HODOGRAPH ANALYSIS AS APPLIED TO THE OCCURRENCE OF CLEAR-AIR TURBULENCE

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Introduction. Several studies of clear-air turbulence, summarized in ICAO (1955), have lead to the conclusion that in different cases this phenomenon must have different causes and that among these causes can be the temporary creation of hydrostatic instability. This idea is confirmed by a recent analysis made by Keitz (1957), based on a series of jet-fighter observations at 30,000 and 35,000 ft obtained during the winter months of 1953–54 over the Eastern United States. With some reasonable reservations, Keitz concluded that "the occurrence of clear-air turbulence is significantly correlated with the change in stability caused by differential advection."

Keitz's method of analysis consists essentially in a comparison of the six-hour isobaric temperature advection at two levels, 300 and 400 mb with reference to the 30,000-ft observations, 200 and 300 mb with reference to the 35,000-ft observations, using the

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published aerological data and constructing smoothed contours of the respective absolute topographies and smoothed isotherms. It is the object of this paper to suggest that a simpler and more universal approach is provided by the hodograph analysis of upper-wind soundings.

Admittedly, any quantitative deductions from the vertical wind profile regarding differential advection require the assumption of geostrophic conditions. This is certainly an important restriction. However, for a broad appreciation, it seems reasonable to assume that in cases of pronounced wind-direction changes with height associated with moderate or high wind speeds the ageostrophic wind component would not be large enough to annul, let alone reverse the sign of, the thermal-advection process. At any rate, this assumption will be made for the following. The daily upper-air analyses prove its validity for layers of 100 mb or greater vertical extent. Whether it can also be made for much thinner layers and for time intervals shorter than those used in synoptic meteorology is somewhat more problematic and remains to be investigated.

The theory of changes in hydrostatic stability resulting from geostrophic advection is briefly considered in the next section. This is followed by some data illustrating the frequency of the phenomenon and by three examples of its connection with clear-air turbulence. Some questions remaining to be answered by further work are posed in the final section of the paper.

Theory. In the following, we consider the horizontal advection of potential temperature which is assumed to be preserved individually. Hence,

$$\frac{d\theta}{dt} = \frac{\partial\theta}{\partial t} + \boldsymbol{v}\cdot\nabla\theta = 0, \tag{1}$$

where ∇ is the horizontal-gradient operator. An expression for $\nabla\theta$ is obtained from the thermal-wind equation which, as shown by Exner (1925, p. 190) takes the same form for the potential temperature as for absolute temperature. Thus, we have

$$\frac{\partial \boldsymbol{v}_g}{\partial z} = \frac{\boldsymbol{v}_g}{\theta}\frac{\partial\theta}{\partial z} + \frac{g}{f\theta}(\boldsymbol{k}\times\nabla\theta) \tag{2}$$

which on vectorial postmultiplication with the vertical unit vector $\boldsymbol{k}$ becomes

$$\nabla\theta = \frac{f}{g}\frac{\partial\theta}{\partial z}(\boldsymbol{k}\times\boldsymbol{v}_g) - \frac{f\theta}{g}\left(\boldsymbol{k}\times\frac{\partial \boldsymbol{v}_g}{\partial z}\right). \tag{3}$$

Combining (1) and (3) yields for the local change of the potential temperature due to horizontal geostrophic advection

$$\frac{\partial\theta}{\partial t} = -\boldsymbol{v}_g\nabla\theta = \frac{f\theta}{g}\left(\boldsymbol{k}\times\frac{\partial \boldsymbol{v}_g}{\partial z}\right)\cdot\boldsymbol{v}_g \tag{4}$$

since the first term on the right in (3) vanishes on scalar multiplication with $\boldsymbol{v}_g$. Essentially the same relation in a different form was already given by Exner (1925, p. 299).

The hydrostatic stability is represented quantitatively by the parameter

$$s = \frac{1}{\theta}\frac{\partial\theta}{\partial z} \tag{5}$$

which measures the Archimedean force resulting from a small displacement in terms of the work done against the geopotential. It is of interest that gs is the numerator of the Richardson number Ri. For the purposes of this paper, changes in Ri shall be assumed to result initially from changes in static stability which bring about turbulence and thereby changes in the vertical-wind profile. Thus, any deductions from the hodograph regarding the time rate of change of static stability would also apply in first approximation to that of the Richardson number.

