AN EVALUATION OF 12-HR STATISTICAL FORECASTS OF THE 1000- TO 500-MB THICKNESS

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ABSTRACT

The usefulness of a number of variables as predictors of the 1000–500 mb thickness is explored. These predictors are utilized, in several combinations, to form multiple regression equations from which four series of 12-hour thickness forecasts are prepared. Verification scores are presented and compared with the scores of 12-hr numerical thickness predictions for the same period.

1. Introduction

Apparently a purely statistical approach to the problem of thickness forecasting has not been tried, although methods involving multiple linear regression have been applied to the prediction of 500-mb heights by White and Palson [1] and to forecasts of sea-level pressures by Malone [2].

The object of the present investigation was to examine simple objective techniques of forecasting the 1000–500 mb thickness. From a number of variables suggested either by physical or empirical considerations, several were selected as predictors to be used in making 12-hr predictions of the thickness. These variables were utilized in several different combinations to form regression equations from which four series of forecasts were prepared on independent data. The following sections describe briefly the development of these equations and present the results of the forecast verifications. The verification scores are compared with the scores of 12-hr thickness predictions, based on a baroclinic model of the atmosphere, produced by the Joint Numerical Weather Prediction Unit at Suitland, Maryland.

2. Forecast equations and verification scores

Physical and empirical considerations.—The theory of thickness changes has been discussed in considerable detail by Sutcliffe [3]. If non-adiabatic temperature changes are neglected, it can be shown [3] that the 1000–500 mb thickness tendency is given approximately by the expression

$$\frac{\partial h}{\partial t} = -\frac{R \ln 2}{g} \overline{\bar{w}(\Gamma - \gamma)} - \bar{V} \cdot \nabla h, \qquad (1)$$

where h is the 1000–500 mb thickness, R is the gas constant for dry air, g is the acceleration of gravity, w is the vertical velocity, Γ and γ are the adiabatic and actual lapse rates, respectively, and V is the horizontal wind velocity. The bars refer to mean values in the 1000–500 mb layers. If it is assumed that the thickness tendency gives some information about the 24-hr thickness change, equation (1) suggests an empirical relation of the form

$$(\Delta_t h)_{24\text{-hr}} = b_1 \bar{w} \overline{(\Gamma - \gamma)} + b_2 \bar{V} \cdot \nabla h. \qquad (2)$$

Miller and others [4] found that the vertical velocity at 700 mb is rather consistently correlated with the meridional component of the velocity. If this relationship holds throughout the lower troposphere, a further approximation to the thickness change can be written in regression form as

$$(\Delta_t h)_{24\text{-hr}} = b_1 \left(\frac{v_{10} + 2v_7 + v_5}{4} \right) \overline{(\Gamma - \gamma)} + b_2 \bar{V} \cdot \nabla h + c. \qquad (3)$$

In the above equation, v is the component of the wind directed toward the north, and the subscripts 10, 7, and 5 refer to the 1000 mb, 700 mb, and 500 mb surfaces, respectively.

The regression coefficients and constants for equations of the form of (3), as well as the squares of the multiple correlation coefficients, were computed at eight stations located in the eastern half of the United States, for the periods November–December 1955, and May 1956 (table 1). The geostrophic wind was used to approximate the actual wind, and the temperature lapse rate was approximated by the difference in thicknesses of the 700–500 mb and 1000–700 mb layers. The 24-hr changes were centered at the 1500 GCT observations.

Although the coefficients of table 1 show that the thickness change can be estimated with some degree of skill by means of equation (3), the correlations are not sufficiently high nor consistent to suggest that the equation can be used in itself as a practical means of forecasting. The relative importance of the several terms—representing the meridional component of velocity, the stability, and the thickness advection—in estimating the thickness change and in predicting the change from independent data, was investigated in some detail [5]. The first term on the right of equation (3) contributes substantially both to estimates and to 12-hr predictions of the thickness change based on advection alone, while the stability accounts for a small but consistent degree of skill in estimating the thickness change. However, correlation coefficients averaging 0.75 between the observed and forecast thickness for 0300 GCT during January–February 1956, and June 1956, and root-mean-square errors somewhat larger than 200 ft, confirmed the impression gained from table 1 that equation (3) cannot be used alone as a reliable means of predicting the thickness even for as short a period as 12 hr.

