THE RELATIVE IMPORTANCE OF DIFFERENT HEAT-EXCHANGE PROCESSES IN THE LOWER STRATOSPHERE[1]

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(Manuscript received 7 July 1958)

ABSTRACT

The temperature variation at a fixed point in the free atmosphere or at a point which moves vertically with an isobaric surface is a result of the combined effect of radiation, horizontal advection of heat, vertical convection of heat, and other heat-exchange processes. A method is presented by which the 12-hr local temperature change that would have resulted from radiative processes alone may be estimated. This method and the conventional method of determining the 12-hr local temperature change due to the horizontal advective process lead to a way of estimating the relative importance of different heat-exchange processes.

Following these methods, the 12-hr local temperature changes in the lower stratosphere over a portion of the United States due to radiation, horizontal advection, and the vertical convective and eddy heat exchange processes combined are separately estimated from many years of radiosonde data and daily isobaric maps. The results of this study show that over the southeastern United States, at least, and probably over a much wider area, the horizontal advective process is more important than radiation in influencing the 12-hr local temperature change at the 200-mb and the 100-mb levels. These two processes are probably of equal importance near the 50-mb level. At the 200-mb level, the vertical convective and the eddy heat-exchange processes combined are estimated to be as important as, or even more important than, the radiative processes over the same area. These conclusions are probably true for both winter and summer, but especially for winter.

1. Introduction

The local temperature variation is here defined either as the temperature variation with time at a fixed point in the free atmosphere or the temperature variation with time at a point which moves vertically with an isobaric surface. In either case, the local temperature variation is a result of the combined effects of radiation, horizontal advection and vertical convection of heat, and other heat exchange processes. A quantitative assessment of the relative importance of these processes causing the local temperature variation in the stratosphere is essential to the understanding of the nature and characteristics of the stratosphere. To the writers' knowledge, such an assessment from observed data has not yet been made. The purpose of this study is, therefore, to determine the relative importance of the major heat-exchange processes in the lower stratosphere, the region from the tropopause to about 20 km level, over the United States of America from observed data.

2. The processes that influence the local temperature variation

The data used in this investigation consist of radiosonde observations, which are reported for standard constant-pressure surfaces, and computations from constant-pressure charts. It is therefore advisable to develop pertinent equations in the x, y, p, and t system.

In this system, the total change of temperature following a particle may be expanded as

$$\frac{dT}{dt} = \left(\frac{\partial T}{\partial t}\right)_p + \vec{v}\cdot(\nabla T)_p + \frac{dp}{dt}\left(\frac{\partial T}{\partial p}\right)_p, \quad (1)$$

where the subscript is simply used to indicate that the x, y, p, and t system is considered. $(\partial T/\partial t)_p$ is the local temperature variation on a constant-pressure surface. $\vec{v}$ is the horizontal velocity vector and is the same vector as in the x, y, and z system. $(\nabla T)_p$ is the temperature gradient along an isobaric surface. dp/dt is the total change of pressure following a particle so that $(-dp/dt)/\rho g$ represents the vertical motion of the particle relative to a constant-pressure surface. $(\partial T/\partial p)_p$ is the change in temperature in the vertical direction per unit pressure change.

The first law of thermodynamics is given by

$$\frac{dT}{dt} = \frac{1}{c_p}\frac{dQ}{dt} + \frac{1}{c_p\rho}\frac{dp}{dt}, \quad (2)$$

where c_p is the specific heat at constant pressure and dQ is the heat per unit mass added to the air particle considered.

From equations (1) and (2), it follows that

$$\left(\frac{\partial T}{\partial t}\right)_p = -\vec{v}\cdot(\nabla T)_p - \omega(\gamma_d - \gamma) + \frac{1}{c_p}\frac{dQ}{dt}, \quad (3)$$

[1] The research reported in this paper has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center under contract AF 19(604)–1755.

where ω is the vertical velocity of the air particle relative to an isobaric surface, γ_d is the dry adiabatic lapse rate, and γ is the lapse rate of air temperature.

Equation (3) shows that the local temperature variation on a constant-pressure surface is brought about by three terms:

(1) The first term on the right represents the contribution due to horizontal temperature advection on an isobaric surface. This term will be designated as A.
(2) The second term on the right represents the contribution due to vertical temperature convection. This term will be designated as C.
(3) The last term represents the contribution due to non-adiabatic processes, such as eddy transport, radiation and conduction of heat. Since the atmosphere is a poor conductor, the contribution due to conduction of heat will be neglected.

It is possible to subdivide each of the above three terms into several parts. For example, the horizontal temperature advection, A, at a location, may possess a seasonal trend—*i.e.*, a tendency of having a net cold advection in one season and a net warm advection in another season. A part of A may therefore be assigned to represent this trend. Here A is subdivided into four parts:

$$A = A_1 + A_2 + A_3 + A_{Np}, \tag{4}$$

where A_1 is the part representing the seasonal trend of A. It is periodic with a period of one year. A_2 is the periodic temperature advection with a period of 24 hr. It may result from the tidal oscillation of the atmosphere. A_3 is the periodic temperature advection of all other periods (*i.e.*, 12 hr, 6 hr, *etc.*). A_{Np} is the non-periodic part of A. It is the major part of A that is associated with the more-or-less-random synoptic temperature variations.

Similarly, the vertical temperature convection, C, is subdivided into four parts:

$$C = C_1 + C_2 + C_3 + C_{Np}. \tag{5}$$

The relation between each part of C and C itself is exactly like that between each corresponding part of A and A itself. The only exception is that C_2 may be induced by the atmospheric tidal oscillation as well as by the vertical motion of the isobaric surface in response to the periodic heating and cooling of the surface by radiative processes.

The last term of equation (3) will be considered to have two parts—*i.e.*,

$$\frac{1}{c_p}\frac{dQ}{dt} = E + R, \tag{6}$$

where E is the portion of $(1/c_p)(dQ/dt)$ brought about by the turbulent eddy exchange of heat and R is the portion of $(1/c_p)(dQ/dt)$ attributed to radiative processes (including the absorption and emission of heat). The contribution to $(1/c_p)(dQ/dt)$ due to the conduction of heat is considered negligible.