The time rate of change of the hydrostatic stability is given by

$$\frac{\partial s}{\partial t} = -\frac{1}{\theta^2}\frac{\partial\theta}{\partial t}\frac{\partial\theta}{\partial z} + \frac{1}{\theta}\frac{\partial}{\partial t}\frac{\partial\theta}{\partial z}. \tag{6}$$

By inverting the order of differentiation in the second term on the right and substituting for $\partial\theta/\partial t$ and $(\partial/\partial z)(\partial\theta/\partial t)$ from equation (4), we obtain the simple expression

$$\frac{\partial s}{\partial t} = \frac{1}{g}\left(f\boldsymbol{k}\times\frac{\partial^2 \boldsymbol{v}_g}{\partial z^2}\right)\cdot\boldsymbol{v}_g. \tag{7}$$

Here the Coriolis parameter has been included in the vector product; in this way, the correct sign for the thermal advection is obtained in both hemispheres provided f is regarded as negative in the southern hemisphere (where $\boldsymbol{k}$ points "downward").

Equation (7) shows that the stability naturally is not affected by uniform advection ($\partial \boldsymbol{v}_g/\partial z$ constant). Stability changes will occur at the boundaries of a layer of uniform advection and also in the presence of a curved wind profile if there is a change in wind direction. In practice, the former case is more important since the curvature of the wind profile can rarely be determined with any degree of assurance from present-day wind observations.[2]

[2] *Note:* It must be pointed out that the mere existence of appreciable curvature in the wind profile is tantamount to ageostrophic conditions. This follows from the equations of horizontal motion with turbulent friction (Sutton 1953, p. 70) which take the vector form

$$K_m\frac{\partial^2 \boldsymbol{v}}{\partial z^2} = f\boldsymbol{k}\times(\boldsymbol{v}-\boldsymbol{v}_g)$$

where K_m is the eddy viscosity and $\boldsymbol{v}$ the actual wind. Hence, $\partial^2\boldsymbol{v}_g/\partial z^2$ must vanish under geostrophic conditions if there is any turbulence present.

Thus, for the practical purpose of an estimate of a change in stability, it suffices to consider the case of quasi-uniform advection in a substantial layer, associated with different (or no) advection in the overlying or underlying layer. For this case of *differential* advection, the hodograph analysis can most conveniently be applied so as to give the change in the mean (virtual) temperature in two or more layers. This change can be written (compare Saucier, 1955) in the form

$$\frac{\partial T_v}{\partial t} = -\frac{fT_v}{g(z_2 - z_1)} \cdot v_n \cdot |\mathbf{\Delta v}| \tag{8}$$

where T_v = mean virtual temperature of the layer between the height levels z_1 and z_2, $\mathbf{\Delta v}$ = thermal wind vector between z_1 and z_2, and v_n = wind component normal to $\mathbf{\Delta v}$.

The possible magnitude of the effect can be appreciated by means of the following example, valid for latitude 32 deg: $z_2 - z_1 = 1000$ m; $T_v = 243$ K; $v_1 = 20$ m per sec, $v_2 = 22$ m per sec, $|\mathbf{\Delta v}| = 9$ m per sec (these latter three values correspond to $v_n = 20$ m per sec and a change in wind direction of 24 deg, approximately);

$$\frac{\partial T_v}{\partial t} = 1.2\text{C per hr},$$

positive or negative according to the sign of the change of wind direction.

Observations. A pronounced thermal advection depends on the simultaneous occurrence of marked wind-direction changes and high wind velocities. An idea of the frequency of this coincidence can be gained from table 1 which gives data for Perth, West Australia, 32S, for the 12 months from November 1955 to October 1956, for the layers 25,000 to 30,000 ft and 30,000 to 35,000 ft. Perth was chosen because it is a station which is almost certainly free of orographic effects on the upper-air currents and for which adequate material was available.

