A PRELIMINARY EVALUATION OF SUTTON'S HYPOTHESIS FOR DIFFUSION FROM A CONTINUOUS POINT SOURCE

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ABSTRACT

Sutton's hypothesis for diffusion from a continuous point source has been evaluated using the data obtained during Project Prairie Grass. It is found that the hypothesis predicts the observed concentration distributions only if there are two values of Sutton's "n," one to characterize lateral diffusion (n_y) and one to characterize vertical diffusion (n_z). Statistical tests indicate that n_y and n_z are invariant with distance between 100 and 800 m of the source, but that the values of n_y and n_z appropriate for these distances exceed the values within 100 m of the source. It is also shown that neither n_y nor n_z can be specified by n_w, the value of n found from a power-law fit to the wind profile in the lowest 8 meters.

1. Introduction

During the summer of 1956, the Geophysics Research Directorate of the Air Force Cambridge Research Center sponsored and directed an experimental program in micrometeorology. The primary objective of the program was to determine the rate of diffusion of a gas, emitted continuously at a point, as a function of meteorological parameters. The field work was conducted in flat, prairie conutry near the town of O'Neill in north central Nebraska. The program was given the name, Project Prairie Grass.

The diffusion measurements and some of the meteorological measurements were made by a team from the Massachusetts Institute of Technology. The other meteorological measurements were made by groups from the Texas A. & M. Research Foundation, the University of Wisconsin, the Air Weather Service, and the Air Force Cambridge Research Center.

In 1947, O. G. Sutton [2] presented a hypothesis on diffusion from a continuous point source. The purpose of this paper is to present an evaluation of the Sutton hypothesis based on the measurements made during Project Prairie Grass.

2. The field site[1]

The section of land on which the diffusion experiments were conducted was particularly flat, as shown in fig. 1. The contour lines in this figure are for one-foot intervals. The gas source was located at the center of five concentric semicircles having radii of 50, 100, 200, 400, and 800 m. North of the E-W line passing through the source, the topography is very flat, being within three feet of 1980 feet above mean sea level. The ground rises gently to the southwest with an average grade of about ten feet per half-mile and to the southeast with an average grade about twenty feet per half-mile.

The vegetative cover was fairly uniform as to grass type. The entire section of land was mowed to a height of about five centimeters prior to the experiments. The grass height changed little during the program; however, scattered shrubs occasionally reaching a height of 18 cm appeared in late August near the end of the field program.

The area upwind from the 800-m arc was kept relatively free of obstructions to air flow. As shown in fig. 1, most of the meteorological measurements were made downwind from the 800-m arc.

3. Experimental procedures[2]

The tracer used in Project Prairie Grass was sulfur dioxide. It was emitted at a height of 46 cm (taken to be 50 cm in the present analysis) during 64 experiments and at a height of 1.5 m for six experiments. Emission was continuous for ten minutes. The gas samplers were spaced at two-degree intervals on each of the first four arcs and at one-degree intervals on the 800-m arc. All of these samplers were mounted at a height of 1.5 m. In addition to this array at 1.5 m, there were six masts erected at the 100-m arc for gas concentration measurements in the vertical. Nine levels between 0.5 m and 17.5 m were sampled on each mast. Thus, 599 samplers were exposed during a typical gas release. Of the 70 gas releases during July and August, there were 34 during lapse conditions and 36 during inversions.

[1] A more complete description of the field site and experimental procedures will appear soon in a Geophysical Research Paper describing Project Prairie Grass and summarizing the significant experimental data.

[2] See footnote 1.

