ϕ_2 and use the finite-difference approximation for $\partial\psi/\partial\lambda$, we find from (5) and (6):

$$J_m = \tfrac{1}{4}\pi g\omega^{-2}(\Delta\varphi)^{-1}\cos\varphi\,\sin^{-2}\varphi \times \sum_{k=1}^{n} k[a_k(\varphi_1, p)b_k(\varphi_2, p) - a_k(\varphi_2, p)b_k(\varphi_1, p)]. \tag{7}$$

Flux of enthalpy.—It was shown by Lorenz (1951) that the poleward flux of sensible heat, or enthalpy, per unit pressure is given by

$$J_h = -\pi\omega^{-1}g^{-1}c_pR^{-1}p\,\sin^{-1}\varphi\,\overline{\frac{\partial\psi}{\partial\lambda}\frac{\partial\psi}{\partial p}}, \tag{8}$$

where

$$\overline{\frac{\partial\psi}{\partial\lambda}\frac{\partial\psi}{\partial p}} = \frac{1}{2\pi}\int_0^{2\pi}\frac{\partial\psi}{\partial\lambda}\frac{\partial\psi}{\partial p}\,d\lambda. \tag{9}$$

Substitution of the appropriate derivatives of the Fourier series for ψ yields the average flux of enthalpy per millibar between p_1 and p_2:

$$J_h = \tfrac{1}{4}\pi\omega^{-1}gc_pR^{-1}\frac{p_2 + p_1}{p_2 - p_1}\sin^{-1}\varphi \times \sum_{k=1}^{n} k[a_k(\varphi, p_1)b_k(\varphi, p_2) - a_k(\varphi, p_2)b_k(\varphi, p_1)]. \tag{10}$$

3. Computed values for January 1951

Contour analysis.—Values were computed for 0300 GCT heights of the 700-mb surface and 1500 GCT heights of the 500-mb surface at selected latitudes for each day of January 1951. Data were interpolated every 15 degrees of longitude. Post-analyzed 700-mb charts based on historical data in the Northern Hemisphere Map Series were supplemented with northern hemisphere maps which were kindly loaned by J. Namias of the U. S. Weather Bureau, Extended Forecast Section. Five-hundred-millibar data were taken from the Northern Hemisphere Historical Maps.

Lat 55N was chosen because more radiosonde data are available near this latitude than any other. The 700-mb surface was chosen as the upper-air surface with the largest amount of data and the smallest errors. The lat 45N and pressure of 500 mb were included so that comparisons could be made with the many theoretical and empirical studies which have concentrated on this region. However, the elevations chosen have previously been found to have relatively small values of angular momentum and enthalpy fluxes.

Twenty-four term harmonic analyses were performed by IBM punched-card machines using normal equations (see, *e.g.*, pp. 264–267, Whitaker and Robinson, 1926) and were checked using the procedures suggested by Whitaker and Robinson (1926), which are essentially partial syntheses of the series. As a further check, a few of the analyses were also performed independently using a Rünge Schedule and a desk calculator. Subsequent calculations were also performed by punched-card machines, using the equations given in the preceding sections.

Kinetic energy of meridional motion.—Figs. 1, 2, and 3 show the daily spectra of the squares of the meridional speeds, and fig. 5 gives their monthly average values. It will be seen that in each case there is a very large interdiurnal variation in the magnitude of the individual harmonics and also in the sum of the harmonics (see table 1). The values at 700 mb and at 500 mb at the same latitude are closely related, both in their daily distributions and in their monthly

FIG. 1. Daily values of kinetic energy of meridional motion of the first 11 harmonics at 700 mb and 55N, v^2 in units of m^2 sec^{-2}.