These figures refer to the appearance of warm or cold advection in one or the other of the two layers. The frequency of *differential* advection can be determined considering the algebraic sum of the changes for both layers, for every sounding with speeds ⩾ 20 kn at the three levels concerned (table 2). It can be seen that strong differential advection as revealed by wind soundings with data given for every 5000 ft is a rather infrequent phenomenon, but, of course, so is heavy clear-air turbulence.

There are not enough observational data available to the authors to justify a statistical analysis of the importance of differential advection for the explanation of clear-air turbulence; but it seems necessary to give at least a few examples which show that the possibility of the effect can be deduced by hodograph analysis. The data refer to a South Australian station at 31 deg lat and were communicated to the Meteorology Department, University of Melbourne, by Mr. G. Trefry.

EXAMPLES

(1) 0130 GCT 22 June 1955: "Moderate turbulence 28,700 to 30,900 ft indicated, very bumpy in patches, lowered to 28,000 to 30,000 ft at 0530 GCT."

Wind sounding 3 hr before 2230 GCT 21 June	Geostrophic temperature advection
25,000 ft 290° 62 m/sec	
30,000 ft 290° 75 m/sec	0C/hr
35,000 ft 290° 91 m/sec	
40,000 ft 310° 73 m/sec	−4C/hr
45,000 ft 320° 50 m/sec	

The radio sounding of 2230 GCT indicates a mean lapse rate of 6.2C per 1000 gpm between 30,000 and 44,000 ft. According to the winds, the turbulent layer should have been extended to 35,000 ft at least. Here it must be kept in mind that there are three hours time interval between sounding and aircraft observations and that the heights of the latter are "indicated" heights without aerological correction.

(2) 1100 GCT 27 July 1955: "Severe turbulence

TABLE 1. Frequency of changes of wind direction per 5000 ft in the upper troposphere, the speed in both levels being ⩾ 20 kn.

Change of direction	Warm-air advection −50°	−40°	−30°	−20°	−10°	0°	10°	Cold-air advection 20°	30°	40°	50°	60°	70°
Number of cases	4	2	13	67	161	308	160	62	15	6	3	1	1
Per cent			11			78				11			

TABLE 2. Frequency of differential advection between 25,000 and 35,000 ft, speeds ⩾ 20 kn, Perth, November '55 to October '56.

Algebraic sum changes	Increase of stability −70°	−60°	−50°	−40°	−30°	−20°	−10°	0°	10°	20°	Decrease of stability 30°	40°	50°	60°	70°	80°
Number of cases	1	3	3	5	13	40	75	98	75	40	16	7	5	–	–	1
Per cent			7			10		65		10			8			

13,500 to 14,500 ft, upper and lower boundaries very marked."

Simultaneous winds	Geostr. temp. adv.
10,000 ft 290° 11 m/sec	
15,000 ft 250° 15 m/sec	+0.5C/hr
20,000 ft 270° 32 m/sec	−0.9C/hr

The mean lapse rate between 10,000 and 20,000 ft was 6.5C per 1000 gpm. Another wind sounding was made 6 hr later; it shows a general WSW current up to at least 25,000 ft (the end of the sounding), indicating that an upper trough must have passed.

(3) 0600 GCT 7 November 1955: "Moderate turbulence of very noticeable type between 39,000 (indicated) and 45,000 ft. Top edge was very marked—*i.e.*, quite turbulent at 45,000 ft, silky smooth 300 ft higher."

Simultaneous winds (unsmoothed as observed)	Geostrophic temperature advection
34,600–36,600 ft 276° 41 m/sec	
36,600–38,200 ft 271° 46 m/sec	
38,200–39,700 ft 270° 45 m/sec	≈0C/hr
39,700–41,500 ft 273° 42 m/sec	
41,500–43,100 ft 269° 30 m/sec	
43,100–44,600 ft 273° 27 m/sec	−2.5C/hr
44,600–46,200 ft 289° 25 m/sec	
46,200–47,800 ft 291° 28 m/sec	≈0C/hr

The mean lapse rate between 39,000 and 48,000 ft was 6C per 1000 gpm.