Statistical considerations.—While it may be said that the approach used to develop equation (2) is basically physical in concept, it is evident that the equation incorporates an idea fundamentally statistical in nature. Thus the instantaneous change of thickness is assumed to persist for 12 hr before and 12 hr after the time for which the tendency is supposed to be valid. This idea of the persistence of the observed thickness trend leads rather naturally to the use of an auto-regressive scheme for predicting the 1000- 500-mb thickness. Because in middle latitudes the more important fluctuations in thickness are of the order of several days, an observed change over a 12-hr period more often than not gives some information about the 12-hr change to follow. In other words, the auto-correlation in the sequence of thickness changes at a point may be used to advantage in

TABLE 1. Regression coefficients and constants for equations of the type

$$(\Delta_t h)_{24\text{-hr}} = b_1\left(\frac{v_{10} + 2v_7 + v_5}{4}\right)\overline{(\Gamma - \gamma)} + b_2\bar{V}\cdot\nabla h + c,$$

when v is expressed in knots, $\overline{(\Gamma - \gamma)}$ is expressed in deg C km^{-1}, and c is in 10's of ft. $R^2_{3.12}$ is the square of the multiple correlation coefficient.

Verifying station	Sample data Nov–Dec 1955 b_1	b_2	c	$R^2_{3.12}$	Sample data May 1956 b_1	b_2	c	$R^2_{3.12}$
Bismarck	−1.23	−0.59	−9.0	0.516	−2.38	−0.70	−3.8	0.557
Green Bay	−1.96	−0.48	−4.8	0.782	−2.37	−0.54	−4.6	0.697
Mitchell AFB	−1.79	−0.53	4.7	0.638	−1.52	−0.65	2.0	0.761
Dodge City	−2.20	−0.56	−2.0	0.485	−4.58	−0.86	3.0	0.462
Nashville	−2.42	−0.44	2.7	0.593	−0.90	−0.68	−2.0	0.540
Charleston	−1.20	−0.28	2.4	0.240	−0.32	−0.60	2.4	0.511
Midland	−2.03	−0.45	−1.0	0.241	−0.86	−0.45	0.0	0.050
Shreveport	−1.64	−0.47	0.5	0.487	−0.98	−0.28	1.8	0.237

TABLE 2. Regression coefficients and constants for equations of the type $h_{+12} = b_1h_0 + b_2h_{-12} + c$, and squares of multiple correlation coefficients ($R^2_{3.12}$). c is in 10's of feet.

Verifying station	Sample data, Nov–Dec 1955 b_1	b_2	c	$R^2_{3.12}$	Sample data, May 1956 b_1	b_2	c	$R^2_{3.12}$
Bismarck	1.328	−0.476	271	0.813	1.273	−0.326	114	0.817
Green Bay	1.618	−0.820	345	0.693	1.408	−0.558	270	0.710
Mitchell AFB	1.185	−0.278	178	0.847	1.075	−0.369	548	0.825
Dodge City	1.741	−0.891	292	0.759	2.121	−0.921	−352	0.818
Nashville	1.596	−0.574	−36	0.873	1.121	−0.254	249	0.641
Charleston	0.979	−0.253	512	0.730	1.341	−0.429	175	0.831
Midland	1.607	−0.614	27	0.747	1.042	−0.020	−28	0.624
Shreveport	1.274	−0.396	233	0.846	0.959	−0.084	236	0.707

TABLE 3. Regression coefficients and constants for equations of the type $h_{+12} = b_1b_0 + b_2h_{-12} + b_3(\Delta_t h)_{24\text{-hr}} + c$, and squares of multiple correlation coefficients ($R^2_{4.123}$). c is in 10's of feet.

Verifying station	Sample data, Nov–Dec 1955 b_1	b_2	b_3	c	$R^2_{4.123}$	Sample data, May 1956 b_1	b_2	b_3	c	$R^2_{4.123}$
Bismarck	1.066	−0.241	0.293	315	0.834	0.959	−0.011	0.440	108	0.846
Green Bay	0.637	0.239	0.688	213	0.845	0.216	0.702	1.015	147	0.815
Mitchell AFB	0.869	0.049	0.394	154	0.871	0.470	0.279	0.573	461	0.889
Dodge City	1.407	−0.540	0.318	257	0.788	1.925	−0.694	0.295	−413	0.854
Nashville	1.262	−0.293	0.397	58	0.888	0.691	0.229	0.602	149	0.709
Charleston	0.807	−0.078	0.314	505	0.739	1.196	−0.280	0.184	166	0.838
Midland	1.557	−0.539	0.142	−20	0.750	1.092	−0.004	0.778	−154	0.639
Shreveport	0.982	−0.112	0.338	246	0.863	1.095	−0.280	−0.717	350	0.724

prediction. This consideration may be expressed in a regression equation of the form