Like A and C, R is further subdivided into four parts—*i.e.*,

$$R = R_1 + R_2 + R_3 + R_{Np}, \tag{7}$$

where R_1, with a period of one year, may result from the seasonal shift of the solar angle. R_2, with a period of 24 hr, is a direct consequence of the diurnal variation of the intensity of solar and terrestrial radiation. Since the ground temperature is, in the mean, higher during the daytime than during the nighttime, terrestrial radiation should have some contribution to R_2. R_3, representing the sum of the periodic part of R with periods of 12 hr, *etc.*, may arise from the fact that the daily temperature maximum and minimum are not exactly 12 hr apart. R_{Np}, representing the non-periodic temperature variation due to radiative processes, may result from the atmospheric absorption of the terrestrial radiation which is made irregular by varying ground conditions, weather conditions, and cloud amounts.

With the above subdivision of the terms A, C, and $(1/c_p)(dQ/dt)$, equation (3) may be written as

$$\left(\frac{\partial T}{\partial t}\right)_p = A_1 + A_2 + A_3 + A_{Np} + C_1 + C_2 + C_3 + C_{Np} + E + R_1 + R_2 + R_3 + R_{Np}. \tag{8}$$

3. Method of separation of the processes that influence the local temperature variation

In order to compare the relative importance of different processes that influence the local temperature variation, it is necessary to determine the magnitude of the local temperature variation that would have been caused by any one of the processes alone. The method of singling out the periodic radiative processes from other processes, as employed by Chiu (1956), and the method of evaluating the horizontal temperature advection, A, will be described here.

Consider the local temperature change between two fixed hours in a day. According to equation (8), this local temperature is brought about by many components, some of them periodic in character, and some of them not. The local temperature change between two fixed hours brought about by those periodic components with a period of 24 hr/l, where l is a positive integer, will be the same day after day. The important components having a period of 24 hr/l are those with $l = 1$ (*i.e.*, periodic components with a period of 24 hr, such as R_2) and those with $l = 2$ (*i.e.*, periodic components with a period of 12 hr, such as part of R_3). Therefore, if the local temperature change between two fixed hours is obtained from radiosonde data for a large number of days and then averaged over the days, the contributions to the local temperature change due to these particular periodic components, such as R_2 and part of R_3, will not be averaged out, but remain of a definite magnitude.

On the other hand, the local temperature change between two fixed hours brought about by those non-periodic components or periodic components with a period other than that of 24 hr/l may vary both in magnitude and sign from one day to the next. The average of this kind of local temperature change for a large number of days will be zero, or very nearly so.

When the time interval between two fixed hours in a day is taken as 12 hr, the contribution to the local temperature change from those periodic components with a period of 24 hr/$2l$, l being a positive integer, is zero. Thus, only those periodic components with a period of 24 hr/s, s being an odd positive integer, will contribute to the average 12-hr local temperature change. If the daily 12-hr temperature change is represented by $(\partial T/\partial t)_p \Delta t$, where $\Delta t = (t_2 - t_1) = 12$ hr, and the averaging process by a bar, then equation (8) takes the following form when averaged over a large number of days (large enough to average out the seasonal trends in temperature variation).

$$\bar{T}(t_2) - \bar{T}(t_1) = \overline{\left(\frac{\partial T}{\partial t}\right)_p \Delta t} = \overline{A_2 \Delta t} + \overline{C_2 \Delta t} + \overline{R_2 \Delta t}, \quad (9)^2$$

where the contributions to $\overline{(\partial T/\partial t)_p \Delta t}$ due to those periodic components with a period of 8 hr or less are considered negligible.

The term C_2, as mentioned before, may be induced by the tidal oscillation of the atmosphere as well as by the vertical motion of the isobaric surface in response to the radiative heating or cooling of the surface. The term A_2, if it exists, is probably induced by the tidal oscillation of the atmosphere and is likely smaller than C_2.

Investigation by Chiu (1959) shows that the possible periodic temperature variation in 12 hr due to the tidal vertical motion of the atmosphere is extremely small. It may easily be shown that the temperature variations on an isobaric surface due to the periodic radiative heating and cooling of the surface in 12 hr is also small compared to that due to the radiative heating and cooling itself.

For this reason, equation (9) may be reduced to the following form without loss of much accuracy:

$$\bar{T}(t_2) - \bar{T}(t_1) = \overline{\left(\frac{\partial T}{\partial t}\right)_p \Delta t} = \overline{R_2 \Delta t}. \quad (10)$$

Thus, $\overline{R_2 \Delta t}$ may be singled out by averaging the daily values of $\overline{(\partial T/\partial t)_p \Delta t}$ over a large number of days. The magnitude of $\overline{R_2 \Delta t}$ depends on what part of the day is chosen for the 12-hr period, Δt. In order to obtain the largest possible magnitude of $\overline{R_2 \Delta t}$, Δt should be chosen to fall entirely within the radiative warming (or cooling) period of the day. Since $R_2 \Delta t$ is the same every day, $\overline{R_2 \Delta t}$ is also equal to $|\overline{R_2 \Delta t}|$.

Of course, the quantity one would like to single out is R, not just R_2. Physical considerations tell us that R_3, representing the radiative temperature variation with a period of 12 hr or any other less prominent periods, is much smaller than R_2. The contribution of R_1 to the temperature change from one season to another could be large. But its average contribution to the temperature change in 12 hr is much smaller than that due to R_2. For example, if R_1 causes a temperature change in the free air over the United States of 10C from mid-summer to mid-winter (which is probably larger than the actual half-year radiative temperature variation over this region), the average temperature change in 12 hr due to R_1 is only about 0.025C. This certainly can be neglected against $\overline{R_2 \Delta t}$ which is about 0.2C or larger in the lower stratosphere as will be shown later. Neglecting R_1 and R_3, equation (7) reduces to

$$R = R_2 + R_{Np}. \quad (11)$$

It may be shown that (see Chiu and Greenfield, 1957), if $\overline{R_{Np} \Delta t} = 0$ is assumed,[3] one may write with confidence that

$$|\overline{R \Delta t}| \cong |\overline{R_2 \Delta t}|. \quad (12)$$

The last equation provides a way for estimating $|\overline{R \Delta t}|$.