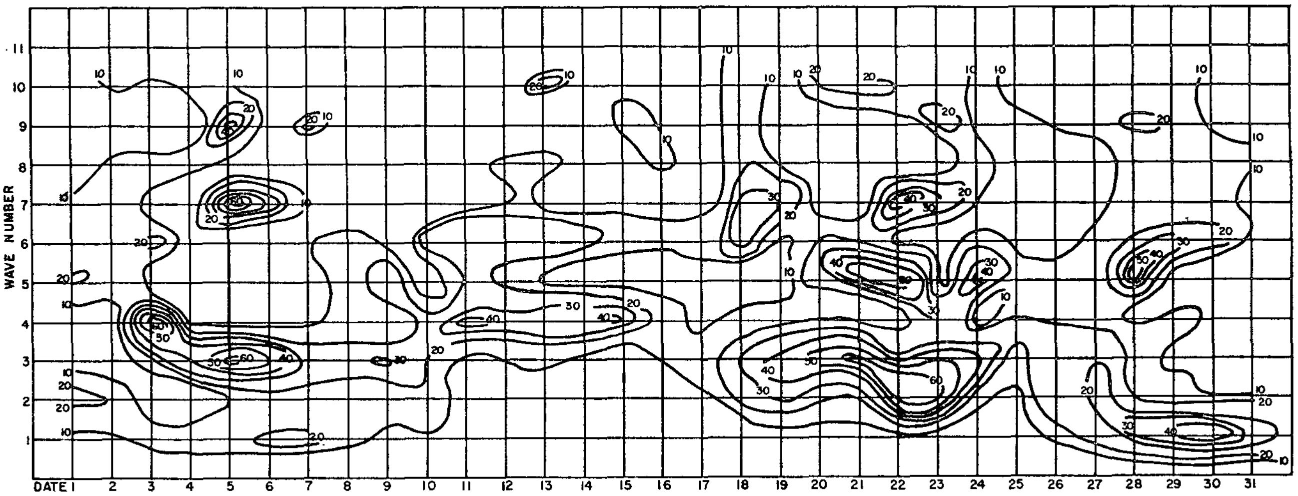

FIG. 2. Daily values of kinetic energy of meridional motion of the first 11 harmonics at 500 mb and 55N, v^2 in units of m^2 sec^{-2}.