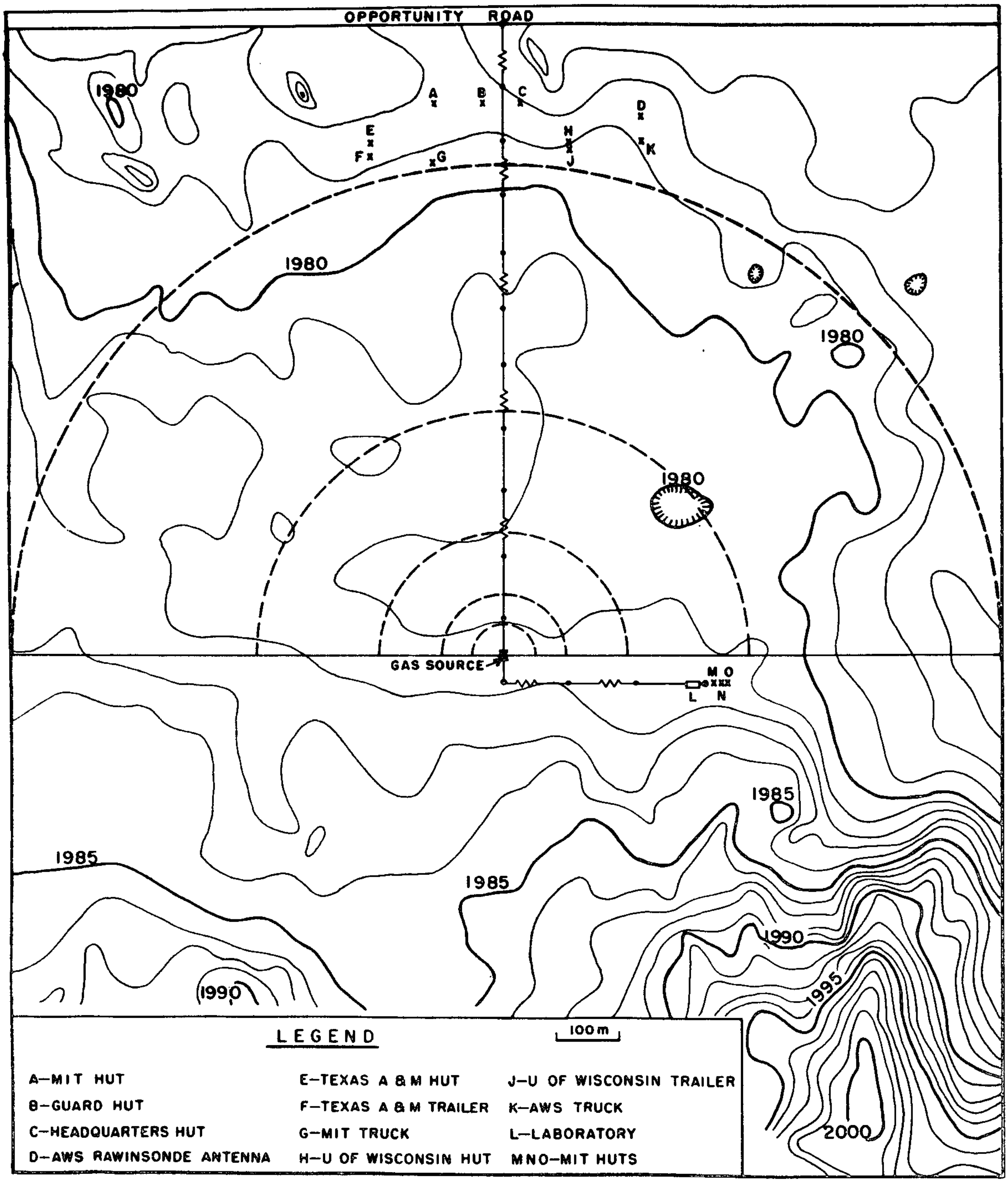

Fig. 1. Topographical map of field site with position of sampling arcs indicated.

4. Sutton's hypothesis

In one of his papers on diffusion, O. G. Sutton [2] presented the following expression for the time-mean concentration of a gas at points downwind from a continuous, elevated point source:

$$\frac{X\bar{V}}{Q}=\frac{\exp(-y^2/C_y^2x^{2-n})}{\pi C_y C_z x^{2-n}}\times\left\{\exp\left[-\frac{(z-h)^2}{C_z^2x^{2-n}}\right]+\exp\left[-\frac{(z+h)^2}{C_z^2x^{2-n}}\right]\right\}. \quad (1)$$

The adjusted mean concentration appears as the left-hand member of (1). Here X is the actual mean concentration; $\bar{V}$, the mean wind speed; Q, the rate of gas emission; x, the direction of the mean wind; y, the direction of the crosswind; z, the vertical direction; C_y and C_z, the lateral and vertical diffusion coefficients, which are assumed invariant with distance; h, the height of the source above ground; and n, a parameter "related to the diffusing power of the turbulence." The bracketed terms in the right-hand member account for reflection of the gas from the surface. Analyses of the data conducted with reflection as well as the difference between source height and sampler height included differ significantly from analyses conducted with these factors neglected. Therefore, the "long form" given in (1) was employed in the analyses described here.

In the present evaluation of Sutton's hypothesis, the diffusion coefficients, C_y and C_z, were assumed to be constant with distance from the source. According to Sutton, n should also be constant with distance in each experiment. This evaluation then begins with an investigation of the constancy of n with distance.

If the cross wind and vertical concentration distributions in the absence of reflection are Gaussian, it follows from (1) that

$$C_y^2 x^{2-n} = 2\sigma_y^2$$
$$C_z^2 x^{2-n} = 2\sigma_z^2, \qquad (2)$$

where σ_y^2 and σ_z^2 are the variances of the crosswind and vertical concentration[3] distributions, respectively. Since these variances at given downwind distances are measurable quantities, the normality statements in (2) permit the expression of C_y and C_z in terms of one unknown, the parameter n, when the observed distributions are Gaussian. From (1) and (2), the adjusted concentration at any downwind distance may be written as

$$\frac{x\bar{V}}{Q} = \frac{\exp(-y^2/2\sigma_y^2)}{2\pi\sigma_y\sigma_z} \times \left\{ \exp\left[-\frac{(z-h)^2}{2\sigma_z^2}\right] + \exp\left[-\frac{(z+h)^2}{2\sigma_z^2}\right] \right\}. \qquad (3)$$

5. Classification of the data

For each arc and each experiment, the first four moments of the distributions of concentration along the arcs were computed. These arcwise concentration distributions were then classified as Gaussian or non-Gaussian on the basis of the computed values of relative skewness and kurtosis and on the degree of linearity of the plots of cumulative concentration on probability paper. Arcs which had non-Gaussian distributions were not employed in this analysis. Of all the distributions classified as Gaussian, the ones considered most Gaussian and least Gaussian are shown in fig. 2. Since Gaussian distributions plot as straight lines on probability paper, it can be seen that the distributions employed in the analysis are very nearly Gaussian. The abscissa is an arbitrary scale chosen for convenience; the zero value along the abscissa is not meant to indicate the position of the mean or the median of the distribution.