The other portion of the local temperature variation that may be accurately singled out is the local temperature variation that would have resulted from the horizontal advective process alone. This portion of the temperature variation is represented by $\vec{v} \cdot (\nabla T)_p$ in equation (3), or A in equation (4). The value of A for

[2] If each of A, C and R has a long-term average value that is not zero, then on the right hand side of equation (9) there should be three additional terms, $\overline{A_{Np} \Delta t} + \overline{C_{Np} \Delta t} + \overline{R_{Np} \Delta t}$. But it will be assumed that the temperature at a point, outside of its periodic fluctuations, is in the long run not changing and so the sum of these three terms is zero or negligible.

[3] This amounts to the assumption that there is no long term average radiative cooling or heating in the lower stratosphere. If this condition is not fulfilled, one has only further to break down R_{Np} into two components, say R_{Np}' and R_{AV}. Then R_{Np}' averages to zero and R_{AV} does not. In that case equation (12) becomes $|\overline{R \Delta t}| \cong |\overline{R_2 \Delta t}| \pm |\overline{R_{AV}}|$. According to Ohring (1958), the average net radiative flux divergence in winter as well as in summer for the whole layer from the tropopause to the 55-km level at latitudes 30N to 40N (the latitudes of the radiosonde stations used in the present study) is about 0 to 10^{-2} calorie per cm^2 per min. This heat flux divergence may cause a temperature change of about 0.1C in 12 hr at these latitudes. As may be seen later, this is smaller than $\overline{R_2 \Delta t}$. For the purpose of this study, equation (12) is sufficient.

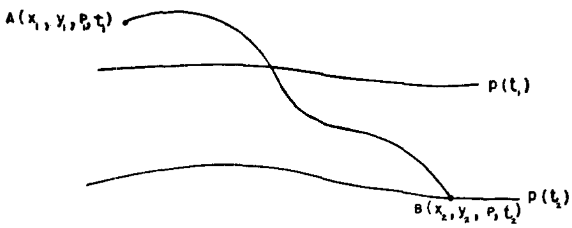

FIG. 1. Positions of an air particle and of an isobaric surface at two different times.

a finite interval of time—*i.e.*, $A\Delta t$, may be evaluated from data on isobaric maps.

In fig. 1, surfaces $p(t_1)$ and $p(t_2)$ represent the hypothetical positions of an isobaric surface at times t_1 and t_2, respectively. $B(x_2, y_2, p, t_2)$ is a point on surface $p(t_2)$ and is directly over the observing station. Suppose an air particle which coincides with B at t_2 were at position $A(x_1, y_1, p_1, t_1)$ at t_1, where in general $p_1 \neq p$. Then the temperature variation following this particle in the time interval from t_1 to t_2 is

$$\begin{aligned}\int_{T \text{ at } A}^{T \text{ at } B} dT &= T(x_2, y_2, p, t_2) - T(x_1, y_1, p_1, t_1)\\ &= T(x_2, y_2, p, t_2) - T(x_2, y_2, p, t_1)\\ &\quad + T(x_2, y_2, p, t_1) - T(x_1, y_1, p, t_1)\\ &\quad + T(x_1, y_1, p, t_1) - T(x_1, y_1, p_1, t_1). \qquad (13)\end{aligned}$$

Comparison of equation (13) and the integral of equation (1) shows that

$$-\int \vec{v}\cdot(\nabla T)_p dt = T(x_1, y_1, p, t_1) - T(x_2, y_2, p, t_1). \qquad (14)$$

In order to evaluate this integral, it is necessary to construct the isobaric trajectory of this particle from time t_1 to t_2. One half of its trajectory, corresponding to $t_1 + \frac{1}{2}(t_2 - t_1)$ to t_2, is constructed upstream from point B according to the geostrophic wind at point B on surface $p(t_2)$. The other half, corresponding to t_1 to $t_1 + \frac{1}{2}(t_2 - t_1)$, is constructed upstream from the terminal point of the first half of the trajectory according to the geostrophic wind at this point on surface $p(t_1)$. Should the wind field change considerably in the distance covered by either half of the trajectory, that half trajectory may be constructed in two steps, each step corresponding to one fourth of the time interval $t_2 - t_1$, in a manner similar to what has just been described, to allow for the wind change.

The integral in equation (14) is obtained by placing the entire isobaric trajectory on surface $p(t_1)$ and then subtracting the temperature at the downstream endpoint (x_2, y_2, p, t_1) from the temperature at the upstream endpoint (x_1, y_1, p, t_1). This difference represents the local temperature variation as a result of the horizontal temperature advection during the chosen time interval. This method of determining the horizontal advection of some quantity is quite standard and has been applied to the determination of the temperature change following a point on an isobaric surface by Miller (1948) and to the determination of the horizontal advection of vorticity by Spar *et al* (1955).

For the comparison of the relative importance of radiative processes and the advective process, the following two points should be noticed:

(1) It is important that the length of the time interval $t_2 - t_1$ chosen for the study of the advective process should be the same as that chosen for the study of radiative processes. But the portion of the day comprising this time interval $t_2 - t_1$ does not have to be the same in these two cases, for in the case of horizontal advection of temperature, there is no preferable hour of the day for the onset or termination of strong or weak, warm or cold advection. Therefore, the average of the absolute magnitudes of temperature advection for a certain length of time, $t_2 - t_1$, will be the same irrespective of the portion of day this time interval occupies. As long as the length of the time interval, $t_2 - t_1$, used in the study of radiative warming (or cooling) is the same as that in the study of advective temperature change, the magnitudes of the contributions may be compared.

(2) Since the temperature variation for a fixed time interval of the day due to the advective process may change both in sign and magnitude from day to day, the average of its absolute magnitudes, but not its algebraic magnitudes, should be used for evaluation of the relative importance of the advective and radiative processes.

According to the method described above, the temperature advection for the time interval $t_2 - t_1$ will be obtained for a large number of days, N, at each of the stations and levels studied. The average of the absolute magnitudes of these daily values of the temperature advection at each station and level will then be computed for the evaluation of the importance of the advective process.

