SOME AIRBORNE REFRACTIVE-INDEX MEASUREMENTS AT 150 MB[1]

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ABSTRACT

Index of refraction data measured by a Crain microwave refractometer mounted in a Boeing 707 jet aircraft on two flights during August 1959 are presented. Emphasis is placed on the horizontal portions of the flights at the 150-mb level, but climbout and descent data are also given. Comparisons are made with theoretical data and index data computed from radiosondes. Some conclusions are drawn about the variability of the index of refraction in the lower stratosphere.

1. Introduction

As prime contractor to the Air Force, the Bendix Systems Division of The Bendix Corporation directed feasibility demonstrations of the AN/AMQ-15 Weather Reconnaissance System in the late summer and fall of 1959. The demonstrations were carried out near Seattle, Washington on a Boeing 707 jet aircraft and at Holloman Air Force Base, New Mexico. This paper contains an account of the data obtained from a refractometer installed on the 707 during two flights which reached an altitude of 43,000 ft.

The effects of refractive index upon guidance systems, satellite- and missile-tracking systems, and aircraft warning systems have generated a need to know more about this variable. The index of refraction can be computed by using eq (1), but the resultant accuracy is dependent on the accuracies of the three sensors used to measure the pressure (P), the temperature (T) and vapor pressure (e). Direct measurement is, therefore, highly desirable especially at higher altitudes where the accuracies of the three sensors are known to be only moderately good.

The order of magnitude of the atmospheric index of refraction lies between unity and about 1.0005. Since these numbers are extremely small, we usually study the quantity N.

$$N = (n - 1)10^6 \qquad \text{(refractive modulus)}$$

where n is the index of refraction of the air. N is given by

$$N = a\frac{P}{T} + b\frac{e}{T^2} \qquad (1)$$

where

$a = 77.6$,
$b = 4810a$,
P = pressure in millibars,
T = temperature in degrees Kelvin, and
e = water-vapor pressure in millibars (mb).

The constants a and b, determined empirically, are those that have been adopted for general use by radio engineers (Smith and Weintraub, 1953).

As can be seen from eq (1), the second term of the equation is a function of water-vapor pressure (or humidity). The major variations in these parameters occur most frequently in the lowest 20,000 ft of the atmosphere. Additionally, the accuracies of the radiosonde observations become less at higher altitudes, and the water-vapor pressure becomes extremely small at altitudes above 20,000 ft. Therefore, in the past, the major emphasis has been placed on obtaining profiles of the refractive index for the lower atmosphere.

Most of the data obtained to date are profiles of the vertical distribution of the index of refraction. Several reasons for this fact are apparent. First, the parameters of eq (1) often vary a considerable amount with height and it is these variations in the vertical index that have been considered to be the most important to radio engineers and others concerned with the atmospheric refractive process. The relative availability of radiosonde data for use in the computation of the refractive index also lends itself to vertical profiles. It is generally believed that the refractive properties of the atmosphere are horizontally stratified in much the same manner as many of the other atmospheric parameters. At higher levels, little is known about the variability of the index in the horizontal except the inference that can be made from other parameters.

Within the past few years, several attempts have been made to establish the values of the refractive index and its variability to heights of about 15 km. M. Dubin (1954) published results of a limited study based on this requirement. By using data gathered on

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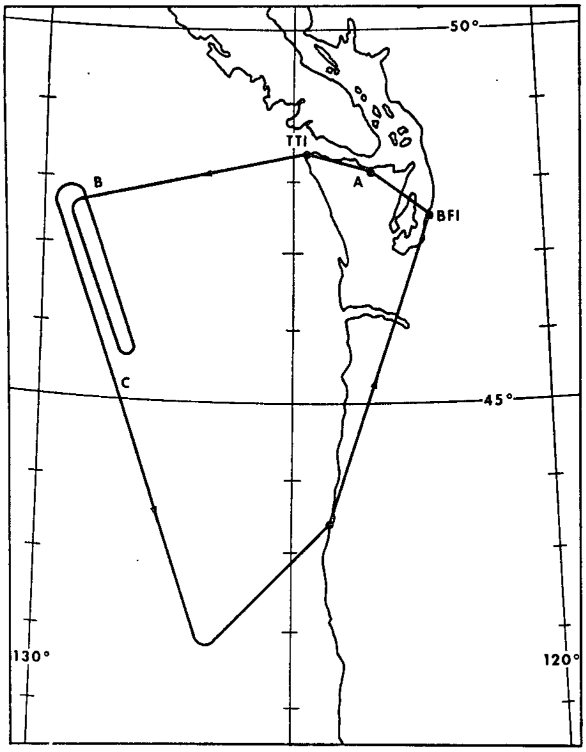

FIG. 1. Flight track for sampling flight of 18 August 1960. Sampling began at point A; the 150-mb level was reached at B and continued to point C, where the descent began. The high-altitude portion was flown between about 1310 and 1400 PST.

