# ENERGY SPECTRA OF 500-MB MERIDIONAL CIRCULATION INDICES

***Byron P. Harper***

Bonneville Power Administration, Portland, Oregon

(Manuscript received 4 October 1960)

## ABSTRACT

Variation of the daily 500-mb hemispheric index of meridional circulation at latitudes 60N and 40N for the period 1 March 1959 to 12 July 1960 is examined by means of the spectrum-analysis method of Tukey. Some significant differences in the spectra at the two latitudes are noted. Characteristics of the spectra during different seasons are investigated by considering relatively small samples throughout the period, and evidence of seasonal variation is discussed.

---

## 1. Introduction

Recent analyses of variation in the hemispheric zonal westerly index, reported by H. A. Panofsky [5], have revealed the existence of a large periodic element in the energy spectrum, usually appearing at wavelengths near 15 and 30 days. Similar investigations, conducted by Chiu [2] and Rosenthal [8], of the energy spectra of zonal and meridional components of several series of upper-level wind observations at various points in the northern hemisphere tend to substantiate the existence of periodic fluctuations in atmospheric circulation patterns.

Most of the research thus far has dealt with the analysis of rather extensive series of daily observations, or consecutive 3-day average values, encompassing one to three years of data. The investigation reported in this paper is a preliminary attempt to determine whether or not periodic activity in hemispheric-circulation indices exhibits any seasonal characteristics when relatively small samples are examined systematically throughout the year. If periodic fluctuations are of more importance in determining the total variance of a circulation parameter during one season than at other times, knowledge of this fact should be of considerable importance to those engaged in medium- or extended-range forecasting. Of equal importance would be any tendency for the frequency of periodic activity to shift to higher or lower values during the different seasons. The daily 500-mb hemispheric meridional index, $V_\phi(t)$, at latitudes 40N and 60N, from 1 March 1959 to 12 July 1960, is the subject of investigation, as described in the following sections.

## 2. Method

The mean meridional gradient at the 500-mb level in feet per mile at a given latitude may be calculated as follows:

$$Z_\phi = \frac{\Sigma \Delta h}{\pi R \cos \phi}, \tag{1}$$

where $\Delta h$ is the difference in height from each maximum to the corresponding minimum to the east along latitude $\phi$, and $R$ is the radius of the earth. Since the sum of north components equals the sum of south components around a latitude circle, only the north components appear in the numerator and a half-circumference in the denominator. This particular meridional index was defined in 1948 by Willett [10] as follows: ". . . east-west pressure gradients (around the latitude circle) are averaged regardless of sign, for their algebraic sum around a latitude circle must be zero." By taking advantage of the latter part of this statement, and by using only the northerly gradients, a considerable reduction of tabulation and calculator work is effected. By combining (1) with the geostrophic-wind equation and the hydrostatic relationship, the following is obtained:

$$V_\phi = \frac{-g \Sigma \Delta h}{R \pi \omega \sin 2\phi}. \tag{2}$$

Here $V_\phi$ is the mean meridional component of circulation around latitude $\phi$, $g$ is the acceleration of gravity, and $\omega$ is the angular velocity of the earth. Daily values of $V_{40}$ and $V_{60}$ in meters per second were calculated from 0000 GCT 500-mb northern-hemispheric charts transmitted over the national facsimile network. Monthly mean values and sample variance, $s^2$, are shown in table 1. The seasonal trend was eliminated from daily values of $V_\phi$, thereby producing a more nearly stationary or homogeneous time series, before proceeding with the analysis [1, pp. 4, 5].

Spectrum analysis was conducted on the resulting

TABLE 1. Summary of 500-mb $V_\phi$.

