

SINGULARITIES IN DAILY TEMPERATURES

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ABSTRACT

The means of daily maximum temperatures using twenty years of record are obtained. The average of these means over n consecutive calendar days is used as a predictor of the daily maximum temperature one and more years in advance. Data from ten stations in the United States for the period 1905 to 1957 are analyzed in this fashion and a series of predictions made for several values of n . If singularities are sufficiently large and persistent, then the averages over only a few days (small n) should be better predictors than averages over a larger number of days. It is found that the prediction errors decrease with increasing n and it is concluded that, whenever averages are used to estimate daily temperature values far in advance, any singularities, even if they exist at all, are much too small to be useful.

1. Introduction

The seasonal march of mean daily temperatures nearly always presents a ragged appearance, no matter how many years of record are used (fig. 1). If one of these daily means or a group of consecutive means departs significantly from a smooth curve drawn or fitted to the means, the claim is often made that a singularity exists. The question of whether these are real meteorological phenomena or simply statistical fluctuations due to the daily means being computed from a finite sample is still controversial. One of the reasons for this disagreement is the inapplicability of the more common statistical methods to the problem, although advanced techniques are available [1; 2]. In fact, many investigators of possible singularities do not attempt to apply statistical methods but assert that the reality of singularities is proven by the appearance of curves such as shown in fig. 1 and by agreement of their singularity dates with those

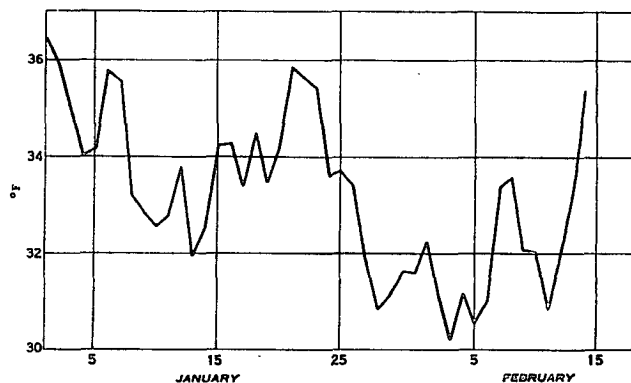


FIG. 1. 70-year (1885-1954) mean daily maximum temperature at Blue Hill, Massachusetts.

¹ Based in part on a thesis submitted to American University, Washington, D. C., in partial fulfillment of the degree of Master of Arts.

found by others or famous in folklore. These assertions have been criticized on the grounds that arbitrarily selecting singularities from a curve is subjective and that the agreement of dates can be explained by the use of data for a similar period of time or by such a multitude of studies that in one or another every day in the year has been chosen as a singularity date.

In studies in which statistical methods have been used, the primary objection is that the validity of choosing a singularity after looking at the data was not considered. (See [2] for a notable exception.) For example, Kangeiser [3] claims a nine-day singularity in January minimum temperatures at Phoenix, Arizona. He states that he noted this singularity while examining a time series in connection with an entirely different problem. But Mr. Kangeiser has been a meteorologist for years and has probably examined many time series. In addition, the nine-day period was almost surely selected because it gave the best results. Unless these two factors of selectivity are considered, the application of any statistical test designed to establish the reality of singularities simply is not valid.

Even if it could be proven statistically that singularities are real, there still remains the problem of their practical importance in particular applications. For example, many forecasters have been asked to give the weather for a single day far beyond the period for which ordinary forecasting methods are applicable (holidays, picnics, special occasions, etc.). Should a value read from a smooth curve be given or should the daily means as shown in fig. 1 be used? Another example is found in the preparation of daily normal values. One of the decisions which must be made is the interval of days to use. Probably the most important use of normals is in estimating the future course of the series so that again the application is one of prediction. Although singularities may be

important in other contexts, the only problem considered here is that of their usefulness in making predictions.