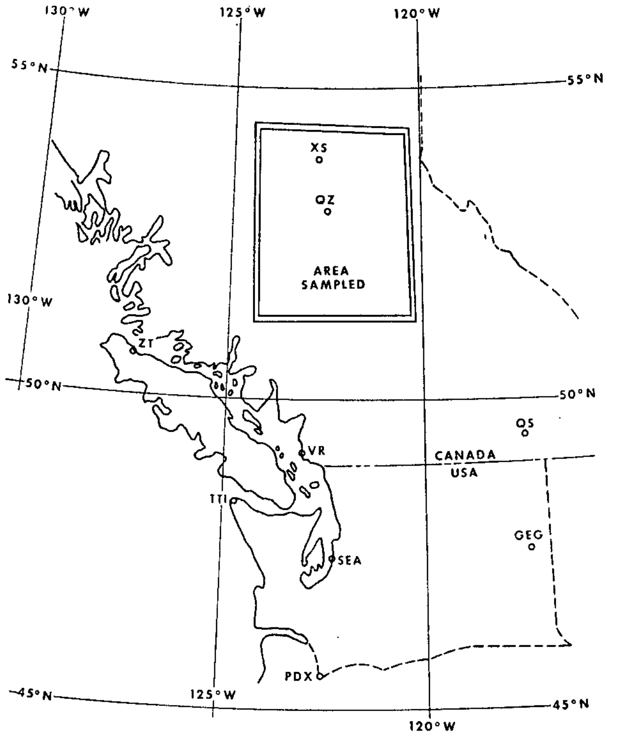

FIG. 2. Flight area for the sampling flight of 21 August 1960. Several passes across the area were made at the 150-mb level between the hours of 1225 and 1425 PST.

three ballon flights in 1949, he computed refractive-index values for heights up to 16 km. Table 1 shows data extracted from a table of results in Dubin's paper. Based on his calculations, he concluded that the density of air is important for altitudes up to 26 km but that the effects of water-vapor content above 20,000 ft may be neglected. Therefore, he recommended "that a standard index of refraction based on a standard density distribution may be adopted for the purpose, since the density distribution is only slightly dependent on latitude, diurnal effects, and atmospheric heating, so that the resultant index of refraction would be a fairly constant standard value within the accuracy required (one part in a hundred thousand)." (It is now recognized that an accuracy of one part per million should be the required goal.)

TABLE 1. Measured refractivity statistics derived from three balloon flights in 1949 (M. Dubin, 1954).

Height (km)	Pressure (mb)	Refractive modulus $[(n-1) \times 10^6]$
6.21	479	138
7.00	432	132
7.89	381	118
9.00	327	106
10.11	276	93
11.10	238	81
12.07	206	72
14.00	152	56
16.04	111	39

Much earlier, Schelling, Burrows and Ferrell (1933), proposed the widely-used earth's-radius model. This concept, inherently a linear model of refractivity, has become known as "4/3 earth" refraction. Bauer, Mason and Wilson (1958) proposed a refractivity model based on a cool exponential atmosphere. This model is shown to better fit the observed data (especially at higher altitudes) than the "4/3 earth model." Bean and Thayer (1959) proposed a model refractivity atmosphere which permits computation of refractive errors of microwave propagation based on the theoretical distribution of the refractive index with height.

In their study on model refractive atmospheres, Bean and Thayer summarized some of the available data on average values and variability of the index up to 14 km. These data are presented in table 2. Particular emphasis was placed on the quasi-constancy at 105 N units of the index at 9 km. Tables 1 and 2 may be used as yardsticks in evaluating the data obtained during the AN/AMQ-15 flight tests presently being discussed.

The AN/AMQ-15 program afforded an opportunity to measure directly the index of refraction above 20,000 ft. It also afforded an excellent opportunity to make the measurements at time intervals suffi-