6. Computation of $\sigma_{z,100}$

The next step was the computation of $\sigma_{z,100}$, the standard deviation of the concentration in the vertical at the 100-m arc were there no ground serving as a boundary. In the computation of $\sigma_{z,100}$, the measured vertical profile of concentration of the 100-m arc was considered to be the sum of two Gaussian distributions, having the same central ordinate and standard deviation, but with one distribution centered at the source height and the other at the same distance below ground, as portrayed in fig. 3. The value of $\sigma_{z,100}$

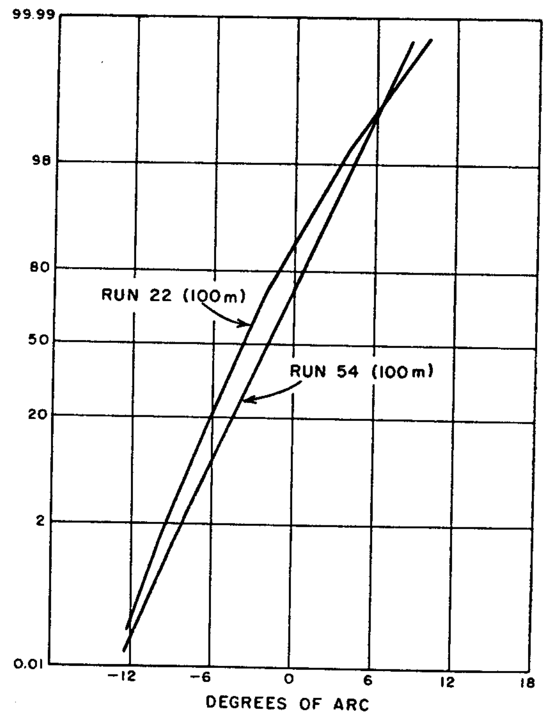

FIG. 2. Probability plots of arcwise integrated concentration.

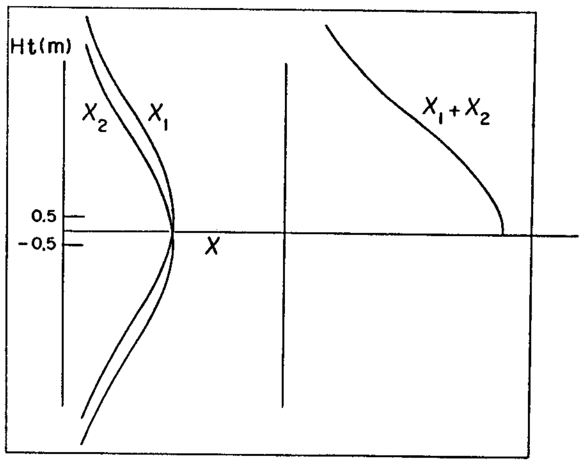

FIG. 3. Method of computation of $\sigma_{z,100}$.

[3] The variance σ_z^2 is that value which would exist were there no ground surface—*i.e.*, if diffusion in the vertical direction were to proceed without the influence of a boundary.

which provided the best fit of the summed Gaussian curves to the observed concentration profile was then computed by trial-and-error methods.

It proves interesting at this point to examine the relationship between $\sigma_{z,100}$ and the stability ratio. Fig. 4 shows a plot of this standard deviation against the stability ratio as determined from measurements made by the Texas A. & M. group. The stability ratio employed in this analysis is the ratio of the temperature difference between the 4-m and 0.5-m heights to the square of the wind speed at the 2-m height. The open circles represent experiments in which the arcwise distributions at 100 m were approximately Gaussian; the closed represent non-Gaussian distributions. Of importance, perhaps, is the indication that the height of the cloud at 100 m is well specified by the stability ratio, particularly in the region of slight to moderate inversions. The graph also serves to bolster one's confidence in the computation of $\sigma_{z,100}$.

7. Computation of n from x_p ratios

A method of computing n will now be described. The diffusion coefficients, C_y and C_z, are assumed to be constants. The ratio of the peak concentrations (X_p) at two radial distances from the source (x_1 and x_2) is then expressed as a function of the vertical diffusion coefficient, C_z, and the parameter n.

$$\frac{x_{p_1}}{x_{p_2}}=\left(\frac{x_2}{x_1}\right)^{2-n} \times \left\{ \frac{\exp\left[-\frac{(z-h)^2}{Cz^2x_1^{2-n}}\right]+\exp\left[-\frac{(z+h)^2}{Cz^2x_1^{2-n}}\right]}{\exp\left[-\frac{(z-h)^2}{C_z^2x_2^{2-n}}\right]+\exp\left[-\frac{(z+h)^2}{C_z^2x_2^{2-n}}\right]} \right\}. \quad (4)$$