2. Procedure

To answer this question the basic procedure was to secure means of daily maximum temperatures using twenty years of record, then to average these over a varying number of days and to use each average to predict the maximum temperature for the central day one, five, and nine years ahead. A twenty-year record was selected since it has been shown that this is best for predicting the next ten years [4] and normal values are revised every ten years. Of the three daily temperatures available (mean, minimum, and maximum), the latter was chosen since it is more representative of the type of large air masses which pass over a station [5]. The periods one, five, and nine years in the future were used since it is felt that these give a good estimate of the next ten years. The predictions were verified by the root-mean-square error, the smallest one indicating the best predictions.

First, the mean daily maximum temperature for a calendar day was used as a predictor of the daily maximum temperature on that day one, five, and nine years in the future. Then the daily means for the three days centered on the same calendar day were averaged and used to predict the same three quantities. This process was continued until eight different predictors, the daily means averaged over 1, 3, 5, 7, 11, 15, 21, and 31 days, were each used to predict the daily maximum temperature on the central day.

In order to secure a large number of predictions, the calendar years were varied. For example, for predictions one year ahead, the mean for the years 1915 to 1934 was used to predict 1935, the mean for 1916 to 1935 predicted 1936, and so on for twenty predictions. Because of the greater spread of years between predictor and predictand, only nineteen predictions were made five years ahead and only fifteen for nine years ahead, a total of fifty-four predictions for each central day selected.

Data from the stations and months listed in table 1 were used. For convenience, these will be referred to as ten stations. They were chosen on the basis of

TABLE 1. List of stations.

Blue Hill, Massachusetts	January
Blue Hill, Massachusetts	July
Duluth, Minnesota	July
Havre, Montana	January
Key West, Florida	January
Key West, Florida	July
New Orleans, Louisiana	July
San Diego, California	January
Tatoosh Island, Washington	January
Tucson, Arizona	January

areal spread and representation of different climatic regimes. In each month, four days—15th, 20th, 25th, and 30th—were selected as central days, a total of forty central days for each of which fifty-four predictions were made.

For each central day, the predictions were verified by the root-mean-square error,

$$E_m = \left\{ \frac{1}{54} \sum_1^{54} (T_k - \hat{T}_{km})^2 \right\}^{\frac{1}{2}}, \quad (1)$$

where T_k is the observed daily maximum temperature and \hat{T}_{km} is a prediction obtained by taking the arithmetic mean of m daily means. That value of m for which E_m is the smallest gives the best predictions and is the optimum number of days over which to smooth the daily means. Eight values of E_m were secured for each of the forty central days and are listed in table 2.

For each of the ten stations, the four values were combined by

$$\bar{E}_m = \left(\frac{1}{4} \sum_1^4 E_m^2 \right)^{\frac{1}{2}} \quad (2)$$

and these are listed in table 2 and plotted in fig. 2. Finally, the ten stations were combined by

$$\bar{\bar{E}}_m = \left(\frac{1}{10} \sum_1^{10} w_i \bar{E}_{mi}^2 \right)^{\frac{1}{2}}, \quad (3)$$

where w_i is proportional to the reciprocal of the variance of the daily maximum temperatures at the i -th station.¹ This insures that each station will contribute equally to the overall mean, $\bar{\bar{E}}_m$, which has been listed in table 2 and plotted in fig. 2.

3. Analysis of root-mean-square errors

The root-mean-square errors can be expressed as

$$E^2 = E_r^2 + E_{st}^2 + E_c^2 + E_{sm}^2 + E_s^2, \quad (4)$$

where E_r is the random component, E_{st} is due to the station used, E_c is due to a change in the location or orientation of the thermometer, E_{sm} is due to the seasonal march of temperatures, and E_s is due to singularities. Considering only a single station E_{st} sets the level of the curve and in no way affects the choosing of the optimum number of days.² E_c has a

¹ The interest here is in the variation of $\bar{\bar{E}}_m$ as m changes. If no weights were used in (3), then the contribution of each station to the variation of $\bar{\bar{E}}_m$ would be directly proportional to the variations of \bar{E}_m . Examination of fig. 2 reveals that \bar{E}_m is more variable at some stations. The weights, w_i , assure that each station contributes equally to the variations of $\bar{\bar{E}}_m$. The weights were $w_i = c\sigma_i^{-2}$ where σ_i^2 is the variance of the temperatures at the i -th station and c is a constant such that

$$\sum_1^{10} w_i = 1.$$

² The method of averaging used in (3) was designed to remove the effects of E_{st} on the mean curve.