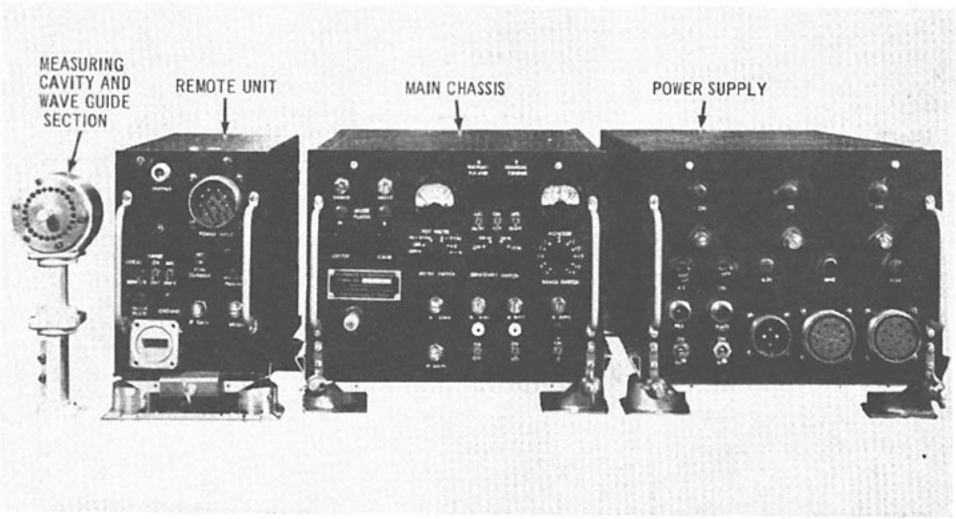

FIG. 3. Electronic components of the Crain Type VIII microwave refractometer. The sampling cavity is shown in the extreme left.

TABLE 2. Refractivity statistics as a function of altitude above sea level as derived from individual radiosonde observations (Bean and Thayer, 1959)

Altitude (km)	$\bar{N}$	Max N	Min N	Range*
4	197.1	209.5	186.5	23.0
5	172.3	184.0	165.0	19.0
6	151.4	161.0	146.0	15.0
7	134.0	139.5	129.5	10.0
8	118.4	121.5	113.3	8.2
9	104.8	108.0	100.0	8.0
10	92.4	97.0	86.0	11.0
11	81.2	86.0	70.0	16.0
12	70.7	76.0	60.5	15.5
14	53.2	60.0	44.5	15.5

* Range = Max N − Min N.

ciently short to reveal the order of magnitude of variability of the refractive modulus in the horizontal.

This paper will present the data gathered during flights on the 18th and 21st of August 1959. Segments of these flights were flown at altitudes near 43,000 ft or at about the 150-mb level. The areas over which the flights were made are shown in figs. 1 and 2. Fig. 1 shows the track for the flight of the 18th of August. At point A, data recording began; at point B, the top of the climb was reached (approximately 150 mb) and the descent to 300 mb was started at C. Fig. 2 shows the area over which the flight of the 21st of August was flown. Precise track data are not available for this flight, but several passes in an approximate east-west direction at the 150-mb level were made across the area shown. The tracks on both days were flown at a speed of about 0.8 mach, and all of the data were gathered between 1200 and 1500 PST.

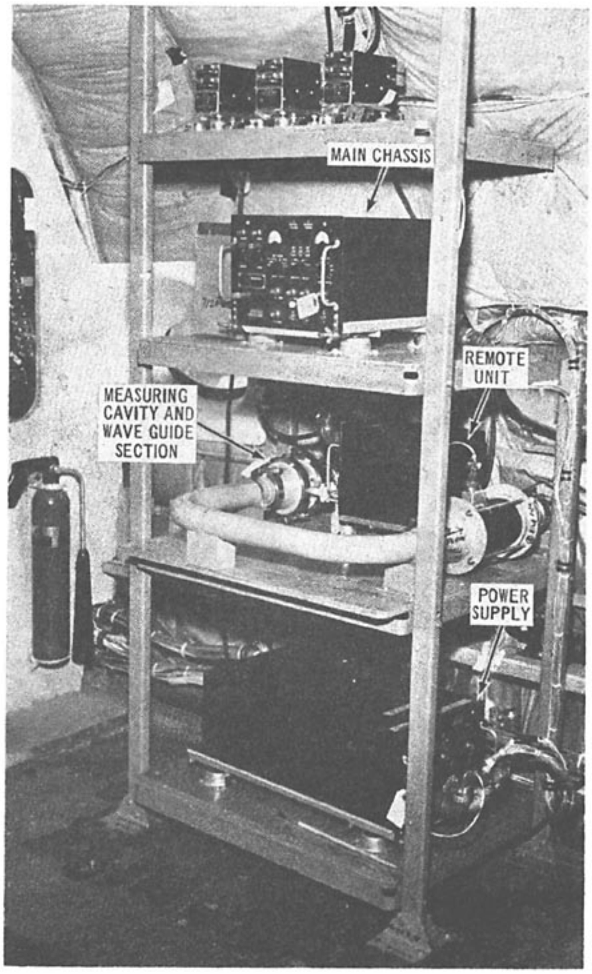

FIG. 4. Crain Type VIII microwave refractometer shown installed in a Boeing 707 jet aircraft as a part of the AN/AMQ-15 data-gathering system