| Year | Month | 40N Mean (mps) | 40N $s^2$ $(\text{mps})^2$ | 60N Mean (mps) | 60N $s^2$ $(\text{mps})^2$ |
|---|---|---|---|---|---|
| 1959 | Mar | 11.25 | 3.21 | 8.14 | 2.38 |
| 1959 | Apr | 10.71 | 1.75 | 8.29 | 1.67 |
| 1959 | May | 9.20 | 3.64 | 8.88 | 2.42 |
| 1959 | Jun | 6.98 | 3.92 | 7.36 | 1.98 |
| 1959 | Jul | 5.19 | 1.66 | 7.15 | 1.87 |
| 1959 | Aug | 5.65 | 1.37 | 6.77 | 1.64 |
| 1959 | Sep | 7.38 | 3.49 | 8.57 | 1.71 |
| 1959 | Oct | 8.94 | 3.25 | 8.69 | 1.77 |
| 1959 | Nov | 10.12 | 3.44 | 9.32 | 1.78 |
| 1959 | Dec | 10.76 | 2.03 | 8.41 | 1.82 |
| 1960 | Jan | 11.51 | 3.38 | 9.41 | 2.04 |
| 1960 | Feb | 11.41 | 1.63 | 9.47 | 1.47 |
| 1960 | Mar | 11.41 | 5.44 | 9.39 | 1.22 |
| 1960 | Apr | 10.06 | 3.95 | 8.33 | 3.48 |
| 1960 | May | 9.55 | 4.73 | 7.52 | 1.86 |
| 1960 | Jun | 6.03 | 1.02 | 7.03 | 0.27 |
| 1960 | Jul | 5.51 | 1.55 | 7.36 | 1.52 |

series by using the method suggested by J. W. Tukey [9]. The following is a brief summary of this method. Coefficients of autocovariance, $R_p$, are calculated as follows:

$$R_p = \frac{\sum_{i=1}^{N-p} (y_i - \bar{y})(y_{i+p} - \bar{y})}{N - p}, \qquad (3)$$

where $y_i$ are the observations taken at equal intervals ($i = 1, 2, 3, \cdots N$), $p$ denotes the consecutively numbered lags ($p = 1, 2, 3, \cdots m$), and $\bar{y}$ is the sample mean. Fourier cosine series transformation of $R(p)$ yields the unsmoothed spectral estimates, $L_h$, as follows:

$$L_0 = \frac{1}{2m}(R_0 + R_m) + \frac{1}{m}\sum_{p=1}^{m-1} R_p, \qquad (4)$$

$$L_h = \frac{1}{m}R_0 + \frac{2}{m}\sum_{p=1}^{m-1} R_p \cos\frac{ph\pi}{m} + \frac{1}{m}R_m \cos(h\pi),$$

$$\text{where } 0 < h < m, \text{ and} \qquad (5)$$

$$L_m = \frac{1}{2m}[R_0 + (-1)^m R_m] + \frac{1}{m}\sum_{p=1}^{m-1} (-1)^p R_p. \qquad (6)$$

The series $L(h)$ is smoothed as follows:

$$U_0 = 0.5(L_0 + L_1), \qquad (7)$$

$$U_h = 0.25L_{h-1} + 0.5L_h + 0.25L_{h+1}, \text{ and} \qquad (8)$$

$$U_m = 0.5(L_{m-1} + L_m). \qquad (9)$$

The resulting values, $U_h$, when plotted as a function of frequency, comprise the energy spectrum of the sample series, $y_i(t)$. The area under the resulting curve equals the variance, $\sigma_y^2$, of the original series. Individual values of $U_h$ represent the portion of $\sigma_y^2$ contributed by periodic oscillations at that particular frequency. More precisely, the individual spectral estimates, $U_h$, denote the variance contributed between the frequencies $2\pi(h - \frac{1}{2})/m$ radians/day and $2\pi(h + \frac{1}{2})/m$ radians/day, except for $U_0$ and $U_m$, where the frequency interval extends from 0 to $\pi/m$ radians/day and from $\pi - \pi/m$ to $\pi$ radians/day. In addition to publications by Blackman and Tukey [1; 9], excellent detailed descriptions of spectrum-analysis techniques have been presented by Panofsky [5; 6] and Chiu [2].