TABLE 2. Root-mean-square-errors for predictions one, five, and nine years ahead.

<i>m</i>	1	3	5	7	11	15	21	31
Date	Blue Hill—January							
15	12.143	12.055	12.035	12.021	12.011	12.011	11.990	12.028
20	12.902	12.870	12.790	12.792	12.696	12.749	12.780	12.805
25	11.643	11.971	11.918	11.944	11.959	11.898	11.849	11.771
30	8.882	8.778	8.747	8.739	8.761	8.784	8.888	9.021
All	11.49	11.52	11.48	11.48	11.459	11.462	11.47	11.49
	Blue Hill—July							
15	6.623	6.471	6.513	6.549	6.482	6.500	6.464	6.440
20	7.141	7.151	7.135	7.078	7.018	7.030	6.980	6.930
25	6.743	5.972	5.785	5.637	5.630	5.662	5.627	5.652
30	7.860	7.851	7.633	7.755	7.462	7.507	7.679	7.837
All	6.98	6.90	6.80	6.80	6.68	6.71	6.73	6.76
	Duluth—July							
15	8.579	9.216	9.159	9.365	9.220	9.276	9.291	9.074
20	7.808	7.968	8.111	8.216	8.295	8.394	8.355	8.231
25	6.497	5.602	5.474	5.491	5.692	5.914	5.948	5.927
30	9.003	8.877	8.770	8.785	8.831	8.936	8.904	8.803
All	8.03	8.04	8.01	8.10	8.13	8.24	8.23	8.10
	Havre—January							
15	22.393	22.516	22.697	22.848	22.866	23.032	22.814	22.646
20	23.658	23.736	23.836	23.861	23.704	23.831	24.002	24.172
25	24.129	24.607	25.263	25.435	25.181	24.963	25.085	25.550
30	18.400	18.172	18.021	17.937	17.967	18.000	17.967	17.940
All	22.26	22.39	22.62	22.69	22.59	22.61	22.63	22.76
	Key West—January							
15	6.624	6.462	6.398	6.393	6.424	6.406	6.395	6.376
20	6.259	6.075	6.029	6.008	5.991	5.955	5.979	5.965
25	6.992	6.852	6.823	6.783	6.735	6.710	6.681	6.622
30	5.989	6.166	6.178	6.142	6.054	5.951	5.869	5.807
All	6.48	6.40	6.36	6.34	6.31	6.26	6.24	6.20
	Key West—July							
15	1.885	1.900	1.939	1.920	1.968	1.972	1.990	1.963
20	2.365	2.453	2.484	2.513	2.487	2.476	2.457	2.430
25	2.426	2.373	2.381	2.373	2.345	2.333	2.321	2.321
30	1.639	1.727	1.732	1.759	1.795	1.851	1.861	1.881
All	2.13	2.16	2.18	2.18	2.19	2.19	2.19	2.18
	New Orleans—July							
15	4.210	4.208	4.181	4.172	4.148	4.132	4.117	4.101
20	4.431	4.491	4.505	4.445	4.403	4.408	4.391	4.382
25	3.759	3.815	3.774	3.771	3.707	3.672	3.634	3.601
30	3.088	3.073	3.064	3.100	3.067	3.052	3.040	3.037
All	3.94	3.97	3.95	3.94	3.90	3.89	3.87	3.85
	San Diego—January							
15	5.590	5.541	5.488	5.457	5.433	5.446	5.431	5.415
20	6.201	6.239	6.139	6.083	6.022	6.007	5.980	5.966
25	7.050	7.075	7.022	7.007	7.083	7.133	7.121	7.045
30	5.755	5.687	5.619	5.478	5.478	5.378	5.405	5.412
All	6.12	6.12	6.04	5.98	5.97	6.08	5.97	5.94
	Tatoosh Island—January							
15	6.449	6.409	6.425	6.446	6.445	6.448	6.438	6.465
20	5.899	5.908	5.825	5.798	5.756	5.720	5.693	5.672
25	6.190	6.166	6.149	6.160	6.143	6.155	6.205	6.115
30	7.238	7.212	7.219	7.214	7.210	7.201	7.149	7.108
All	6.46	6.44	6.43	6.43	6.41	6.40	6.39	6.36
	Tucson—January							
15	7.810	7.589	7.502	7.461	7.450	7.431	7.402	7.384
20	10.837	10.677	10.618	10.583	10.500	10.427	10.384	10.394
25	9.644	9.593	9.344	9.343	9.535	9.353	9.294	9.316
30	8.827	8.807	8.771	8.781	8.620	8.541	8.485	8.509
All	9.35	9.17	9.13	9.11	9.10	9.01	8.96	8.97
All stations	5.036	5.027	5.010	5.010	4.990	4.990	4.983	4.970