Elimination of C_z is possible through use of (2) and the computed value of $\sigma_{z,100}$.

$$C_z^2 = 2\sigma_{z,100}^2/100^{2-n}. \quad (5)$$

Thus the ratio of peak concentrations at two distances can be used to find a value of n appropriate for that pair of distances. The n-value for one pair of distances can be compared with the n-value for each of the other pairs of distances. If, in the comparison of two n-values, no distance appears more than once, fifteen comparisons, albeit not independent, can be made with the five arcs of data available in each of the gas releases. Application of Student's t-test to fourteen gas releases with stabilities ranging from slight lapse to extreme inversion conditions indicates that the difference in n's is not significantly different from zero in all of the fifteen possible comparisons. When only slight to moderate inversions are examined, it is found that in twelve of the fifteen comparisons of n, the difference in n's is not significantly different from zero. In the remaining three comparisons the probabilities of finding, by chance, absolute mean differences as large as those observed are between one and five per cent. The conclusion, then, is that the parameter n computed from peak concentration ratios does not vary significantly with distance within 800 m of the source over a wide range of stabilities.

8. Computation of n from arcwise distributions of concentration

The data collected during Project Prairie Grass also permit the computation of n values from the arcwise distributions of concentration. Actually, if there were crosswind data rather than arcwise data, n could be computed from (2).

$$\frac{\sigma_{y_2}}{\sigma_{y_1}} = \left(\frac{x_2}{x_1}\right)^{(2-n)/2}. \quad (6)$$

However, since the angles of diffusion in the experiments studied are relatively small, the ratio of the standard deviations along the arcs ($\sigma_{\theta_2}/\sigma_{\theta_1}$) can be substituted for the ratio $\sigma_{y_2}/\sigma_{y_1}$. It is important to note, at this point, that the values of n computed from the ratios $\sigma_{\theta_2}/\sigma_{\theta_1}$ differ considerably from those computed from the peak concentration data (table 1). The values of n computed from the $\sigma_{\theta_2}/\sigma_{\theta_1}$ are denoted as n_y to distinguish them from the values of n computed from the peak concentrations, denoted as n_p.

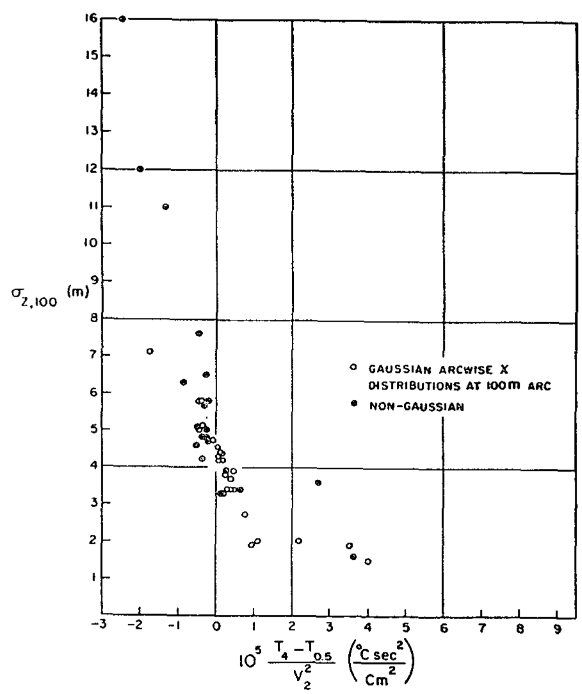

FIG. 4. $\sigma_{z,100}$ vs. stability ratio.

TABLE 1. Values of n_y, n_p, and n_z computed from observed ratios of appropriate diffusion parameters for the cases with Gaussian arcwise concentration distributions identified by run number and the computed values of stability ratio (10^5 °C sec²/cm²) and $\sigma_{z,100}$(m).