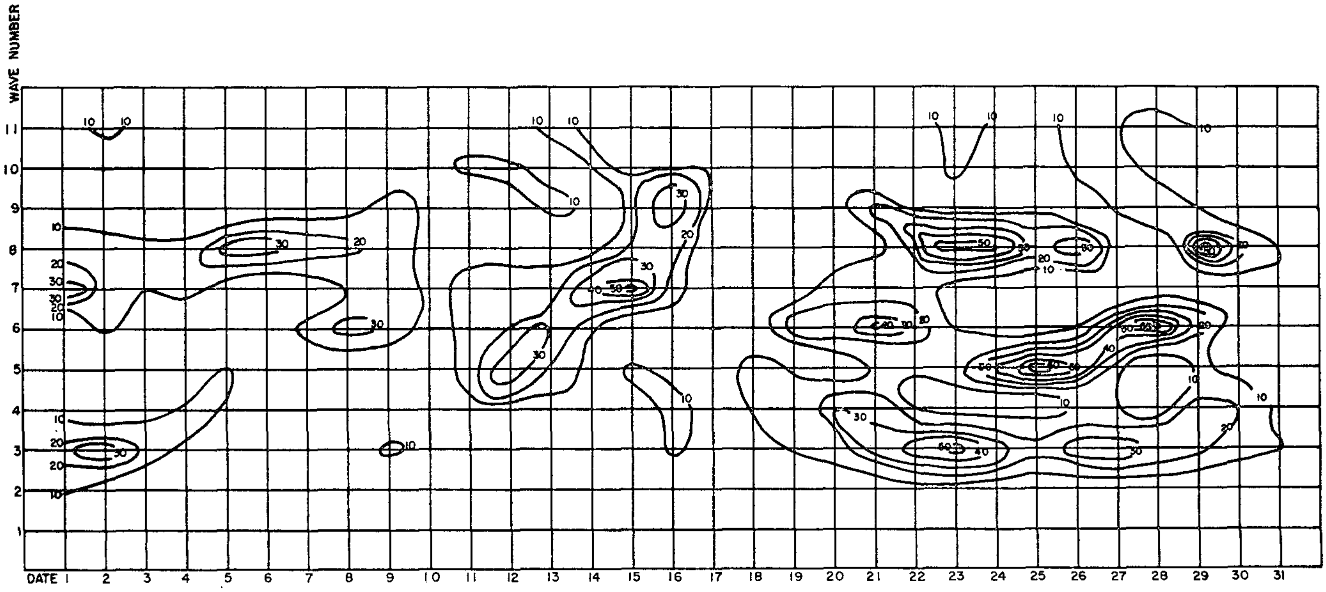

FIG. 3. Daily values of kinetic energy of meridional motion of the first 11 harmonics at 500 mb and 45N, v^2 in units of m^2 sec^{-2}.

means, with the 700-mb values averaging about 0.6 of the values at 500 mb.

In contrast, the distributions of the monthly means at the same pressure but different latitudes are strikingly different: at latitude 55N a single maximum appears at wave number four, whereas a minimum is found at this wave number at 45N. Maxima at 45N appear at wave numbers three and eight. Surprisingly, it is the 55N values rather than those for 45N which resemble most closely the means found by White and Cooley (1956) for latitude 45N for the winter months of 1949–1950.

The monthly mean values differ markedly from the spectrum presented by Charney (1951) although individual days and short periods may be found which resemble Charney's. Syōno's means of the harmonics of contour-height deviations for January of 1949 (Syōno *et al*, 1955) and his values for January 1946 (Syōno *et al*, 1950; Syōno and Gambo, 1952) and the averages of the harmonics of northward displacement of a particular contour given by Graham (1955)

TABLE 1. Values of the mean zonal current, the total geostrophic eddy energy of zonal and meridional motion, and the total geostrophic eddy flux of angular momentum and sensible heat, for each day of January 1951 at 55N and 700 mb. The momentum flux is an average for 50N to 55N, and the heat flux an average for 500 mb to 700 mb.

Date	$\bar{u}$ msec^{-1}	$\sum_{k=1}^{n} u_k^2$ m^{2}sec^{-2}	$\sum_{k=1}^{n} v_k^2$ m^{2}sec^{-2}	J_m 10^{21} g cm^2 mb^{-1}	J_h 10^{10} cal mb^{-1} sec^{-1}
1	8	60	62	− 27	6
2	6	44	53	− 73	66
3	6	95	59	− 94	71
4	9	99	58	−170	187
5	6	65	57	−116	96
6	6	51	65	− 15	194
7	8	92	61	− 36	101
8	9	47	55	−190	156
9	10	48	62	+177	71
10	10	70	60	−188	142
11	11	102	51	−134	120
12	12	165	90	−169	71
13	10	62	86	− 20	93
14	9	38	95	+ 6	30
15	10	67	58	−101	80
16	10	91	87	+109	114
17	11	55	52	−106	124
18	10	71	68	− 14	89
19	6	50	87	+ 77	97
20	6	44	133	+ 72	51
21	4	127	170	+227	91
22	8	67	134	+205	113
23	13	148	168	−265	175
24	5	96	129	−168	69
25	7	68	84	− 25	106
26	7	71	42	− 29	89
27	4	95	62	+302	11
28	3	152	126	− 93	133
29	4	128	93	− 81	105
30	5	96	85	+ 87	139
31	5	93	67	− 12	147