similar effect, since moving the thermometer serves only to raise the entire curve and not to change its shape.³ The seasonal influence, E_{sm} , is quite small. It

³ In preparing daily normals, it is, of course, desirable to remove this effect. This can often be done by using information from nearby stations.

arises from the fact that the arithmetic mean of a number of daily means is being used to approximate the true seasonal march of temperatures. However, it is well known that the annual variation of temperatures is closely approximated by a cosine curve, at least in temperate latitudes [6]. It has been shown (pp. 28–31 of [7]) that the formation of predictors by taking averages of daily means instead of fitting a single harmonic has little effect on the root-mean-square errors. Thus the seasonal influence on E can be neglected. Therefore, in equation (4), the variations of E with changing m are due primarily to E_r and E_s .

Consider the case of no singularities—that is, E_s equal to zero. Then the only problem is to estimate the true smooth curve. With a finite sample, this cannot be done exactly since

$$E_r^2 = E_p^2 + E_m^2, \tag{5}$$

where E_p is due to the variation of the temperatures about the true mean and E_m is due to the error in estimating the true mean. E_p is an irremovable random variation and it is the value which a curve in fig. 2 should approach asymptotically as the degrees of freedom used in estimating the true mean approach infinity. If only one daily mean is used in this estimation, then, with a twenty year record, there are about twenty degrees of freedom. Using nearby daily means increases the number of degrees of freedom, although not by twenty for each mean because of the correlation between daily temperature values. Thus, with only random influences, curves such as those in fig. 2 would decrease monotonically with increasing number of days of smoothing and approach some positive value asymptotically.

Even if singularities are a true meteorological phenomenon, in order to be of practical importance in predicting daily temperatures one, five, and nine years ahead, they must be large enough for their influence, as measured by E_s , to overcome the reduction in error due to increasing the degrees of freedom.

The all-station curve is based on the predictions at all stations for all days and thus has the widest applicability. It decreases monotonically with increasing number of days of smoothing, indicating that singularities are, on the average, not useful in prediction. This curve shows that in forecasting daily temperatures far in the future it is better to use a smoothed curve of daily means. In the preparation of daily normal values, a national weather service must use the same procedure at all stations; otherwise, it would be difficult to make climatological comparisons between stations. The all-station curve indicates that a large interval of days should be used in the preparation of daily normal values.