Run No.		50/100	50/200	50/400	50/800	100/200	100/400	100/800	200/400	200/800	400/800	Stability ratio	$\sigma_{z,100}$
17	n_y	0.79	0.73	0.70	0.65	0.66	0.65	0.60	0.64	0.57	0.51		
	n_p	0.46	0.40	0.38	0.40	0.35	0.36	0.38	0.36	0.40	0.44	0.37	3.7
	n_z	−0.48	−0.08	−0.01	0.11	0.02	0.06	0.16	0.10	0.23	0.38		
18	n_y	0.41	0.38			0.35							
	n_p	0.05	0.23			0.32						0.77	2.7
	n_z		−0.14			0.29							
21	n_y	0.47	0.43	0.44		0.40	0.43		0.44				
	n_p	0.33	0.28	0.27		0.25	0.25		0.26			0.09	4.2
	n_z	0.11	0.09	0.08		0.08	0.07		0.07				
22	n_y	0.61	0.59	0.64	0.69	0.57	0.65	0.71	0.73	0.78	0.84		
	n_p	0.32	0.32	0.33	0.30	0.32	0.33	0.30	0.33	0.28	0.24	0.07	4.3
	n_z	−0.25	−0.04	−0.05	−0.13	0.07	0.00	−0.12	−0.06	−0.21	−0.35		
24	n_y	0.55	0.53	0.57	0.62	0.52	0.58	0.64	0.64	0.71	0.79		
	n_p	0.31	0.34	0.37	0.35	0.36	0.39	0.36	0.42	0.36	0.28	0.06	4.6
	n_z	−0.10	0.09	0.13	0.04	0.20	0.21	0.07	0.21	0.01	−0.20		
28*	n_y	0.49	0.42	0.56		0.34	0.60		0.86			0.71	
35_s	n_y	0.49	0.44	0.43	0.53	0.40	0.41	0.55	0.42	0.62	0.83		
	n_p	0.27	0.25	0.30	0.34	0.25	0.31	0.36	0.37	0.43	0.47	0.30	3.4
	n_z		−0.05	0.12	0.12	0.08	0.21	0.18	0.32	0.22	0.12		
37	n_y	0.34	0.85			0.25							
	n_p	0.29	0.23			0.18						0.18	4.2
	n_z	0.23	0.16			0.12							
38	n_y	0.82	0.74	0.74	0.73	0.66	0.70	0.70	0.74	0.72	0.66		
	n_p	0.43	0.40	0.42	0.44	0.37	0.42	0.45	0.46	0.49	0.51	0.27	3.8
	n_z	−0.65	−0.08	0.03	0.13	0.06	0.13	0.20	0.19	0.26	0.34		
39*	n_y	0.54	0.54	0.66		0.53	0.71		0.90			3.30	
41	n_y	0.61	0.61	0.61	0.68	0.61	0.62	0.70	0.62	0.75	0.87		
	n_p	0.47	0.45	0.48	0.53	0.43	0.48	0.54	0.54	0.60	0.66	0.39	3.4
	n_z	0.14	0.20	0.32	0.33	0.24	0.35	0.39	0.46	0.46	0.45		
42	n_y	0.56		0.34			0.23						
	n_p	0.29		0.21			0.17					0.13	4.2
	n_z	−0.23		0.05			0.12						
53*	n_y	0.94	0.88	0.80	0.78	0.83	0.73	0.73	0.62	0.68	0.74	5.79	
54	n_y	0.71	0.69			0.65							
	n_p	0.50	0.50			0.44						0.44	3.9
	n_z	0.70	0.24			0.32							
55	n_y	0.76	0.66	0.63	0.60	0.57	0.56	0.55	0.56	0.55	0.53		
	n_p	0.43	0.41	0.32	0.33	0.37	0.26	0.30	0.15	0.26	0.38	0.12	4.4
	n_z	−0.10	0.07	−0.06	0.06	0.17	−0.04	0.04	−0.25	0.01	0.23		
56	n_y	0.60	0.56	0.62	0.68	0.53	0.63	0.71	0.72	0.80	0.83		
	n_p	0.38	0.35	0.39	0.46	0.33	0.38	0.46	0.45	0.54	0.62	0.20	4.35
	n_z	0.02	0.08	0.12	0.20	0.13	0.15	0.23	0.18	0.27	0.37		
58	n_y	1.13	1.05	0.99	0.92	0.97	0.91	0.85	0.86	0.79	0.72		
	n_p	0.94	0.55	0.58	0.65	0.45	0.55	0.62	0.61	0.68	0.75	4.05	1.5
	n_z	1.94			0.10	−0.26	0.14	0.37	0.40	0.59	0.78		
59	n_y	1.10	1.00	0.99	0.93	0.91	0.94	0.87	0.96	0.85	0.75		
	n_p	1.12	0.75	0.72	0.67	0.55	0.62	0.60	0.66	0.63	0.58	2.19	2.0
	n_z			0.19	0.85	0.12	0.27	0.32	0.39	0.41	0.44		
60	n_y		0.66	0.67	0.69				0.71	0.72	0.74		
	n_p		0.48	0.46	0.49				0.39	0.50	0.60	0.27	3.9
	n_z		0.24	0.18	0.27				0.08	0.27	0.47		
65	n_y	0.77	0.73	0.73	0.67	0.68	0.70	0.64	0.73	0.62	0.51		
	n_p	0.48	0.44	0.49	0.48	0.42	0.49	0.48	0.56	0.51	0.45	0.31	3.4
	n_z	−0.19	0.02	0.18	0.25	0.10	0.26	0.31	0.39	0.40	0.40		