$$h_{+12} = b_1 h_0 + b_2 h_{-12} + c, \qquad (4)$$

where h_0 is the thickness at 1500 GCT on the date of the forecast. Equation (4) makes use of the thickness at a given point, but at different times; in this respect it differs from equation (3) and also from the methods employed in [1] and [2] where space functions of the height field are employed. There is at least a good possibility that spatial relations among the height or thickness values are best taken into account in the physical or dynamical equations leading logically to the technique of numerical integration. However the dynamical equations do not take into account what has occurred in the past (*e.g.*, h_{-12} in the equation above), and theoretically there is no reason why they should. In this sense, equation (4) may contain information not contained in equation (3) nor in the abbreviated dynamical equations forming the basis for numerical prediction.

Evaluation of a statistical forecast.—Regression equations of the type of equation (4) were used to make 12-hr thickness predictions for 0300 GCT at each of the eight stations. The regression coefficients and constants, computed from data for November–December 1955, and May 1956, are listed in table 2. The squares of the multiple correlation coefficients, suggesting how well the thickness is specified by equation (4) and offering a suggestion of the "predictability" of the independent variables, are also shown in table 2. In table 5 (column I), the correlations between observed and predicted thickness values for the periods January–February 1956, and June 1956, and the root-mean-square errors of the forecasts, are indicated for each of the eight stations.

A combined equation.—The predictors employed separately in equations (3) and (4) may be combined into a single regression equation of the type

$$h_{+12} = b_1 h_0 + b_2 h_{-12} + b_3 (\Delta_t h)_{24\text{-hr}} + c, \qquad (5)$$

where $(\Delta_t h)_{24\text{-hr}}$ is the estimated 24-hr thickness change obtained from equation (3). Regression coefficients and constants were again computed from the data for November–December 1955, and May 1956, and are listed in table 3. In table 5 (column II), the correlation coefficients between observed and predicted thickness values for 0300 GCT for the periods January–February 1956, and May 1956, and the root-mean-square errors of the forecasts, are shown

TABLE 4. Regression coefficients and constants for equations of the type $h_{+12} = b_1 h_0 + b_2 h_{-12} + b_3(JNWP) + c$, and squares of multiple correlation coefficients ($R^2_{4.123}$). c is in 10's of feet.

Verifying station	Sample data, Nov–Dec 1955					Sample data, May 1956				
	b_1	b_2	b_3	c	$R^2_{4.123}$	b_1	b_2	b_3	c	$R^2_{4.123}$
Bismarck	0.001	−0.037	0.986	92	0.936	0.122	−0.077	0.931	60	0.883
Green Bay	−0.162	0.072	0.963	218	0.931	−0.664	0.342	1.266	99	0.942
Mitchell AFB	0.245	0.016	0.720	46	0.948	0.174	0.034	0.658	256	0.957
Dodge City	0.138	−0.247	1.011	192	0.846	0.619	−0.521	1.098	−350	0.868
Nashville	0.448	−0.138	0.629	94	0.942	−0.459	0.264	1.245	−91	0.848
Charleston	−0.014	0.185	0.636	363	0.883	0.444	−0.026	0.563	45	0.876
Midland	0.894	−0.468	0.657	−139	0.791	−0.479	−0.027	1.662	−279	0.736
Shreveport	−0.121	0.141	0.831	280	0.912	0.231	0.054	0.683	62	0.746

TABLE 5. Correlation coefficients (r) between observed and 12-hr forecast thickness values, and root-mean-square errors (RSME, 10's of feet) of the predictions. The form of the prediction formula tested is shown in the heading of each subdivision of the table.