## 3. Long-period spectra of $V_{40}$ and $V_{60}$

The energy spectra of $V_{40}$ and $V_{60}$ were calculated for the 500-day period, 1 March 1959 to 12 July 1960, by using 72 values of $R_p$ ($N = 500$, $m = 72$). The resulting values of $U_h$, plotted as a function of both frequency and period, are presented in fig. 1. The well-defined maximum at 40N at 4 cycles per 144 days, or a 36-day wavelength, accounts for 7.3 per cent of the variance in the 500-day series. In general, the spectra at both latitudes are quite similar to those reported by Panofsky [5] of long-period samples of zonal indices. At 60N, the maximum in the spectrum is found at 16 days, and this value of $U_h$ accounts for 6.7 per cent of the variance in the series. A rather prominent secondary peak is found at the 29-day wavelength. A similar characteristic has been noted in Panofsky's data. When comparisons of series taken simultaneously at high and middle latitudes are possible, the energy concentrations at the higher latitude frequently appear at higher frequencies than those at middle latitudes. If the data published by Chiu [2] are examined from this standpoint, the relationship between frequency and latitude suggested above appears to change sign when lower latitudes are considered. The spectrum of the meridional component of circulation over Belmar, New Jersey has energy concentrations at periods near 20 and 7 days, while at Cocoa, Florida the peaks in the spectrum appear at 10 and 5 days. In the spectra of meridional components of circulation at 40,000 ft over several points in the Pacific at low latitudes, reported by Rosenthal [8], maximum points are at periods near 10 days or less, with one exception. At Marcus Island, the northernmost location considered, the maximum is near 16 days.

According to Tukey [9], the ratio of individual spectral estimates or observed powers at each frequency to the long-run averages or population values for these frequencies is distributed as Chi-squared divided by its mean or Chi-squared divided by the number of degrees of freedom. In other words,

$$\frac{\text{Observed value}}{\text{Average value}} = \frac{\chi^2}{k}, \qquad (10)$$