TABLE 1—*Continued.*

Run No.		50/100	50/200	50/400	50/800	100/200	100/400	100/800	200/400	200/800	400/800	Stability ratio	$\sigma_{z,100}$
	n_y	0.52	0.54	0.57	0.57	0.55	0.59	0.58	0.62	0.60	0.57		
67	n_p	0.37	0.39	0.42	0.44	0.40	0.44	0.45	0.48	0.48	0.48	0.23	3.3
	n_z	0.04	0.17	0.24	0.34	0.23	0.29	0.33	0.34	0.37	0.40		
6*	n_y	0.18	0.19	0.23	0.22	0.19	0.26	0.23	0.33	0.26	0.19	−0.18	
11*	n_y	0.43	0.42	0.37	0.40	0.41	0.34	0.39	0.27	0.37	0.48	−0.22	
	n_y	0.18	0.23	0.32		0.27	0.39		0.51				
27	n_p	0.06	0.09	0.02		0.12	0.01		−0.10			−0.47	5.8
	n_z	−0.12	−0.06	−0.31		−0.03	−0.37		−0.71				
	n_y	0.31	0.26	0.22		0.21	0.17		0.13				
30	n_p	0.12	0.05	−0.11		−0.01	−0.21		−0.40			−0.36	5.1
	n_z	−0.19	−0.21	−0.50		−0.22	−0.58		−0.93				
	n_y	0.52	0.47	0.46	0.48	0.41	0.43	0.46	0.45	0.49	0.53		
34	n_p	0.30	0.28	0.19	0.14	0.28	0.14	0.10	0.02	0.02	0.01	−0.21	4.7
	n_z	−0.05	0.06	−0.13	−0.24	0.14	−0.15	−0.26	−0.41	−0.46	−0.50		
	n_y	0.32	0.23	0.19	0.28	0.15	0.13	0.26	0.11	0.32	0.53		
57	n_p	−0.10	0.18	0.11	0.08	0.42	0.19	0.13	−0.04	−0.01	0.02	−0.08	4.7
	n_z	−1.36	0.12	0.01	−0.14	0.70	0.25	0.00	−0.17	−0.33	−0.49		
	n_y					0.65	0.68		0.71				
62	n_p					0.27	0.23		0.18			−0.41	5.8
	n_z					−0.12	−0.23		−0.35				

* No $\sigma_{z,100}$ for these runs.

Since n_y is dependent on the lateral diffusion of the gas while n_p is dependent on the vertical as well as the lateral diffusion of the gas, the fact that these two sets of n differ indicates that there must be two different sets of power indices to describe the diffusion, one for the lateral direction and the other for the vertical direction.

From the expression for the crosswind integrated concentration (CIC),

$$\frac{(\mathrm{CIC})\bar{V}}{Q}=\frac{1}{\sqrt{\pi}C_z x^{(2-n_z)/2}} \times\left\{\exp\left[-\frac{(z-h)^2}{C_z^2x^{2-n_z}}\right]+\exp\left[-\frac{(z+h)^2}{C_z^2x^{2-n_z}}\right]\right\} \quad (7)$$

a third set of n values, dependent on vertical diffusion alone, can be computed. These n's are denoted as n_z. Actually the values of n_z were computed with the arcwise integrated concentration (AIC) substituted for the crosswind integrated concentration in (7). As indicated earlier in this paper, the difference between the AIC and CIC values is negligible in the gas releases employed in this analysis.

The extent to which n_z differs from n_p and n_y is shown in table 1. In general, the individual value of n_p is the arithmetic mean of the corresponding values of n_y and n_z. Departure of n_p from the exact mean is due to the fact that the height of the source, h, is not zero.

9. Variation of n_y and n_z with distance

The dependence of n_y and n_z on distance from the source was investigated by the use of Student's t-test in the manner employed in the test of the dependence of n_p on distance. Here, too, it was found that neither n_y nor n_z varies significantly with distance beyond 100 m. However, for stabilities ranging from slight lapse to strong inversions, the tests indicate weakly that the n_y and n_z values appropriate for distances greater than 100 m exceed the values appropriate for distances less than 100 m.

The variation of σ_y with distance for the slight to moderate inversion cases is shown in fig. 5. This figure shows a logarithmic plot (solid curve) of the mean value of σ_y *vs.* distance from the source. The values of σ_y have been normalized by division by the value of σ_y at 100 m. A rough idea of the variability of σ_y from run to run can be obtained by noting the observed range of σ_y at each distance; the range is marked off by horizontal dashes at each distance. If n_y were a constant, this plot would be a straight line. The variation of σ_z with distance for the slight to moderate inversion cases is shown in fig. 6. The individual, normalized values of σ_z were found from (2) to be $(x/100)^{(2-n_z/2)}$, where x is in meters and n_z is the appropriate value computed from (7). These individual, normalized values of σ_z were then used to determine the mean relationship shown in fig. 6.

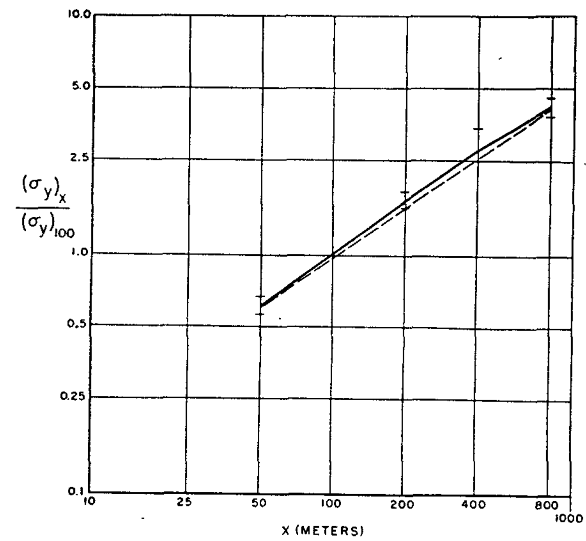

FIG. 5. Mean $(\sigma_y)_x/(\sigma_y)_{100}$ *vs.* distance for slight to moderate inversion cases.