To obtain an idea of the importance of the vertical convective and the eddy heat-transport processes, the average absolute magnitude of the temperature variation in the time interval $t_2 - t_1$ due to all the heat exchange processes should first be obtained. From equations (3) and (6) it follows that

$$\begin{aligned}\overline{|(\partial T/\partial t)_p \Delta t|} &= \overline{|A\Delta t + C\Delta t + E\Delta t + R\Delta t|}\\ &\leqslant \overline{|A\Delta t|} + \overline{|C\Delta t + E\Delta t|} + \overline{|R\Delta t|}. \qquad (15)\end{aligned}$$

Therefore, if every term except the convective term and the eddy term in equation (15) is known, a rough estimate of the combined importance of the convective term and the eddy term may be made.

In the discussion that led to equation (9), it was assumed that $\overline{(\partial T/\partial t)_p \Delta t}$ is averaged over a large number of days, large enough to average out the terms representing its seasonal trends, such as A_1, C_1, and R_1 in equation (8). If $(\partial T/\partial t)_p \Delta t$ is averaged over a season or over the same season for a number of years, then, instead of equation (9), equation (8) should lead to

$$\overline{(\partial T/\partial t)_p \Delta t} = \overline{A_1 \Delta t} + \overline{C_1 \Delta t} + \overline{R_1 \Delta t} + \overline{A_2 \Delta t} + \overline{C_2 \Delta t} + \overline{R_2 \Delta t}. \quad (9a)$$

A study by Chiu (1959, paragraph 3.4) indicates that when $(\partial T/\partial t)_p \Delta t$ is so averaged the terms representing the seasonal trends have a tendency to cancel each other. Whatever residual effects remain, due to the first three terms on the right of equation (9a) that are not cancelled, should show themselves in the mean seasonal temperature change. This mean seasonal temperature change may be large from winter to summer. But its average contribution to the temperature change in 12 hr is small by comparison with $\overline{R_2 \Delta t}$, and so equation (9) is valid to a very good approximation.

4. Estimate of the magnitude of $|\overline{R\Delta t}|$

From about five years (1952 to 1956) of radiosonde data over the United States, taken by fifteen radiosonde stations that had been conducting four observations (one every 6 hr) a day, the mean temperatures at the four observational hours were obtained separately for winter and summer at the 200-mb, 150-mb, 100-mb, and 50-mb levels of the stratosphere. For any season and level, the difference in the mean temperatures at any two observational hours, which are separated by 12 hr, is $\overline{(\partial T/\partial t)_p \Delta t}$ for that season and level.

From the discussion given in section 3, it is clear that the magnitude of $\overline{(\partial T/\partial t)_p \Delta t}$, which largely represents $\overline{R_2 \Delta t}$, would depend on how the 12-hr period, Δt, is chosen from the day. In order to obtain the largest possible value of $\overline{(\partial T/\partial t)_p \Delta t}$, at each season and level the mean temperatures at different observational hours for all the stations were plotted on one diagram according to the local time of each station observation. A diurnal temperature curve was clearly evident in each of these diagrams. By applying the method of harmonic analysis, the amplitude of the periodic diurnal temperature variation for each season and level was determined. The details of the determination of the mean temperatures at different observational hours and of the determination of the amplitudes of the periodic diurnal temperature variation at different seasons and levels are fully described in another paper (Chiu, 1958). After the possible errors in $\overline{(\partial T/\partial t)_p \Delta t}$ due to direct solar radiation on the temperature sensing element and due to other causes were allowed for, it was found that $|\overline{R_2 \Delta t}|$ has the following magnitudes.

TABLE 1. The magnitude of $|\overline{R_2 \Delta t}|$.

	Amplitude in C, winter	Amplitude in C, summer
200 mb	0.15	0.2
150 mb	0.15	0.2
100 mb	0.2	0.25
50 mb	—	0.35

From equation (12) the magnitude of $|\overline{R_2 \Delta t}|$ given above will be considered as the representative value of $|\overline{R\Delta t}|$ in future discussions.

5. Determination of the magnitude of $|\overline{A\Delta t}|$

a. Maps used and sample stations selected for this determination. Two types of maps were used for this determination.

(1) The 200-mb charts, covering the United States and its immediate surroundings only, were prepared by the Weather Bureau Analysis Center for transmission on the facsimile circuit. Two maps each day, 0300 and 1500 GMT, for the following periods were used in this study:

For winter	*For summer*
January through February 1953	June through August 1953
January through February 1954	June through August 1954
December 1954 through Feb. 1955	June through August 1955
December 1955 through Feb. 1956	June through August 1956

(2) The 100-mb and 50-mb charts, covering the whole northern hemisphere, were prepared by the Weather Bureau on a project sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center. These maps were available for only seven months (not all consecutive) of 1953, and only the 0300 GMT chart was prepared. Consequently, only the maps for January and February 1953 were used in this study. For easier construction of air trajectories on these maps, the original version of these maps (about 27 in × 32 in) obtained from the U. S. Weather Bureau was used. A smaller edition of these maps was published by the U. S. Weather Bureau (1956).

Since the construction of trajectories was quite a laborious and time-consuming operation, it was practical to limit the study of the horizontal temperature advection to a few sample stations in the United States. The stations chosen were:

Rome, New York,
Washington, D. C.,
Montgomery, Alabama,
Ft. Leavenworth, Kansas.