Verifying station and elevation in m	Column I $h_{+12} = b_1h_0 + b_2h_{-12} + c$				Column II $h_{+12} = b_1h_0 + b_2h_{-12} + b_3(\Delta_t h)_{24\text{-hr}} + c$				Column III $h_{+12} = JNWP$				Column IV $h_{+12} = b_1h_0 + b_2h_{-12} + b_3(JNWP) + c$			
	Jan–Feb 1956		June 1956		Jan–Feb 1956		June 1956		Jan–Feb 1956		June 1956		Jan–Feb 1956		June 1956	
	r	*RMSE*	r	*RMSE*	r	*RMSE*	r	*RMSE*	r	*RMSE*	r	*RMSE*	r	*RMSE*	r	*RMSE*
Bismarck, 505	0.94	14	0.90	10	0.95	13	0.89	12	0.97	9	0.92	20	0.97	9	0.92	9
Green Bay, 210	0.85	16	0.66	21	0.92	13	0.67	17	0.94	10	0.74	17	0.94	10	0.74	18
Mitchell AFB, 24	0.80*	15*	0.87	10	0.83*	15*	0.84	12	0.89*	17*	0.86	13	0.89*	12*	0.89	8
Dodge City, 792	0.80	23	0.58	14	0.82	22	0.66	12	0.91	15	0.77	18	0.91	19	0.65	12
Nashville, 177	0.89	14	0.94	8	0.88	15	0.95	8	0.97	9	0.97	6	0.96	20	0.95	7
Charleston, 13	0.86	16	0.91	8	0.89	15	0.91	7	0.96	15	0.94	8	0.94	12	0.93	7
Midland, 871	0.87	21	0.76	6	0.87	21	0.80	6	0.92	16	0.75	18	0.91	18	0.72	8
Shreveport, 53	0.89	12	0.88	5	0.91	12	0.86	5	0.95	10	0.83	6	0.95	11	0.86	5
Average	0.864	16.7	0.821	11.3	0.884	16.1	0.828	10.6	0.939	13.0	0.850	14.3	0.934	14.5	0.839	10.0

* January data only.

for each of the eight stations. It is evident from the verification scores that inclusion of the physical variable does not add markedly to the accuracy of the forecasts, although there is a fairly consistent improvement over the scores shown in column I.

3. Comparison of results with numerical predictions of thickness

The forecast potentialities of equations (3), (4), and (5) may profitably be compared with the performance of the numerical prediction technique. In table 5 (column III), the correlation between the observed thickness and the 12-hr thickness predictions made by the Joint Numerical Weather Prediction Unit, at each of the eight points for 0300 GCT January–February 1956, and June 1956, is listed, as are the root-mean-square errors of the numerical forecasts. A study of table 5 suggests that the auto-regressive prediction equation as well as equation (5) may have some advantage over the numerical technique in the summer months, inasmuch as the RMS errors of the forecasts were somewhat reduced. In comparing the scores at the several stations, it should be borne in mind that at the higher stations (for example, Midland, Texas) a large portion of the 1000–500 mb thickness is fictitious. During the winter months, the numerically produced forecasts are noticeably superior to those based on the regression equations.

4. Combined autoregressive and numerical forecasts

One further possibility of improving the numerical forecasts was investigated. The numerical forecast, as well as the initial and earlier thickness values, may itself be used as a predictor in an equation of the type

$$h_{+12} = b_1 h_0 + b_2 h_{-12} + b_3(JNWP) + c, \qquad (6)$$

where the third independent variable is the numerically predicted thickness. Once again, regression coefficients and constants were computed from the sample data (November–December 1955, and May 1956); these are shown in table 4. Forecasts were made for 0300 GCT for the periods January–February 1956, and June 1956; the correlations between observed and predicted thickness values, as well as the root-mean-square errors of the forecasts, are shown in table 5 (column IV).

5. Summary and conclusions

The results of the study, summarized in table 5, may be briefly recapitulated.

Twelve-hour thickness forecasts were prepared, by four different objective methods, for a winter period and a summer period at eight points in the eastern part of the United States. The first method, involving space functions of the height fields, produced rather poor results which were greatly improved upon by a simple auto-regressive scheme utilizing the thickness at the initial time and the thickness 12 hr earlier. The latter method represents logically a statistical extrapolation of the observed 12-hr thickness trend. This simple method gave results in the summer month which compared favorably with 12-hr numerical forecasts of thickness.

An even simpler prediction model might be based upon the regression toward normal of the *thickness* rather than the thickness *trend*; thus an equation of the type

$$h_{+12} = b_1 h_0 + c \qquad (7)$$

could be used to predict the thickness 12 hr in advance. However, application of the F-test showed that the additional term $b_2 h_{-12}$ in equation (4) contributed materially to estimates of the thickness.

A third method, represented by a combination of the auto-regressive scheme and the first technique incorporating physical and empirical considerations, showed little improvement over the auto-regressive model.

The fourth method, utilizing the "thickness trend" and in addition the numerical forecast, produced results like the simpler auto-regressive scheme which in the summer month compared favorably with the numerical predictions.

The overall results of the study indicate that, in certain seasons and certain localities (notably in the summer months and in mountainous regions, and perhaps also in oceanic areas where upper-air data are sparse), an auto-regressive technique utilizing current and past thickness values may prove to be a useful objective forecasting method supplementing the technique of numerical integration. However, additional tests based on more extensive data, and forecasts for 24 and perhaps 36 hr, would be required to verify this conclusion. In any such projected study, it might be advisable to determine the influence of the thickness trend at distant points on the future change at the forecast point.

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