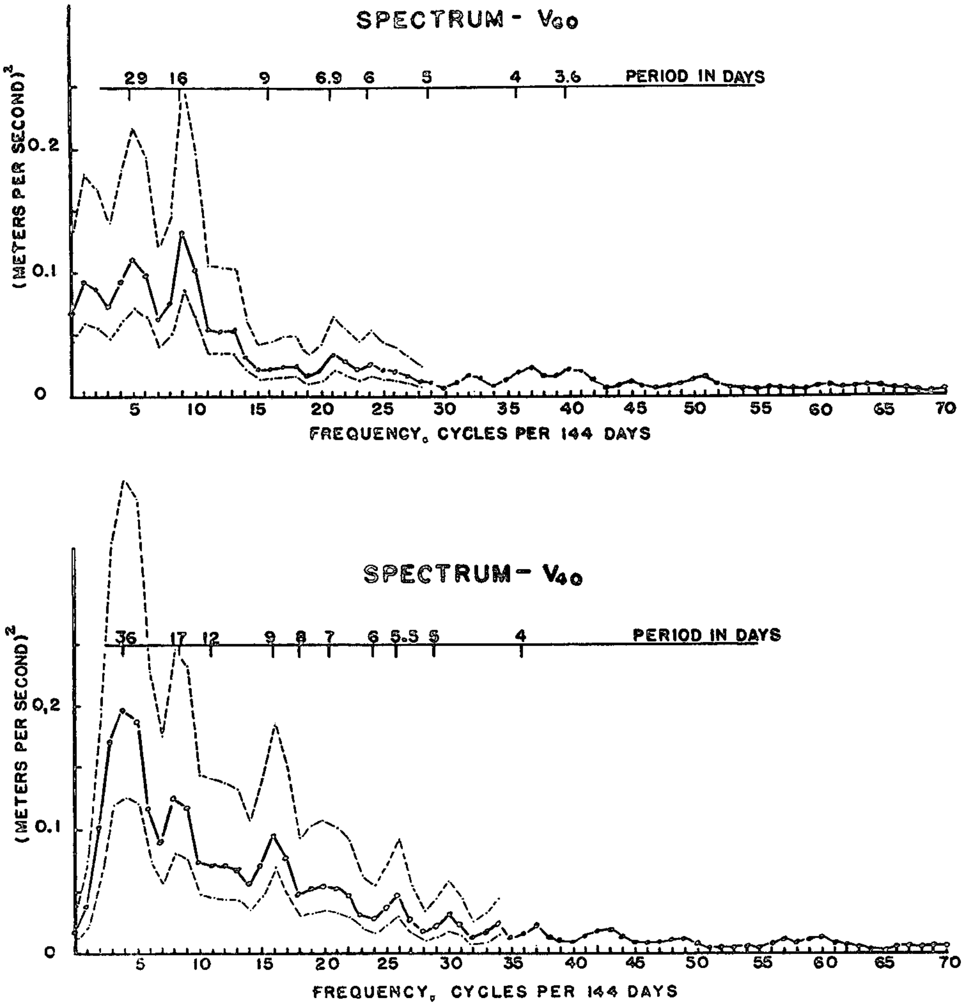

FIG. 1. Energy spectra of daily 500-mb $V_\phi$, 1 March 1959 to 12 July 1960. $N = 500$, $m = 72$. 80% confidence limits are indicated by the dashed curve.

where $k$ is the number of degrees of freedom. For individual spectral estimates,

$$k = \frac{2N - \frac{3}{2}m}{m}. \qquad (11)$$

For the samples of $V_{40}$ and $V_{60}$, $k = 12$. The significance of peaks in the sample spectra may be examined by assuming that the true population spectra are essentially flat, with uniform or randomly distributed powers centered on the mean value. Then the fact that any particular frequency contributes more to the over-all variance than another is attributed to sampling fluctuations. The hypothesis that the prominent peaks in the sample spectra do not depart significantly from the assumed population mean may be tested by using the ratio defined by (10). For this purpose, a two-tailed test is appropriate, with a critical region in the distribution falling below the mean as well as above it. However, interest is directed to the maxima in this case. If the 1 per cent significance level is adopted as the criterion for rejection of the null hypothesis, the critical regions for values of the ratio (observed value)/(average value), are as follows:

peaks; $\chi^2/k > 2.35829$, with 12 d.f., and

gaps; $\chi^2/k < 0.2562$, with 12 d.f.

For rejection of the hypothesis concerning peak values, the product, (average value)·(2.35829), must be exceded by the observed value of $U_h$ at the peak in question. The average of the line powers, or $\bar{U}_h$, for the sample spectra of $V_{40}$ is 0.0369 (mps)$^2$, and for $V_{60}$ it is 0.0271 (mps)$^2$. Individual values of $U_h$ at the principle low-frequency peaks in the sample spectra and the critical products are presented in table 2.

TABLE 2. Results of test of hypothesis.

| 40N | | | 60N | | |
|---|---|---|---|---|---|
| Frequency (cy/144 days) | Peak $U_h$ | $\bar{U}_h\chi^2/k$ | Frequency (cy/144 days) | Peak $U_h$ | $\bar{U}_h\chi^2/k$ |
| 4 | 0.1975 | 0.0870 | 2 | 0.0920 | 0.06391 |
| 8 | 0.1260 | 0.0870 | 5 | 0.1113 | 0.06391 |
| 16 | 0.0954 | 0.0870 | 9 | 0.1332 | 0.06391 |

In each case, the critical values are exceeded; therefore, the hypothesis that the peaks are due to sampling fluctuations in essentially uniform spectra must be rejected. The probability that a Type I error has occurred is less than 0.01 in all cases.

Another method of treating these data is to assume that individual values of $U_h$ are estimates of true population means at each frequency and, by again using the ratio of observed-to-average values and the $\chi^2/k$ distribution, to calculate the confidence interval for the true population value at each frequency. The 80 per cent confidence limits have been calculated for the low-frequency portion of the spectra by using the method of Tukey [1] and are shown as the dashed curves in fig. 1. A smaller confidence interval would have been obtained if a smaller value of $m$, the maximum lag, had been used. This would increase the value of $k$ and, accordingly, diminish the variability of $\chi^2/k$. However, as the ratio of $N$ to $m$ is increased, the resolution of the powers at low frequencies is reduced. Low-frequency values with periods from 10 to 60 days were of primary interest in this study. Undue importance should not be attached to the resulting confidence interval of the long-term spectra shown in fig. 1, since the data published by Panofsky [5] suggest that there may be a significant variability from year to year in the spectra of large-scale-circulation indices. The confidence limits are presented here in an attempt to indicate the effects of sample size, and the ratio $N/m$, on reliability of the sample estimates. The spectra are not intended to be representative of conditions universally.