It would be extremely difficult to determine the

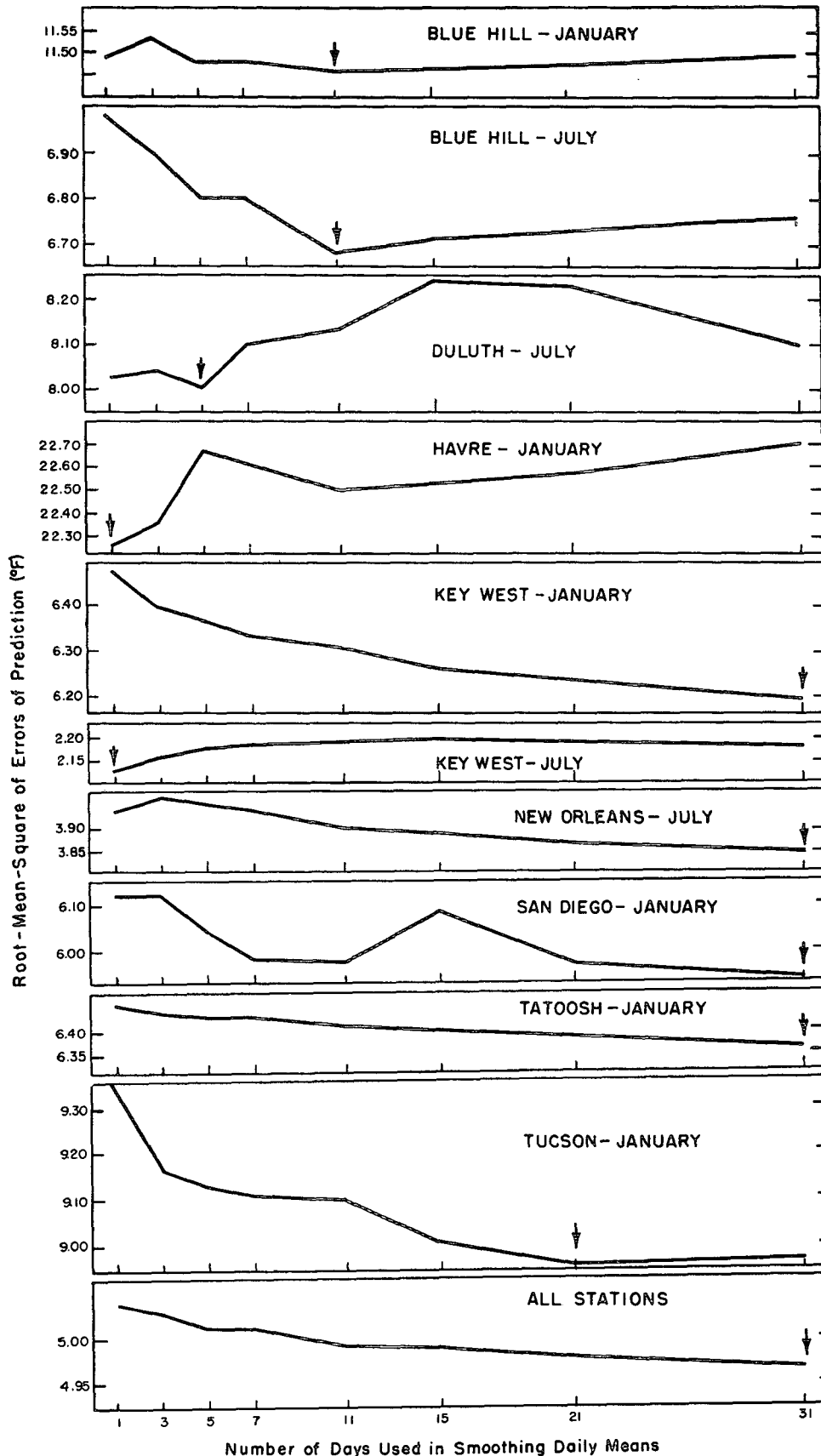


FIG. 2. Root-mean-square of errors of prediction one, five, and nine years in the future (°F). Based on 216 predictions at each station for each *m*.

statistical significance of the all-station root-mean-square error values (\bar{E}_m). The tests depend upon the spatial and temporal relationships among the daily maximum temperatures, and these may be quite complex. However, each point on the curve is based on 2160 predictions (54 at each of 4 days at 10 stations). Even though the predictors are not independent of each other, it is probably true that the variations in \bar{E}_m are not due to small sample size.

A striking feature of the curve is its extreme flatness. The range is only 0.08 degree Fahrenheit, and this is less than two per cent of the average magnitude of the curve. Thus, from a practical view point, it makes very little difference whether singularities are or are not considered in predictions of daily maximum temperatures far into the future. Practical considerations of economy and availability of the smoothed or unsmoothed values should be the primary governing factors.