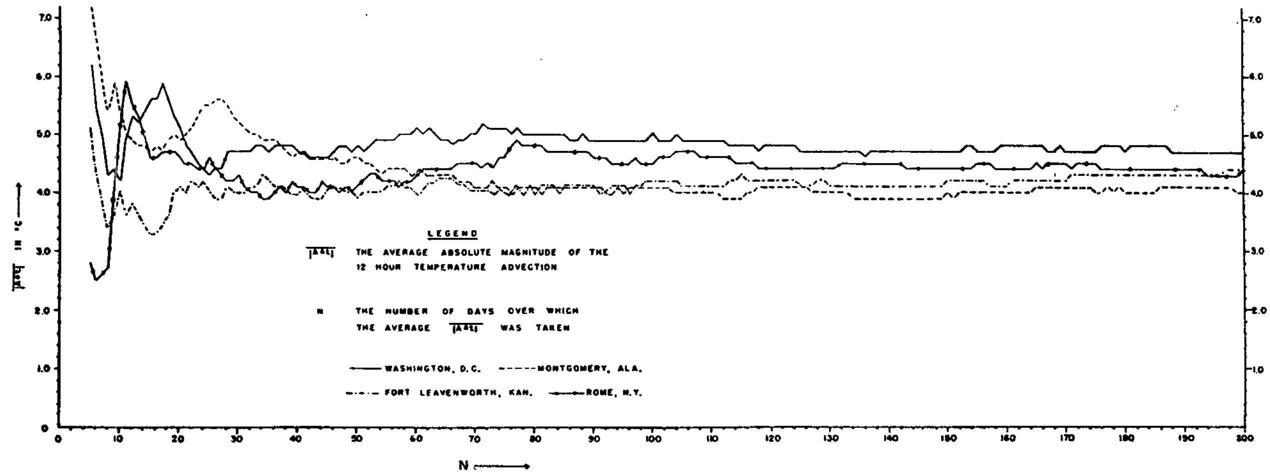

FIG. 2. The successive average absolute magnitude of $A\Delta t$ at the 200-mb level in winter.

No stations near the west coast of the United States were chosen because trajectories from these stations often led to regions near or beyond the limits of the 200-mb charts.

All the above stations were used for the determination of the temperature advection at the 200-mb surface in winter. Only the last three stations were used for the determination of the temperature advection at the 100-mb and 50-mb levels in winter.

At the 200-mb level, Washington, D. C., Montgomery, and Ft. Leavenworth were often under the influence of a closed circulation in the summer time. The construction of 12-hr trajectories would not be accurate in this situation. Therefore, only Rome, which is not hampered by this limitation, was used for the study of the temperature advection at the 200-mb level in summer.

b. Results of this determination. Following the procedures described in section 3, one 12-hr isobaric trajectory, from 0300 to 1500 GMT, was constructed for every day (with only a few exceptions) of the period chosen at each of the stations selected. The small number of days eliminated from the study were those days on which the wind and temperature fields were such that they could lead to considerable errors in the construction of trajectories or in the interpolation of temperature at the endpoints of the trajectories. At the 100-mb and 50-mb levels, since only one map per day was available, the whole 12-hr trajectory was constructed from data on one map.

For each trajectory constructed, a corresponding 12-hr local temperature advection, $A\Delta t$, was determined from the temperature data on the isobaric map. In this manner, $A\Delta t$ was determined for almost every day of the chosen periods at the selected stations and levels shown by table 2. The average absolute magnitude of $A\Delta t$ was then computed from these daily values.

In order to demonstrate that the average absolute magnitude obtained is a good representation of the true average absolute magnitude, the following procedures were followed. At each station and level, the absolute magnitude of $A\Delta t$ for different days of the chosen periods were tabulated in chronological succession, from the first 12-hr temperature advection in 1953 to the last one in 1956 for the 200-mb level. Separate tabulations were made for winter and summer at each level. From these tabulations, the successive cumulative absolute magnitudes were obtained. Then successive average absolute magnitudes were obtained by dividing each cumulative absolute magnitude by the corresponding number of days of accumulation.

In figs. 2 and 3, the successive average absolute magnitude of $A\Delta t$ on the 200-mb surfaces is plotted

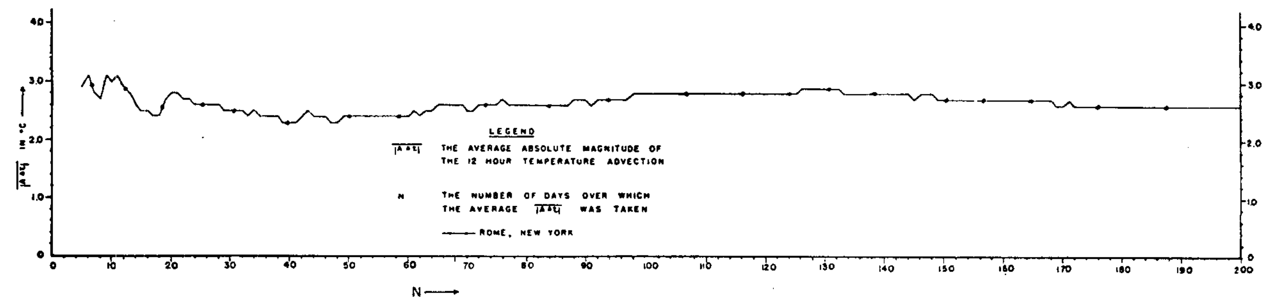

FIG. 3. The successive average absolute magnitude of $A\Delta t$ at the 200-mb level in summer.