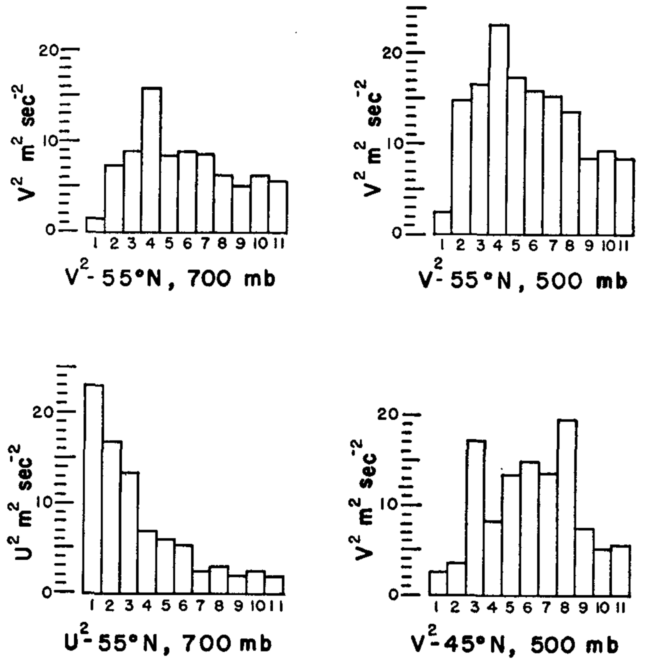

FIG. 5. Monthly average spectra of kinetic energy.

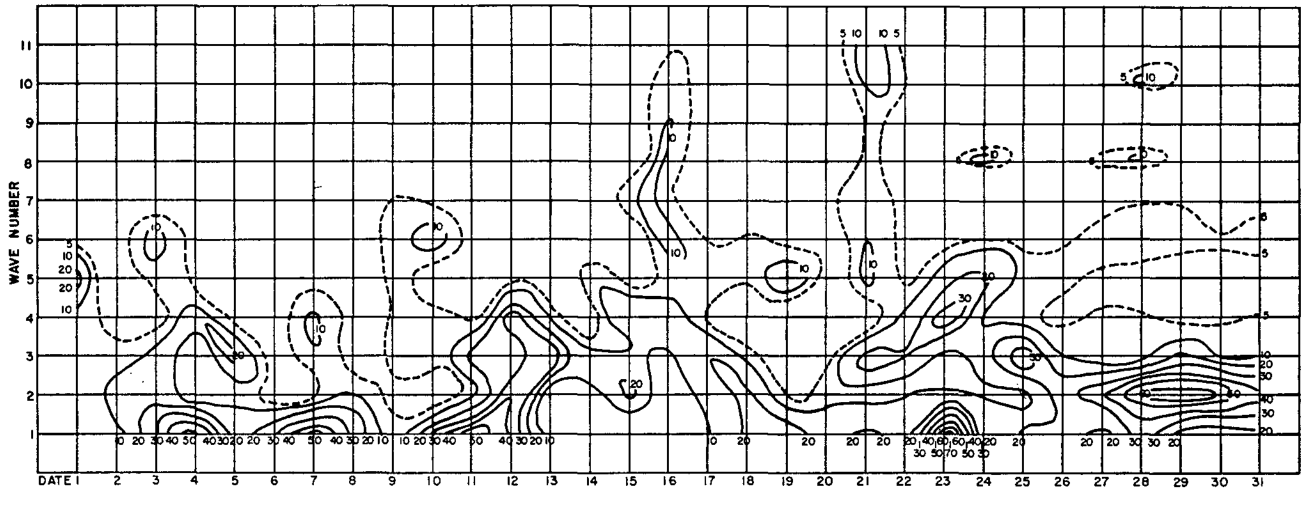

FIG. 4. Daily values of kinetic energy of zonal motion of the first 11 harmonics at 700 mb and 55N, v^2 in units of m^2 sec^{-2}.

cannot be compared directly with the means of kinetic energy, but qualitatively they appear to be in general agreement with the values in this study and those of White and Cooley and not in agreement with those of Charney.