As a preliminary step in the investigation of possible seasonal characteristics in fluctuation of the hemispheric meridional index, energy spectra were calculated from the attenuated series, 1 August 1959 to 31 May 1960 ($N = 305$, $m = 45$). Note that the June and July data for both summer periods, as well as that for the spring season of 1959, were eliminated. The resulting series consists primarily of fall, winter and spring data. These spectra appear in fig. 2. Visual comparison of figs. 1 and 2 reveals little over-all change in the spectrum of $V_{40}$, although the maximum near 17 days has increased somewhat in prominence. However, the peak at 2 cycles per 90 days has increased in importance and now contributes 9.1 per cent of the variance. By contrast, a definite change has occurred in the spectrum of $V_{60}$. The secondary prominence near 29 days has disappeared altogether, and the maximum near 15 days has increased considerably in importance. 9.6 per cent of the variance in the series is now accounted for by activity at this wavelength. Apparently, the character of periodic activity at 60N is subject to greater changes from summer to winter than that at 40N.

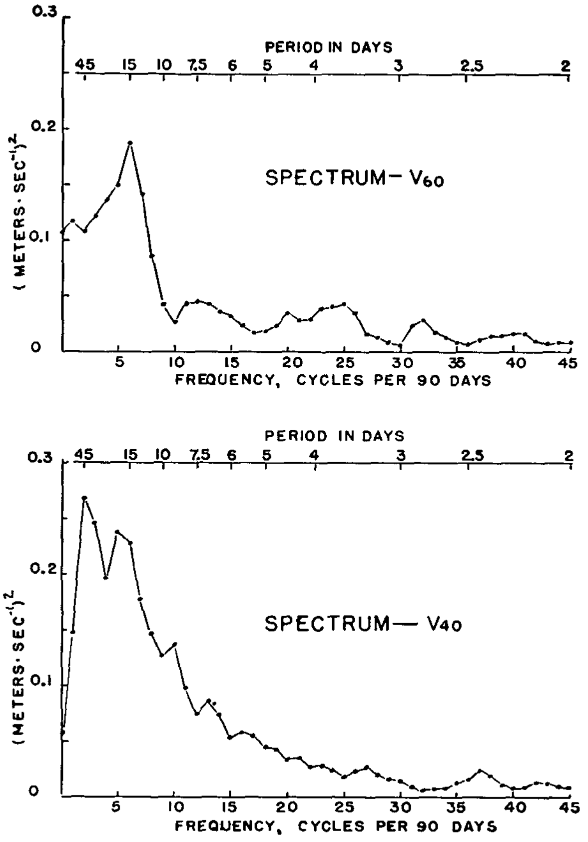

FIG. 2. Energy spectra of daily 500-mb $V_\phi$, 1 August 1959 to 31 May 1960. $N = 305$, $m = 45$.

## 4. Seasonal variation in the spectra of $V_\phi$

In order to examine the energy spectra of meridional circulation in more detail during all seasons of the year, energy spectra were calculated from samples consisting of 120 observations by using 30 values of $R_p$ ($N = 120$, $m = 30$). 13 spectra were calculated at each latitude in this manner from 120-day samples beginning 15 March 1959, 15 April 1959, . . . and 15 March 1960. Since the basic characteristics of the spectra for the entire period have been established in the preceding section, any serious departures from the patterns in figs. 1 and 2 that may occur due to the reduction in sample size should be readily apparent. The resulting spectra at 60N and 40N are presented in figs. 3a and 3b.

Confidence intervals and significance tests have not been calculated for these spectra, since the object of the analysis was to determine whether or not the prominent features of the spectra in figs. 1 and 2 appear consistently throughout the different seasons and to investigate tendencies for seasonal shifting of important frequencies that have long been suspected. The spectra are not intended to be representative of circulation conditions in all years. The ratio $N/m$ has been reduced considerably, and only 6 deg of freedom