In these three examples as in any other case of the same type, the calculated differential advection should be interpreted more in a qualitative sense than in a quantitative sense, considering the possible magnitude of the ageostrophic wind components. Particularly in the first example and also to a lesser degree in the third, it is not the point that there should have really occurred a local diminution of temperature at the rate of several degrees per hour. It should only be deduced that considerable advection of colder air was taking place in a certain layer while there was no such change in the underlying one, thus originating an irregular unstable layer with pronounced turbulent vertical exchange. In such cases, it would be extremely interesting to obtain detailed wind soundings at short time intervals (*e.g.*, every 10 min) in order to study the transient changes in the vertical wind structure in cases with increasing instability by differential advection.

It should be stressed here that the hodograph-analysis gives indeed something more than the instantaneous *Ri* number: the consideration of differential advection reveals the way in which *Ri* is likely to change (i.e., its tendency). This seems of particular importance in view of the fact that flight observations of clear-air turbulence and wind soundings will in most cases not be perfectly simultaneous. It follows that different cases should be classified according to the corresponding hodographs before applying the Richardson criterion.

Concluding remarks. General considerations indicate that differential advection can be an important factor in the formation of clear-air turbulence only when the initial lapse rate is not too small, because the warm advection below a certain level or the cold advection above it can hardly be strong enough to sufficiently modify the vertical structure of a very stable stratified atmosphere. This means that the conception of differential advection as possible cause of clear-air turbulence certainly does not apply to those cases in which turbulence is observed in inversion layers—and there can be no doubt that such cases exist.

On the other hand, the additional condition of steep lapse rate makes it understandable that more reports of turbulence refer to the upper troposphere than to the middle troposphere. Furthermore, the conception of differential advection leads to an explanation of the following widely observed phenomenon: in several cases, moderate or heavy clear-air turbulence has been found in the vicinity of, and particularly a few 100 mi ahead of, an upper-air trough. In that sector of the upper-wave pattern, the appearance of differential advection is most likely, and the positive vertical motion of the prefrontal air mass can have contributed to produce a steep lapse rate.

Nevertheless, the following circumstances must also be considered: (1) heavy clear-air turbulence appears generally in rather shallow layers, of the order of 1000 m thick, and over quite short horizontal distances, of the order of 100 mi; (2) the modern radar sets which allow for an almost continuous tracking of the path of the balloon-borne reflector with a high degree of exactness show that the vertical structure of horizontal winds is often much more irregular than would appear from linear interpolation of the wind data given for the conventional aerological levels.

These new and better wind soundings which it is hoped will soon become available to meteorological research workers will open, and make necessary, a new approach to the clear-air turbulence problem. It also seems likely that the explanation of the small extent of the turbulence zones mentioned under (1) will be found there. However, several questions arise immediately:

(a) What persistence can be attributed to the finer vertical structure of the winds in the free atmosphere?
(b) How far can the geostrophic assumption be applied to these apparently much more erratic air currents?
(c) Does there exist a sufficiently strong correlation between the irregularity of the vertical wind

structure and the occurrence of clear-air turbulence?

(d) Is it really a not infrequent event, as some soundings and flight observations suggest, that in the vicinity of a "frontal surface" (or rather a transition layer between different air masses in the free atmosphere) parcels of one air mass are intruding in discrete layers into the other one, far ahead of the main transition zone?

Many years ago, W. Peppler (1930), when analysing the captive balloon soundings made from shipboard over Lake Constance on occasions of heavy turbulence in the free atmosphere, found evidence for a phenomenon which he called "Verzahnung von Luftmassen," the "meshing" of air masses. Peppler had in mind exactly what is now suggested by some cases of clear-air turbulence and by the modern detailed wind soundings. His very interesting paper (which unfortunately was not mentioned in the special bibliography on atmospheric turbulence in the METEOROLOGICAL ABSTRACTS AND BIBLIOGRAPHY, 1952) contains some experiences concerning the aerological conditions associated with the appearance of clear-air turbulence which are now being repeated in the modern papers on this subject.

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