The daily maxima and minima do not seem to favor strongly any particular wave number, nor does there appear to be any fixed number of maxima. There are periods where there appears to be a natural division between "long" and "short" waves, but for the month as a whole such a division would be quite arbitrary.

At the outset of the study it was thought that a systematic transfer of kinetic energy from low to high wave number might be found, such as has been reported for turbulent eddies of a vastly smaller scale, or that perhaps the direction of transfer between harmonics might be reversed for waves of planetary scale, with systematic transfer from high to low wave number and thence to the mean current. Such a systematic transfer would be evidenced in figs. 1 to 4 by a sloping axis of the regions of maximum and minimum, up and to the right for transfer from low to high wave number, and down to the right for high to low. Examination of the figures does not reveal a marked tendency in either direction, although in figs. 1 and 3 there is a suggestion of low to high wave number transfer. This suggests that more complex processes than simple transfer from one wave number to another are at work.

The sums of the values of v^2 contributed by the various harmonics may be interpreted as a sort of meridional "index." It differs from the meridional index defined by Willett (1948) in that the square of the meridional velocity component replaces the absolute value of this component. Daily values of this sum are given in table 1.

Kinetic energy of zonal motion.—The distribution of kinetic energy of zonal motion, as shown in figs. 4 and 5, differs markedly from that of meridional motion, with the largest value at wave number one and a rapid decrease with wave number. This may be related to the fact that, for geostrophic wind, the energy of meridional motion of a harmonic as given by equation [2] is proportional to the square of the wave number, while the wave number does not appear in equation (4) for the energy of zonal motion.

It is interesting to note that despite the great difference in the spectral distribution of the energies of meridional and of zonal motion the average total daily perturbation energy of zonal motion was 83 $m^2\ sec^{-2}$, identical with the average value of kinetic energy of meridional motion, although there were considerable variations between individual daily values of the two quantities.

As with the energy of meridional motion, there are large interdiurnal variations both in the individual harmonics and in the total perturbation energy. Wave number one has the maximum energy for slightly less than half the month. The rapid drop of energy with increasing wave number, of course, precludes frequent occurrence of multiple maxima.

An interesting feature of the time variation of both zonal and meridional energies is the occurrence of simultaneous minima at virtually all numbers on several occasions during the month. These are to be found on the second, ninth, seventeenth, and twenty-sixth of the month, and there are indications of another occurrence on the last day of the month. Between these periods a more or less homogeneous wave-number state exists, with a somewhat irregular growth and decay in the harmonics which are predominant during the period. This suggests that the state of the atmosphere at the time of the kinetic