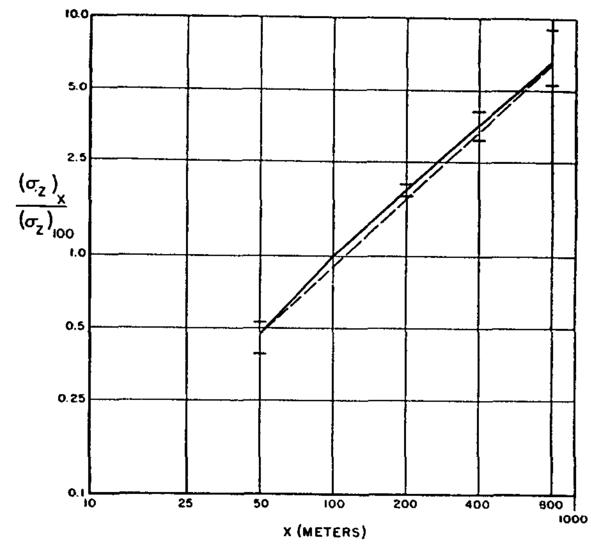

FIG. 6. Mean $(\sigma_z)_x/(\sigma_z)_{100}$ *vs.* distance for slight to moderate inversion cases.

10. Determination of *n* from the wind profile

One of the features of the Sutton work which has made his formulae attractive to persons engaged in solving practical diffusion problems is the assertion [1] that the formulae "have the merit of depending only upon observable meteorological quantities and do not rely in any way on quantities derived from observations of diffusion." Sutton [1] suggests, for example, that for the value of n required in such formulae as (1) one use the value of n_w found from the power law profile,

$$\frac{\bar{V}_2}{\bar{V}_1} = \left(\frac{z_2}{z_1}\right)^{n_w/(2-n_w)}, \qquad (8)$$

where $\bar{V}_2$ and $\bar{V}_1$ are mean speeds at the heights z_2 and z_1, respectively. Therefore, it is pertinent in the present evaluation to calculate n_w from (8) and to compare this value with the values of n_y, n_p, and n_z computed from the observations of diffusion.

For each experiment the exponent in (8) was found as the slope of a least-squares fit of log $\bar{V}$ *vs.* log z for values of $\bar{V}$ at the following heights above ground: 0.25, 0.50, 1, 2, 4 and 8 m. Since n values computed from the diffusion data are independent of distance beyond 100 m, mean values of n_y, n_p, and n_z for the the region beyond 100 m only are plotted against the n_w values in fig. 7. It is fairly evident, from fig. 7, that n_w cannot be used to specify the appropriate mean value of n_y, n_p, or n_z. Although the power law profile fails to provide reasonable values of n_y, n_p, or n_z, it should not necessarily be concluded at this time that these diffusion parameters cannot be specified in terms of meteorological parameters other than n_w. The degree to which such specification is possible, however, is the subject of another investigation under way.

11. Reliability of results

A few statements concerning the reliability of these results are required. A detailed discussion of the errors which enter in the determination of the parameters employed in this analysis will be published elsewhere.[4] It will suffice here, perhaps, to summarize this discussion of errors. The rate of gas emission is believed accurate to within one per cent. The absolute values of mean concentration are believed accurate to within ten per cent, the relative values to within five per cent.

The hypothesis being evaluated is based on the assumption that the ground is a perfect reflector. It is important then to consider the possibility that there was a significant loss of SO_2 to the ground or the prairie vegetation. Therefore, computations were made of the rate of transport of SO_2 through a vertical cylindrical surface at the 100-m arc. Only those gas releases which satisfied at 100 m the rather rigid Gaussian criteria described in Section 5 and had well established values of σ_z were used in these computations. The computed rate of transfer at the 100-m arc (Q_c) was determined from

$$Q_c = \int_0^\infty \int_{-\infty}^\infty X\bar{V}dydz. \qquad (9)$$

The integration was performed numerically with appropriate values of $\bar{V}$ taken from the mean wind profiles measured by Texas A. & M. about 900 m from the source. Eighteen nighttime and five daytime

[4] See footnote 1.