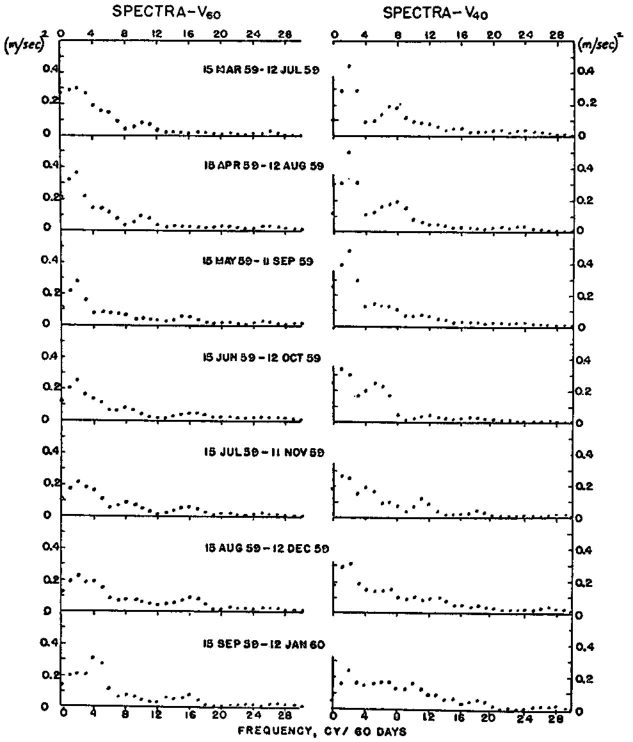

Fig. 3a. Energy spectra of daily 500-mb $V_\phi$. $N = 120$, $m = 30$.

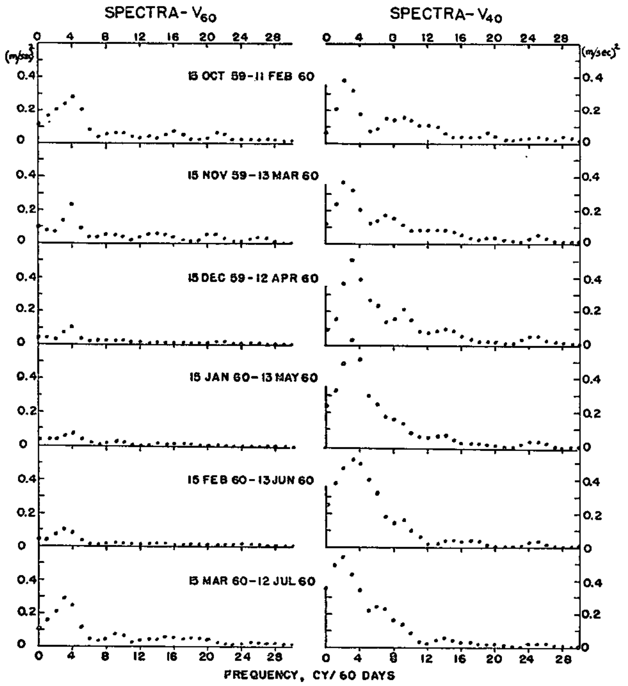

Fig. 3b. Same as fig. 3a.

would be available for the calculation of $\chi^2$. It is assumed that this quantity is too small to be used in making reliable statistical inferences about long-term averages. On the other hand, some interesting relationships within the framework of the large sample are suggested by the study of small overlapping samples during this period.

Examination of the spectra reveals the existence of a prominent low-frequency oscillation throughout the year at both high and middle latitudes, varying between 1 and 4 cycles per 60 days. Secondary prominences appear at higher frequencies from time to time, but they are obviously subject to far greater variation than the major low-frequency oscillation. Beginning in March 1959 at 40N, periodic activity at 7 to 8 cycles per 60 days can be followed successively through the periods beginning in April, May, June and July, although it becomes quite weak in the third period, 15 May to 11 September. Evidently there is a tendency for this oscillation to drift gradually toward low frequency. A similar tendency can be observed simultaneously in the major low-frequency oscillation around 2 to 3 cycles. At 60N, the minor energy concentration at 10 cycles, beginning in March 1959, appears to shift gradually to 8 cycles in the July-to-November period, but this pattern is not clearly defined. These seem to be the only instances of consistently identifiable peaks in the middle- or high-frequency range of the spectra, although other prominences appear randomly throughout the analysis period, indicating short-lived periodic activity.