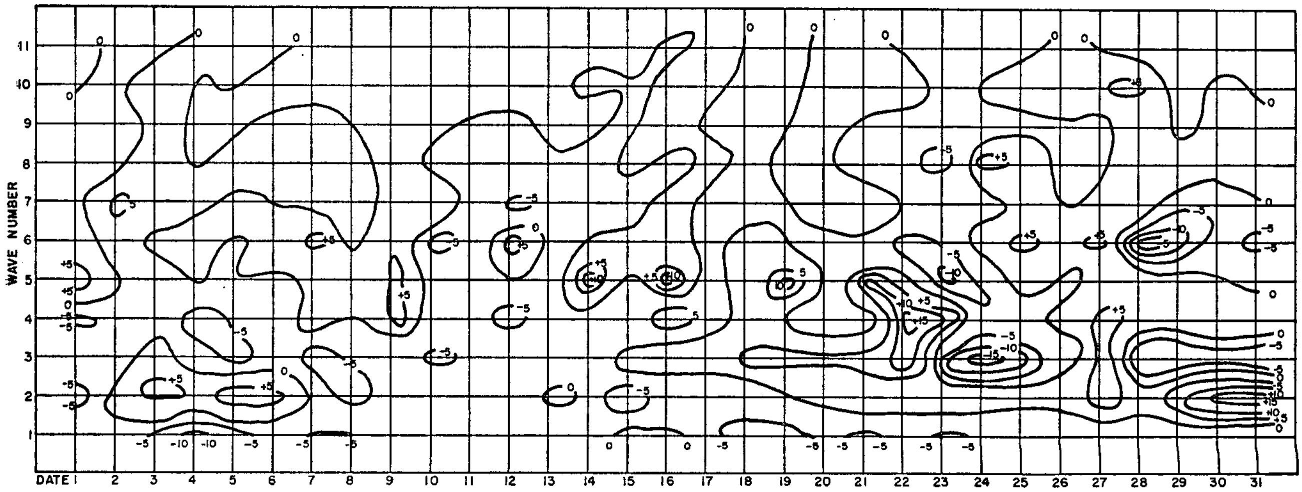

Fig. 6. Daily values of the contribution of poleward flux of angular momentum by the first 11 harmonics in units of 10^{22} gm $cm^2\ sec^{-1}$/mb sec.

energy minimum may largely represent the wave-number state for several days following.

Flux of angular momentum.—The daily values of the contribution of each harmonic to the northward flux of angular momentum across a vertical strip one millibar in height extending around the latitude circle at 700 mb are given in fig. 6. These figures represent an average value between 50N and 55N. Here, again, large interdiurnal variations are to be seen with frequent changes in the direction of flux at all wave numbers.

The magnitudes of the contributions of the short waves are not only small in their mean values, as shown in fig. 8, but all of the daily contributions of these high wave numbers are likewise small, the major contribution to angular momentum flux, at this latitude at least, coming from the very long, or planetary, waves.

While no figures are available for individual harmonics for other years, the total geostrophic eddy flux of angular momentum has been computed by several authors. The average total value of -0.3×10^{23} g cm^2 sec^{-1}/sec mb for January 1951 compares to -6.0×10^{23} computed by Mintz (1951) for January 1949, -0.1×10^{23} for January 1946 found by Widger (1949), and -0.2×10^{23} given by Lorenz (1951) for the winter of 1945–1946. The interannual variation of individual harmonics certainly seems to be at least as large percentagewise as that of the total. The relative variability might be smaller, however, at lower latitudes where mean values two orders of magnitude greater are found. The spectral distribution, also, might be quite different at other latitudes.

Flux of sensible heat (*enthalpy*).—In order to compute heat flux from the non-simultaneous data at 500 mb and 700 mb, some adjustment must be made in one of the sets of data. The procedure adopted was to average the Fourier coefficients of the geopotential-height series for 500 mb for 12 hr, preceding and following the time of the 700-mb observations. This has the same effect as averaging the 500-mb height at each point on the latitude circle and then performing the Fourier analysis. It is thought that this procedure should yield an adequate approximation to the 0300 GCT 500-mb-height profile, although there will be some reduction in the amplitudes of the faster moving waves.

The day-by-day values of the northward flux of enthalpy, shown in fig. 7, exhibit considerable interdiurnal variation, with the exception of wave number two where persistence is quite strong, with a marked

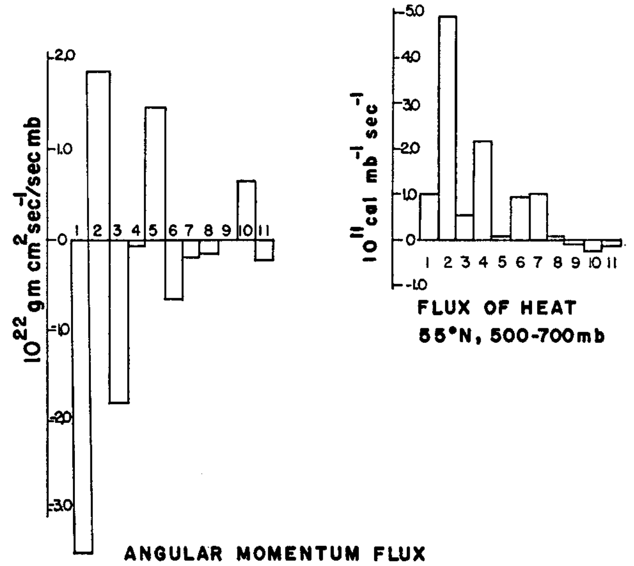

FIG. 8. Monthly average spectra of angular momentum flux and heat flux.