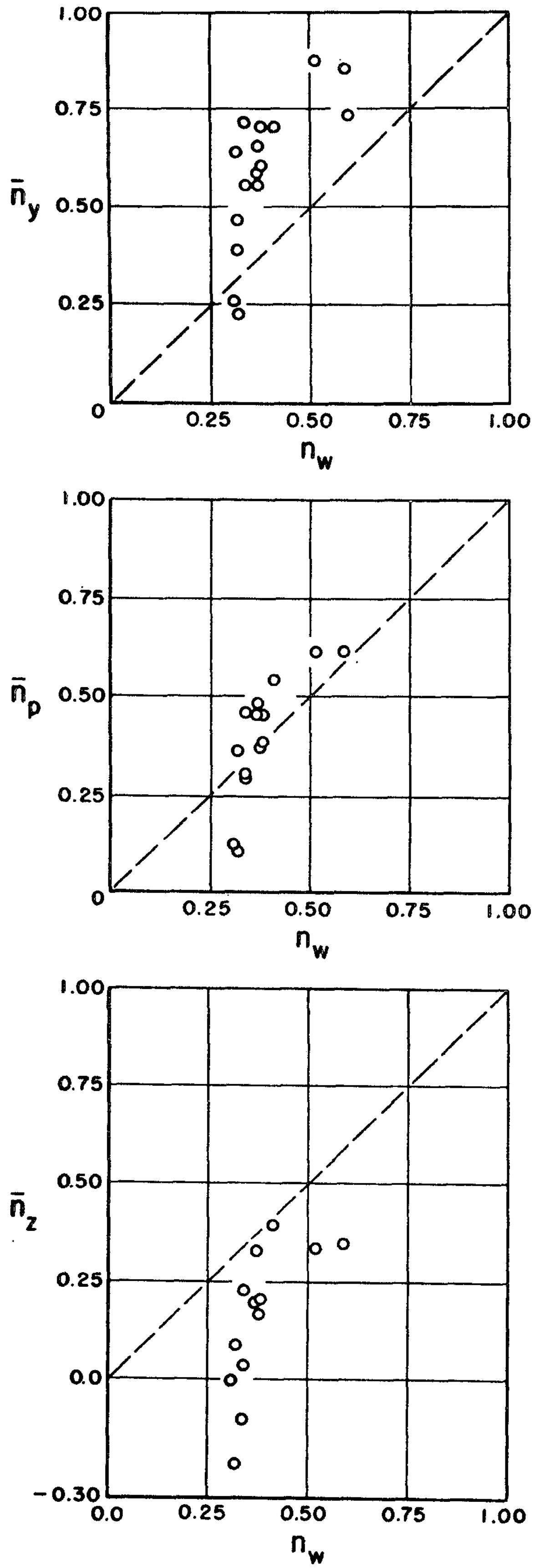

FIG. 7. Diffusion indices, $\bar{n}_y$, $\bar{n}_p$, and $\bar{n}_z$ *vs.* wind profile index: 7a. $\bar{n}_y$ *vs.* n_w. 7b. $\bar{n}_p$ *vs.* n_w. 7c. $\bar{n}_z$ *vs.* n_w.

releases were studied. In only one of the 23 releases is the value of Q_c less than Q, the actual rate of emission. The median value of $(Q_c - Q)/Q$ at night is 10 per cent, in the daytime 19 per cent. The computation was repeated with values of $\bar{V}$ measured by Massachusetts Institute of Technology a few yards west of the source. Again, in only one of the 23 releases is the value of Q_c less than Q. The median value of $(Q_c - Q)/Q$ at night is 10 per cent, in the daytime 16 per cent. Thus these continuity calculations indicate no significant loss of SO_2 within the first 100 m of travel. Nor is there reason to believe that losses of SO_2 were greater beyond the 100-m arc.

An attempt to account for the magnitude of the excess of Q_c over Q is in order. About one-third of the excess is believed ascribable to evaporation of water in the samplers, a factor not included in the determination of the concentrations used in this analysis. The fact that cup anemometers, even though carefully calibrated in wind tunnels, may overestimate mean wind speed in the outdoor, more turbulent atmosphere is another factor which may contribute to the excess. Undoubtedly part of the excess, particularly in the daytime experiments, can be ascribed to the fact that the method of computing Q_c has errors which arise because the arcwise distributions were not perfectly Gaussian and/or the observed profiles of concentration on the masts at 100 m were not perfectly represented as the sum of two Gaussian distributions.

12. Concluding remarks

It is to be noted that the computed values of n_p and n_z given in this paper are primarily dependent on the value of the standard deviation of the vertical distribution, which was computed for the 100-m arc only. Therefore, the variation—or nonvariation—of n_p and n_z with distance must be considered somewhat tentative until such time as experiments are conducted with sufficient vertical sampling on more than one arc.

In spite of the limited vertical sampling, however, it is quite certain that diffusion patterns cannot be characterized by a single turbulence parameter n; instead, this analysis indicates that a basically binormal distribution of concentration (1) satisfies the diffusion data only if there are two values of n, one to characterize lateral diffusion and one to characterize vertical diffusion. Thus the form

$$\frac{x\bar{V}}{Q} = \frac{\exp(-y^2/C_y^2 x^{2-n_y})}{\pi C_y C_z x^{2-(n_y+n_z)/2}} \times \left\{ \exp\left[-\frac{(z-h)^2}{C_z^2 x^{2-n_z}}\right] + \exp\left[-\frac{(z+h)^2}{C_z^2 x^{2-n_z}}\right] \right\} \quad (10)$$

provides a better fit to the cases having Gaussian distributions than does (1).

This analysis also indicates that neither n_y nor n_z can be specified by n_w, the value of n found from a power-law fit to the wind profile in the lowest 8 meters. If the form given in (10) is employed, the specification of n_y and n_z in terms of observable meteorological parameters must be sought elsewhere.

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