A graphical summary of some characteristics of the major low-frequency oscillations at both latitudes is presented in fig. 4. For each sample, the portion of the

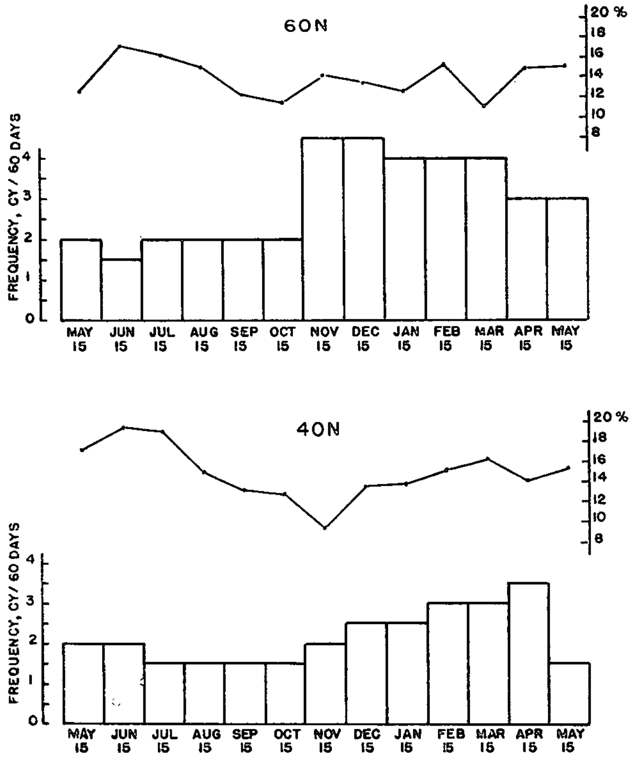

Fig. 4. Percentage of variance accounted for by the major low-frequency oscillation in each 120-day sample, and frequency of the oscillation. Dates indicated below the block diagrams are approximate center dates of each 120-day period.

total variance accounted for by the maximum value of $U_h$ at low frequencies appears in the upper graph. The accompanying block diagram represents the frequency, in cycles per 60 days, of these peaks in the spectra. Frequency values were interpolated to the nearest $\frac{1}{2}$ cycle by considering the energy values at adjacent frequencies. Dates indicated below the block diagrams are the approximate center dates of the series of 120-day periods. A seasonal trend is suggested in the predominant low-frequency oscillation at both latitudes, involving both the frequency and the strength of the activity. Higher-frequency fluctuations are prominent during the winter months and the reverse during summer. Seasonal variations appear to be somewhat out of phase, with maximum high-frequency activity at 60N in November and December, while at 40N the maximum occurs in March. A similar lag appears during the summer at the low-frequency points. At 40N, the data suggest a gradual increase in frequency during the fall and winter, followed by an abrupt change to low frequency around May. At 60N, something like a complementary pattern is observed: an abrupt change from low to high frequency in October or November, followed by a gradual decrease in frequency during the winter and spring. This leads to an interesting pattern of circulation fluctuations in late fall and early winter, when major oscillations in the meridional circulation are occurring at high latitudes with a wavelength of 12 to 15 days, while at middle latitudes the principle fluctuation is occurring around a 25- to 30-day wavelength.

Strongest oscillatory tendencies, or more nearly periodic fluctuations, at both latitudes apparently occur around June and July, as indicated by the portion of variance contributed by individual values of $U_h$. During the late summer and fall, periodic activity at low frequencies makes its smallest contribution to the over-all variance. Here again there appears to be a lag between the activity at different latitudes, with high latitude activity preceding that at middle latitudes by a month or two.