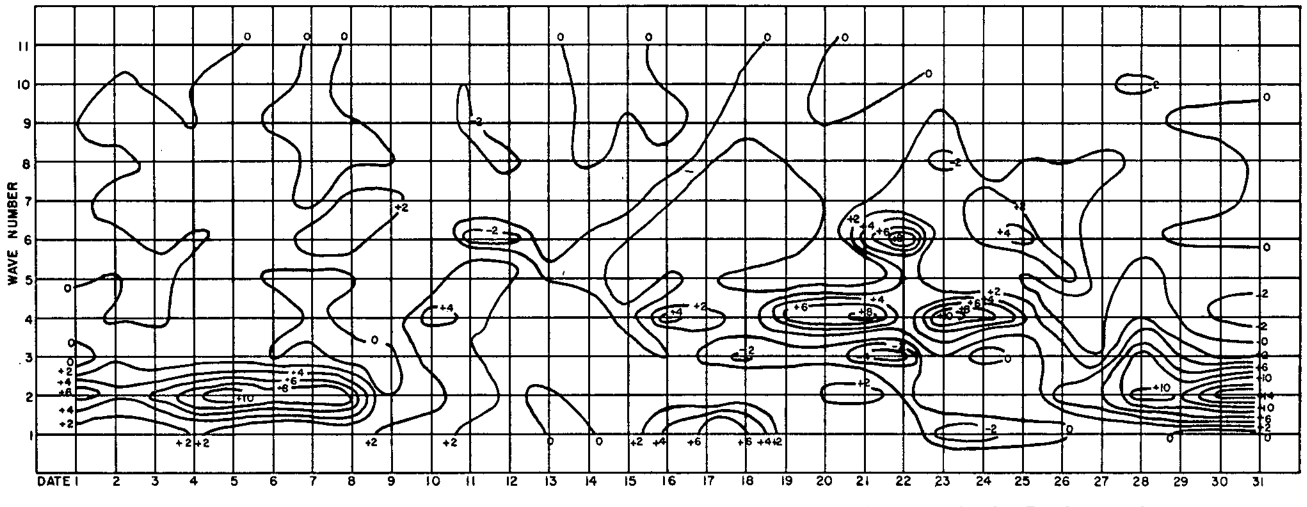

FIG. 7. Daily values of the contribution to the poleward flux of sensible heat by the first 11 harmonics in units of 10^{11} cal sec^{-1} mb^{-1}.

positive maximum at this wave number for nearly half the month, and positive values for all but two days of the month. This is reflected in the monthly averages (fig. 8) by a large maximum value at wave number two. Consideration was given to the possibility that wave number two might be influenced by inclusion of the semidiurnal pressure wave to which the geostrophic assumption should not apply. A computation using data given by Haurwitz (1947) for the Caribbean, where its effect might be expected to be larger than at 50N to 55N, yielded a value for the semidiurnal pressure wave two orders of magnitude smaller than the mean value for wave number two. Therefore, this possibility must be rejected.

In view of the small magnitudes of heat flux in the short waves, no significance is attached to their negative sign. The total geostrophic eddy flux of heat agrees closely with that reported for January 1946 and for the winter season of 1945–1946 by Lorenz (1951). There were no days during the month having a negative value of the total geostrophic eddy flux of enthalpy.