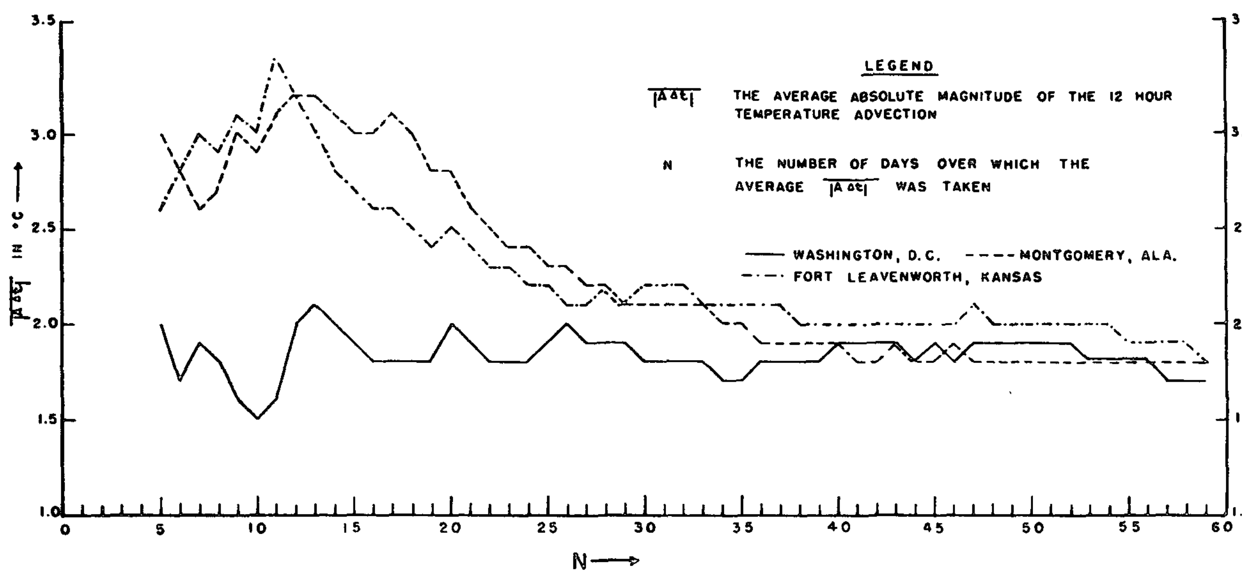

FIG. 4. The successive average absolute magnitude of $A\Delta t$ at the 100-mb level in winter.

against the total number of days, N, over which it is averaged. (The first four points are omitted in each case.) Fig. 2 presents results for winter conditions; fig. 3 is for summer.

From these figures, it is seen that the average absolute magnitude of $A\Delta t$ at the stations and levels shown becomes quite stable as N increases above 30 and remains practically at a constant value when N is 120 days or more. (Notice that the fluctuation of this average absolute magnitude after N reaches 120 is only about 0.1C. This is entirely negligible compared to its general magnitude of about 4.5C in winter and 2.6C in summer.) This indicates that in this case the average absolute magnitude of $A\Delta t$ taken at any N when $N > 120$ days should be a good estimate of the true average absolute magnitude of $A\Delta t$. Here its magnitude at $N = 200$ is taken as the representative absolute magnitude of $A\Delta t$.[4]

Figs. 4 and 5, in a scale twice as large as that of figs. 2 and 3, show the same quantity in winter at the 100-mb and 50-mb levels, respectively, over Washington, D. C., Montgomery, and Fort Leavenworth. Even though only two months' data were used at these two levels, the average absolute magnitude of $A\Delta t$ rapidly approaches a stable value as N increases. The average absolute magnitude at the end of each curve in these figures is taken as the representative absolute magnitude of $A\Delta t$ at the station and level for which the curve was constructed. These representative absolute magnitudes at different stations, levels, and seasons are summarized in the following table.

TABLE 2. The average absolute magnitude of the 12-hr temperature advection, $\overline{|A\Delta t|}$, and the standard deviation of the absolute magnitudes of the 12-hr temperature advection, $\sigma_{|A\Delta t|}$, in the lower stratosphere.

	Magnitude in C in winter						Magnitude in C, summer	
	200 mb		100 mb		50 mb		200 mb	
	$\overline{\lvert A\Delta t\rvert}$	$\sigma_{\lvert A\Delta t\rvert}$	$\overline{\lvert A\Delta t\rvert}$	$\sigma_{\lvert A\Delta t\rvert}$	$\overline{\lvert A\Delta t\rvert}$	$\sigma_{\lvert A\Delta t\rvert}$	$\overline{\lvert A\Delta t\rvert}$	$\sigma_{\lvert A\Delta t\rvert}$
Washington, D. C.	4.7	3.5	1.7	1.6	1.0	0.7	—	—
Montgomery	4.0	3.4	1.8	1.4	1.0	0.8	—	—
Ft. Leavenworth	4.4	3.3	1.8	1.6	0.8	0.8	—	—
Rome	4.4	3.5	—	—	—	—	2.6	2.1

[4] The calculation of the average absolute magnitudes of $A\Delta t$ at the stations shown by figs. 2 and 3 was carried out beyond $N = 300$. Since these magnitudes for $N > 200$ remain practically the same as for $N = 200$, they are not shown for $N > 200$.

c. Reliability of the above results. In the course of this study, it was necessary to divide the work of constructing trajectories and obtaining 12-hr temperature advections at different stations among five assistants in order to complete the study in a reasonable length of time. Because of the different backgrounds of the assistants and of other factors, some of the $A\Delta t$ obtained in this study may be in error by two or more degrees Centigrade. But the average absolute magnitudes of $A\Delta t$ as given in table 2 are quite accurate. This is indicated by the fact that each curve in figs. 2 through 5 becomes very stable as N becomes large. This is also borne out by a test conducted in this study. In this test, two assistants whose qualifications for this work differed by about the largest possible amount in the group worked independently on the construction of the 12-hr temperature advection at the 200-mb level over three stations from the same two months of maps. The average absolute magnitude of $A\Delta t$ at each station was obtained by each person in the same manner as before. The results of this test show that their average absolute magnitudes of $A\Delta t$ at the 200-mb level become the same at two stations and differ by only 0.1C at another station when N reaches 59. Should N be even larger, these differences are expected to be