From fig. 2, it can be seen that the optimum interval of days of smoothing varies considerably from station to station. The stations were combined into groupings such as Summer-Winter, North-South, East-Central-West, *etc.*, and the root-mean-square errors of the individual central days (table 2) were intensively examined in an attempt to discover some possible explanation of the differences. No definite pattern was found. For example, the only two stations for which no smoothing is required are Havre—January and Key West—July, and it would be difficult to find two more climatologically diverse stations in the United States. Also, both Blue Hill—January and Blue Hill—July require the same amount of smoothing, indicating no seasonal difference; but Key West—January and Key West—July require a maximum and minimum interval of days, respectively, indicating a seasonal difference. The two midcontinent stations of Duluth and Havre both require small intervals of days, and it is possible that there is a continental-maritime difference. However, simply by taking enough combinations, one of them is sure to indicate a pattern so that many more stations would be required to establish a relationship.

The individual station curves are also rather flat. For example, the error introduced by using a thirty-one day interval as a predictor rather than the optimum interval indicated by the station curves is listed in table 3. The greatest difference is only one-half degree Fahrenheit at Havre, Montana, which is 2.2 per cent of the average magnitude of the Havre curve. This is rather small, and for many applications the interval of days of smoothing may not be too important.

Six of the ten station root-mean-square error curves have minima at less than thirty-one days. Does this

TABLE 3. Comparison of skill in predictability using a thirty-one day average and the best average.

	Interval of days giving best predictions (minimum \bar{E}_m)	Difference between E_{31} and minimum \bar{E}_m (deg F)	Ratio of difference to average \bar{E}_m (in per cent)
Blue Hill—January	11	0.03	0.3
Blue Hill—July	11	0.05	0.7
Duluth—July	5	0.09	1.1
Havre—January	1	0.50	2.2
Key West—January	31	0.00	0.0
Key West—July	1	0.05	2.3
New Orleans—July	31	0.00	0.0
San Diego—January	31	0.01	0.0
Tatoosh Island—January	31	0.00	0.0
Tucson—January	21	0.01	0.1

necessarily mean there are singularities at these stations and not at the other four, or could chance fluctuations give this result? As noted above, an answer to this question would require a knowledge of the sampling fluctuations of each curve. For example, there are eight values on each curve, and it would be necessary to know the probability of any one of them being as low or lower than the root-mean-square error for thirty-one days simply by chance. In addition, the six stations have been selected out of ten and the probability of this occurring by chance must also be known. Statistical problems of this type are extremely difficult, and solutions have been obtained only in very special cases with many simplifying assumptions. However, the results in table 3 show that even if singularities exist at individual stations they are relatively unimportant as far as predictability is concerned.

Of the forty individual station-days (table 2), twenty-three have minimum root-mean-square errors at less than thirty-one days. In particular, the values of July 20 at Duluth, January 15, 20, and 25 at Havre, and July 15, 20, and 30 at Key West are all generally increasing with increasing interval of days. The same problems of sampling variability and selectivity apply, and it simply is not possible to decide which, if any, of these or other singularities are real.

4. Conclusion

Although it is a difficult task to establish statistically whether or not singularities in daily maximum temperatures are real meteorological phenomena, it has been shown that it is possible to answer the simpler question of how useful they are for predicting daily values far into the future. The all-station curve of fig. 2 indicates that any singularities in daily maximum temperatures which may possibly exist are not sufficiently large and wide spread, in both space and time, to be of use in prediction. This certainly

does not necessarily prove that there are no singularities, since combination of the data into one curve will tend to mask a few singularities at individual stations on specific days.

At specific stations and days, it was not possible to make conclusive statements about the reality of singularities. Table 3 gives an indication of the maximum prediction error made by using smoothed values when, in fact, singularities are real. It is concluded that, from a practical view point, it makes very little difference whether or not singularities are real. Table 3 is also useful in making decisions about how much improvement in forecasting, if any, can possibly be made by using singularities, and thus it may serve as a guide for the amount of effort to be expended in searching for them.

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