4. Summary and conclusions

The methods of computation derived in section 2 appear to yield valid results when applied to existing map data. Confidence in this validity is strengthened by the close similarity between 700-mb and 500-mb values even though they are based on completely independent observations and analyses, one for 0300 GGT and the other for 1500 GCT.

The spectral distribution of kinetic energy of meridional motion was found to differ only slightly in the vertical but to vary greatly with time and significantly with latitude. Comparison to the results of others indicates that monthly means vary significantly from year to year. One or more maxima appear between wave number two and eight. Values in all published studies drop off at wave number one and at wave numbers above eight.

The distribution of the kinetic energy of zonal motion was vastly different from that of meridional motion, with the maximum in the very long waves.

At the latitude investigated, fluxes of angular momentum and sensible heat were accomplished principally by the very long or planetary waves, with a distinctly minor contribution from the short waves. This is a very surprising result. If true at other latitudes and for other months, then the role played by the very long waves in maintaining the heat and momentum balance of the general circulation may be more important than was previously believed.

The large interdiurnal variability of values associated with individual wave numbers means that certain features of the monthly mean spectra are not statistically significant. Perhaps the best resolution of this uncertainty can be obtained by repeating these analyses for a period longer than one month.

All studies of these quantities published to date have concentrated on the winter months, especially the month of January. It would be desirable to extend the scope to other seasons as well as to other latitudes. One item of particular interest in an extension to additional latitudes would be evaluation of convergence of flux of angular momentum in a particular harmonic and its relation to the growth, decay, and movement of the corresponding harmonic of the height field.

Acknowledgment.—Thanks are due to Mr. Frank Luppino of the Florida State University Computing Center for his assistance in the numerous IBM computations involved.

REFERENCES

Charney. J. G., 1951: *Compendium meteor.* Boston, Amer. meteor. Soc., 470–482.

Graham, R. D., 1955: An empirical study of planetary waves by means of harmonic analysis. *J. Meteor.*, **12**, 298–307.

Haurwitz, B , 1947: Harmonic analysis of the diurnal variations of pressure and temperature aloft in the eastern Caribbean. *Bull. Amer. meteor. Soc.*, **28**, 319–323.

Kaplan, W., 1953: *Advanced calculus.* Cambridge, Mass., Addison-Wesley, 679 pp.

LaSeur, N. E., 1954: On the asymmetry of the middle-latitude circumpolar current. *J. Meteor.*, **11**, 43–57.

Lorenz, E. N., 1951: *Computations of the balance of angular momentum and the poleward transport of heat.* Rep. No. **6**, Gnrl. Circ. Proj. No. AF19(122)-153, Mass. Inst. Tech., 38 pp.

Mintz, Y., 1951: The geostrophic poleward flux of angular momentum in the month of January, 1949. *Tellus,* **3**, 195–200.

Syōno, S., and K. Gambo, 1952: On numerical prediction II, III. *J. meteor. Soc. Jap.*, Second Ser., **30**, 264–271, 273–280.

——, ——, Y. Sasaki, and A. Koide, 1950: On numerical prediction (1). *J. meteor. Soc. Jap.*, Second Ser., **28**, 1–23 or 77–79.

Syōno, S., A. Kasahara, and Y. Sekiguti, 1955: Some statistical properties of the atmospheric disturbance on 500 mb level. *J. meteor. Soc. Jap.*, **33**, 23–30.

White, R. M., and D. S. Cooley, 1956: Kinetic-energy spectrum of meridional motion in the mid-troposphere. *J. Meteor.*, **13**, 67–69.

Whittaker, E. T., and G. Robinson, 1926: *The calculus of observations.* London, Blackie, 395 pp.

Widger, W. K., 1949: A study of the flow of angular momentum in the atmosphere. *J. Meteor.*, **6**, 291–299.

Willett, H. C., 1948: Patterns of world weather changes. *Trans. Amer. geophys. Union*, **29**, 803–809.