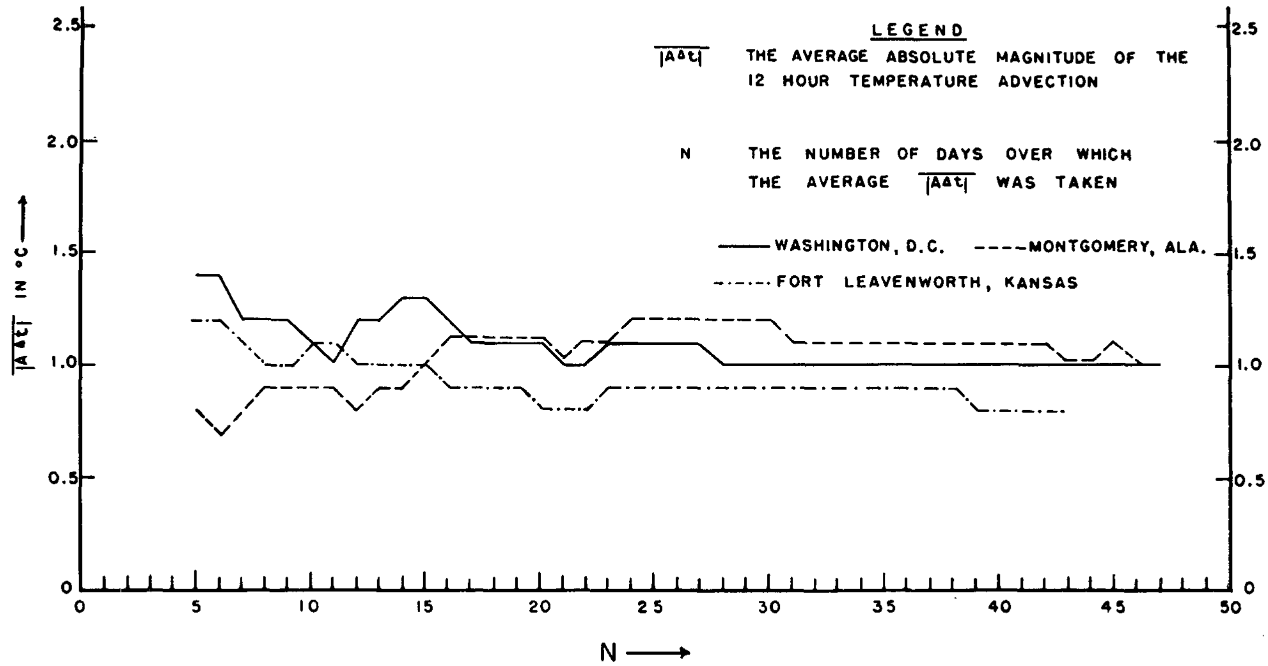

FIG. 5. The successive average absolute magnitude of $A\,\Delta t$ at the 50-mb level in winter.

still smaller. From this and from the fact that each curve in figs. 2 through 5 oscillates by only about 0.1C, it may be concluded that the values as given by table 2 are accurate to about 0.2C. This accuracy is more than adequate for the purposes of this study.

d. The standard deviation of the absolute magnitudes of $A\,\Delta t$ in the lower stratosphere. The standard deviation of the absolute magnitudes of $A\,\Delta t$ denoted as $\sigma_{|A\Delta t|}$ is, by definition, given by

$$\sigma_{|A\Delta t|} = [\overline{|\vec{v}\cdot(\nabla T)|^2} - |\overline{\vec{v}\cdot(\nabla T)}|^2]^{\frac{1}{2}}. \qquad (16)$$

From the same daily values of $A\,\Delta t$ used in section 5b, $\sigma_{|A\Delta t|}$ was calculated for the same stations, levels, and seasons as was $|\overline{A\,\Delta t}|$. The results of this calculation are also summarized in table 2.

These standard deviations serve as an indication of the scattering of the absolute magnitudes of $A\,\Delta t$ at each station and level about their average absolute magnitude given by the same table. It is seen that at each level the standard deviations are nearly as large as their corresponding average absolute magnitudes. This means that at each level the absolute magnitudes of $A\,\Delta t$ could sometimes be twice as large as their average absolute magnitude. Since $R_2\Delta t$ remains the same every day, the relative importance of $A\,\Delta t$ in comparison with $R_2\Delta t$ for any individual day could sometimes be far more significant than the comparison between the average absolute magnitudes of $R_2\Delta t$ and $A\,\Delta t$ would indicate.

6. Determination of the magnitude of $\overline{|(\partial T/\partial t)_p\Delta t|}$

From the radiosonde data for the same winter periods listed in section 5a, the 12-hr temperature change from 0300 to 1500 GMT, at the 200-mb, 100-mb, and 50-mb levels over Washington, D. C., and Montgomery, Alabama, were obtained for every day within the chosen period that had both 0300- and 1500-GMT observations in the same day.[5]

Following the same procedures used to obtain the average absolute magnitudes of $A\,\Delta t$, the average absolute magnitudes of $(\partial T/\partial t)_p\Delta t$ were computed at each station and level.

In fig. 6, the average absolute magnitudes of $(\partial T/\partial t)_p\Delta t$ in winter at the 200-mb surface over Washington, D. C., and Montgomery are plotted against N. It is seen that the average absolute magnitude reaches a rather stable value when N is 50 or more. At the 100-mb and 50-mb levels, this average absolute magnitude, not shown here, reaches a stable value just as fast. For this reason, the amount of data used is considered sufficient for the determination of the average absolute magnitudes of $(\partial T/\partial t)_p\Delta t$ at the stations and levels mentioned. The result of this determination is summarized in table 3.

7. Estimate of the magnitude of $\overline{|C\Delta t + E\Delta t|}$

At the 200-mb level over Washington, D. C., in winter, $\overline{|(\partial T/\partial t)_p\Delta t|}$ is given by table 3 as 3.5 and $\overline{|A\,\Delta t|}$ by table 2 as 4.7. Therefore, according to equation (15), $\overline{|A\Delta t + C\Delta t + E\Delta t + R\Delta t|}$ is slightly smaller than $\overline{|A\,\Delta t|}$. This can be accomplished only if $A\,\Delta t$ and $(C\Delta t + E\Delta t + R\Delta t)$ are in most cases different in sign and not too different in magnitude. (For if they are very much different in magnitude,

[5] A day here means the time from 0000 to 2400 GMT.

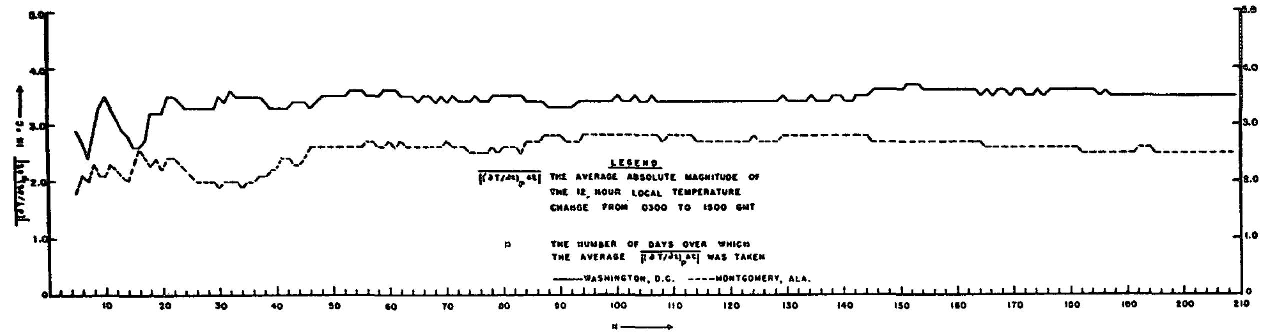

FIG. 6. The successive average absolute magnitude of $(\partial T/\partial t)_p \Delta t$ at the 200-mb level in winter.