It is interesting to note that the major low-frequency oscillation in the variation of the meridional index assumes more importance in its contribution to the over-all variance as the sample is diminished from 500 to 305 days and then to 120 days. This apparently holds true at both latitudes. At 40N, the low-frequency peak in the spectrum of the 500-day series accounts for 7.3 per cent of $\sigma_v^2$. In the 305-day series, this ratio increases to 9.1 per cent. When the group of 120-day series is considered, the comparable values range from 9.6 to 19.5 per cent, with a mean value of 14.9 per cent. A similar relationship exists at 60N. This suggests that basically periodic fluctuations in hemispheric-circulation patterns are of primary importance over periods of 3 or 4 months' duration, particularly in certain seasons of the year, but shifting of the basic frequencies with season results in a gradual reduction of significance at a particular frequency as the length of the series under consideration is increased. This phenomenon would seem to be closely related to the forecasting technique sometimes referred to as "Riehl's trends," developed at the University of Chicago by Herbert Riehl and others [7] and based on the systematic north or south drift of the hemispheric jet, as well as many of the medium-range forecasting techniques employed in Europe, described by Hofmann, Flohn and others [3; 4].

## 5. Summary

No physical explanation of the characteristics of periodic activity exhibited by the spectra of hemispheric-circulation indices investigated in this study will be attempted here. It must be emphasized that the data examined were only of 16 months' duration. Similar investigations should be conducted over a period of several years to determine the stability of relationships suggested by this particular sample. In brief, these are summarized as follows:

(a) Maximum energy concentrations in long-period spectra of zonal and meridional indices at 60N tend to occur at higher frequencies than in similar spectra at 40N.

(b) There is some evidence suggesting that this relationship reverses at lower latitudes; *i.e.*, periodic activity at higher frequencies may be more important at latitudes from 5N to 25N than at middle latitudes.

(c) Periodic activity appearing at low frequencies (*i.e.*, between 1 and 4 cycles per 60 days) in long-period spectra at middle and high latitudes can be identified throughout all seasons of the year when small samples are examined. The activity seems to make its strongest contribution to the over-all variance of the hemispheric-circulation index over periods of the order of 3 or 4 months.

(d) Evidence of shifting of the basic frequencies with season has been found. In general, activity at relatively low frequencies predominates in the summer, and higher frequencies, or shorter wavelengths, predominate during the winter.

(e) Energy concentrations in the spectra appear to be strongest in the early summer and relatively weak in the fall.

*Acknowledgments.* I wish to thank Mrs. V. M. Green and Mr. Stuart McAlpine who tabulated the synoptic

data and performed the numerous calculator operations. I am also indebted to Mr. D. D. Wong, Mr. J. S. Clubb and Mr. Rodney J. Brown for assistance with the programming and operation of the Bendix G15 and IBM 650 computers used in this study.

## REFERENCES

1. Blackman, R. B., and J. W. Tukey, 1958: *The measurement of power spectra.* New York, Dover Publications, Inc., 190 pp.
2. Chiu, Wan-cheng, 1959: *Wind and temperature spectra of the upper troposphere and lower stratosphere over North America.* New York, Dept. of Meteor. and Oceanography, New York Univ., 53 pp.
3. Flohn, H., and A. Hoffman, 1956: Methods of medium and long range forecasting and the problems of their future development. *Meteor. Rundshau*, **9**, 5–10.
4. Hofmann, A., H. Diehl, E. Dinies, G. Seidel, and H. Trenkle, 1957: *Medium range forecasting techniques for Central Europe.* Offenbach/Main, Forschungsabteilung des Deutschen Wetterdienstes, 42 pp.
5. Panofsky, H. A., and Mae D. Lethbridge, 1958: *Research directed toward the study of the relation of solar energy variations to changes of the tropospheric circulations.* University Park, Dept. of Meteor., Penn. State Univ., 45 pp.
6. Panofsky, H. A., and G. W. Brier, 1958: *Some applications of statistics to meteorology.* University Park, Penn. State Univ., 224 pp.
7. Riehl, Herbert, and collaborators, 1952: *Forecasting in middle latitudes.* Meteor. Monogr., **1**, No. 5, 80 pp.
8. Rosenthal, Stanley L., 1960: Some estimates of the power spectra of large-scale disturbances in low latitudes. *J. Meteor.*, **17**, 259–263.
9. Tukey, J. W., 1950: The sampling theory of power spectrum estimates, in *Symposium on applications of autocorrelation analysis to physical problems.* Off. Nav. Res., Washington, D. C., 47–67.
10. Willett, Hurd C., 1948: Patterns of world weather changes. *Trans. Amer. geophys. Union*, **29**, 803–809.