

Some Comments on "Hodograph Analysis as Applied to the Occurrence of Clear-Air Turbulence"

RICHARD J. REED

University of Washington

10 November 1959

In recent articles by Keitz¹ and Schwerdtfeger and Radok² it has been suggested that differential advection is a significant factor in the formation of clear-air turbulence. The supposition is that the differential advection reduced the hydrostatic stability, $1/\theta \partial\theta/\partial z$, and thereby the Richardson number,

$$Ri = \frac{g}{\theta} \frac{\frac{\partial\theta}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2}, \quad (1)$$

below the critical value required for the onset of turbulence. The vertical wind shear is assumed to remain constant or to change only after the release of the turbulence.

Keitz has shown that a significant relationship, in the *statistical* sense, exists between the occurrence of clear-air turbulence in an area and the differential advection measured from synoptic charts. However, the relationship is so slight that one is led to inquire whether or not statistical significance has much to do with physical significance. If clear-air turbulence bears any relationship whatsoever to the synoptic pattern, it may be expected to be correlated, at least weakly, with various meteorological parameters, even though there may be little or no physical connection.

The purpose of this note is to present arguments to the effect that (1) differential advection is probably not an important factor in the *formation* of clear-air turbulence and that (2) decreased hydrostatic stability may just as well lead to an increase in the Richardson number as a decrease.

(1) In order to assess the importance of differential advection in the problem of stability change, we first write the complete expression for the local change of $\partial\theta/\partial z$.

$$\frac{\partial}{\partial t} \left(\frac{\partial\theta}{\partial z} \right) = \frac{\partial}{\partial z} \left(\frac{d\theta}{dt} \right) - \frac{\partial}{\partial z} (V \cdot \nabla\theta) - \frac{w\partial}{\partial z} \left(\frac{\partial\theta}{\partial z} \right) - \frac{\partial w}{\partial z} \frac{\partial\theta}{\partial z} \quad (2)$$

where V is the horizontal wind vector, w the vertical

velocity, and ∇ is the horizontal del-operator. The second term on the right-hand side of eq (2) represents, by definition, the differential advection. The remaining terms, which represent the effects of differential heating, vertical advection, and essentially the horizontal divergence, respectively, in bringing about stability change, are not mentioned in Schwerdtfeger's and Radok's treatment; yet, it may easily be shown that all of these terms are of the same order of magnitude as the local change and the differential advection. On this basis alone, one must question an approach which estimates stability change solely from the differential advection.

But now, for the sake of argument, let us suppose that these neglected terms are indeed negligible. Then, upon differentiation and introduction of the thermal wind relationship eq (2) takes the form

$$\frac{\partial}{\partial t} \left(\frac{\partial\theta}{\partial z} \right) = -V \cdot \nabla \frac{\partial\theta}{\partial z} - \frac{\partial V'}{\partial z} \cdot \nabla\theta \quad (3)$$

where V' is the geostrophic deviation. Under these conditions, the local change depends on the horizontal advection of stability and the rotation of the gradient vector by the geostrophic deviations. The magnitude of the final term in eq (3) is difficult to determine. We will follow Schwerdtfeger and Radok and neglect it. We arrive then finally at the expression

$$\frac{\partial}{\partial t} \left(\frac{\partial\theta}{\partial z} \right) = -V \cdot \nabla \frac{\partial\theta}{\partial z} \quad (4)$$

or

$$\frac{d}{dt} \left(\frac{\partial\theta}{\partial z} \right) = 0. \quad (5)$$

Thus, under the various assumptions made by these authors, the static stability is conserved, or, to put it differently, the differential advection in a thin layer simply equals the advection of pre-existing stability by the wind in the layer. Such a process can, therefore, hardly be thought of as an important mechanism in the *formation* of turbulence, though it may, of course, advect turbulence into a region where it did not previously exist.

Our conclusion is that differential advection is only one of several factors which must be considered in evaluating local stability change and that under geostrophic conditions it has no effect whatsoever on individual stability change.³

The role of differential advection in changing the moisture stratification and, hence, the convective instability, is omitted from this discussion.⁴ There can

¹ Keitz, E. L., 1959: Differential advection as a factor in clear-air turbulence. *J. Meteor.*, 16, 57-62.

² Schwerdtfeger, W., and U. Radok, 1959: Hodograph analysis as applied to the occurrence of clear-air turbulence. *J. Meteor.*, 16, 588-592.

³ The second part of this conclusion is strictly true only for infinitely thin layers, but it is probably nearly correct for the shallow layers in which clear-air turbulence occurs.

⁴ Miller, J. E., 1955: Intensification of precipitation by differential advection. *J. Meteor.*, 12, 472-477.

be little doubt that this mechanism is of importance in the problem of severe convective storms.

(2) Upon substitution of the thermal-wind relationship, eq (1) may be written as

$$Ri = \frac{f^2 \theta}{g \tan^2 \alpha} \frac{1}{\partial \theta / \partial z} \quad (6)$$

where $\tan \alpha$ is the slope of the isentropic surface. From this expression, we state the proposition that the Richardson number decreases when the static stability increases, provided, of course, that the slope of the potential-temperature surfaces remains constant. It is not meant to suggest seriously that this proposition is generally true. However, it does bring out the fact that low Richardson number may occasionally be associated with large stability, as in frontal zones, and *a priori* it is as acceptable as the hypothesis that the vertical wind shear tends to remain constant while the stability changes. A reasonable conclusion would seem to be that there is no unique relationship between stability change and change in Richardson number.