their difference may be larger than $\overline{A\Delta t}$ itself in magnitude.) This means that $\overline{|C\Delta t + E\Delta t + R\Delta t|}$ should be similar to $\overline{|A\Delta t|}$ in magnitude. Since $\overline{|R\Delta t|} \cong \overline{|R_2\Delta t|}$ is quite a bit smaller than $\overline{|A\Delta t|}$ for the station, level, and season considered,[6] $\overline{|C\Delta t + E\Delta t|}$ is probably as large as, or even larger than, $\overline{|R\Delta t|}$ for this particular station, level, and season.

TABLE 3. The average absolute magnitude of the 12-hr temperature change. $\overline{|(\partial T/\partial t)_p\Delta t|}$, for winter in the lower stratosphere, in degrees Centigrade.

	200 mb	100 mb	50 mb
Washington, D. C.	3.5	2.1	1.4
Montgomery	2.5	1.7	1.8

Similar arguments lead to the same conclusion as above at the 200-mb level over Montgomery in winter. This kind of argument becomes less effective at higher levels as the difference between $\overline{|A\Delta t|}$ and $\overline{|R\Delta t|}$ becomes small. No estimate of $\overline{|C\Delta t + E\Delta t|}$ at the 100-mb and 50-mb levels was attempted.

8. Discussion of results

According to table 1, the maximum value of local temperature variation in 12 hr due to radiative processes ranges from about 0.15C at the 200-mb level in winter to about 0.35C at the 50-mb level in summer. These magnitudes apply to all the stations studied.

Table 2 shows that in winter the average absolute magnitude of the 12-hr temperature advection in the lower stratosphere over southeastern United States stations amounts to about 4.5C at the 200-mb level. It decreases rather rapidly with height and amounts to about 1.8C at the 100-mb level and about 1.0C at the 50-mb level. In summer, its magnitude at the 200-mb level over Rome, N. Y., is 2.6C.

Table 2 shows that the standard deviation of the absolute magnitude of the 12-hr temperature advection in the lower stratosphere over the same southeastern United States stations decreases steadily with height in winter. However, its magnitude at each level is very much the same at different stations. In winter, it is about 3.5C at the 200-mb level, 1.6C at the 100-mb level, and 0.8C at the 50-mb level. In summer, it is 2.1C at the 200-mb level over Rome, N. Y. Comparison of these magnitudes with the average absolute magnitudes of the 12-hr temperature advection shows that on any individual day the 12-hr temperature advection could be two or more times as large in magnitude as that indicated by its average magnitude.

The average absolute magnitude of the 12-hr temperature change due to the vertical convective process and eddy heat exchange process is estimated, according to the arguments given, in section 7, to be likely as large as, or even much larger than, the maximum 12-hr temperature change due to radiative processes.

9. Conclusion

From the above discussions, the following conclusions are reached for the winter situation. These conclusions are specifically applicable to the lower stratosphere over the southeastern United States. Although, judging from the constancy of the average absolute magnitude of the 12-hr temperature advection at different stations, it is probably applicable to the lower stratosphere over much wider areas.

(a) At the 200-mb and 100-mb levels, the horizontal advective process is more important than the radiative processes in influencing the 12-hr local temperature change (about seven times as important at the 200-mb level, and twice as important at the 100-mb level). At the 200-mb level the vertical convective and the eddy heat-exchange processes are estimated to be as important as, or even more important than, the radiative processes over the same area.

(b) The importance of the horizontal advection decreases rather rapidly with height, while the importance of the radiation increases with height. They are probably of equal importance near the 50-mb

[6] Because of the way $\overline{|R_2\Delta t|}$ was determined, $\overline{|R_2\Delta t|}$ is applicable to Washington, D. C., as well as to many other stations in the United States.

level, provided that the radiative processes increase in importance with height in winter in the same fashion as in summer.

The small amount of information available for the summer indicates that the above conclusions probably apply to the summer situation as well. However, this should be considered as only a guess at the present time.

10. A final remark

After this work was completed, it was discovered that the geostrophic wind scale, which was copied from the Air Weather Service Manual 105-32 entitled "Use of gradient wind scale," was slightly out of scale. A subsequent manual issued by the Air Weather Service, AWSM 105-32A, that contained the necessary changes from the former manual, had unfortunately escaped our attention.

The scale used in this study is a bit smaller than the correct size. As a result, each trajectory constructed in this study is a bit shorter than it should be. Therefore the horizontal temperature advection given in this study probably represents more closely an 11-hr temperature advection rather than a 12-hr temperature advection. A sample study of the 12-hr temperature advection at the 200-mb level over Rome from one summer and one winter month data with the correct geostrophic wind scale was made. The result of this study shows that the average absolute magnitude of the 12-hr temperature advection obtained from the correct scale is about half a degree larger than the corresponding value obtained from the incorrect scale for the winter month. It also shows that these two quantities are the same for the summer month. This coincides with the expectation that the average absolute magnitude of the 12-hr temperature advection should be similar to, but probably slightly larger than, the corresponding magnitude of the 11-hr temperature advection. In any case, the conclusions reached in the last section would not be invalidated but rather would be strengthened by the changes in the average absolute magnitudes of the 12-hr temperature advection if the correct geostrophic wind scale would have been used.

Acknowledgments.—The writers wish to express their sincere appreciation to Professors Julius London and Bernhard Haurwitz for the valuable suggestions given to this study and for their careful reading of the manuscript of this report; to Messrs. Carl J. Rice and Edward G. Albert, to Mrs. Gladys Eisener and Miss Miriam Wertheimer for their help in carrying out many phases of this study without which it would not have been possible; and to Mrs. Sadelle Wladaver for typing this report.

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