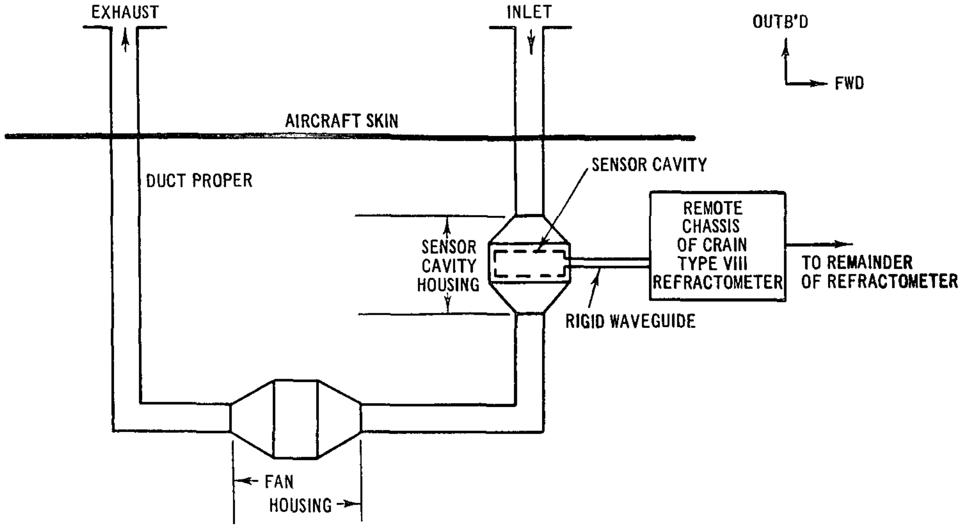

Fig. 5. Schematic of refractometer installation and air duct.

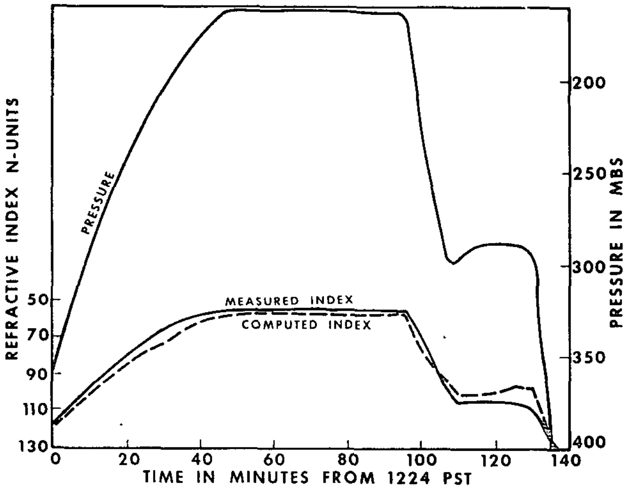

Fig. 6. Index of refraction *vs* time. Data gathered on 18 August 1959. "Measured index" values obtained from a Crain microwave refractometer mounted in a Boeing 707 jet aircraft. "Computed index" was derived from pressure, temperature, and frost-point data gathered simultaneously with the "measured index." Static pressure is plotted for reference purposes.

2. Sensor description

The sensor used to gather the index-of-refraction data was a Type VIII Crain microwave (X-band) refractometer. The manufacturer states that this instrument is accurate to within ±2 N units, has a resolution of 0.5 N unit, and has a response time of 1 sec for 90 per cent of a step change. The principle of operation is as follows. The dielectric constant of ambient air introduced without change into the chamber of a cavity oscillator determines the resonant frequency of the oscillator. The square root of the dielectric constant is equal to the refractive index. The instrument essentially records the changes in frequency of the oscillator, and these can be used to indicate changes in index of refraction. A bead thermistor installed in the duct carrying ambient air to the sensor verified that the temperature remained at ambient values.

Figs. 3 and 4 are photographs of the sensor. Fig. 3 shows the components of the sensor. The sampling cavity and wave-guide section are at the extreme left of the photograph. The other units from left to right are the remote unit, the main chassis and the power supply. Fig. 4 shows the refractometer installed in the 707 aircraft. The relative size of the assembly can be established by comparison with the standard fire extinguisher also shown in the photograph. Fig. 5 is a schematic drawing of the duct necessary to transport the ambient air (from outside the 707 aircraft) past the refractometer sensor and to exhaust it outside the aircraft. The air was drawn through the ports in the side of the aircraft which were parallel to the skin of the aircraft. The air was drawn through the duct by a pump rather than through use of ram effect of the aircraft's motion through the atmosphere; thus, a constant air flow through the sensor was achieved.

3. Results

The flight of 18 August 1959 consisted of a leg approximately 50 min. in length near the 150-mb level. The data were processed at 30-sec intervals. Fig. 6 shows the index-of-refraction values plotted at approximately 5-min intervals for the entire flight. The static pressure at the flight level is plotted for reference purposes and a computed index of refraction curve is also plotted on this figure. The computation was made at 5-min intervals from the pressure, temperature and frost-point data which were recorded simultaneously.

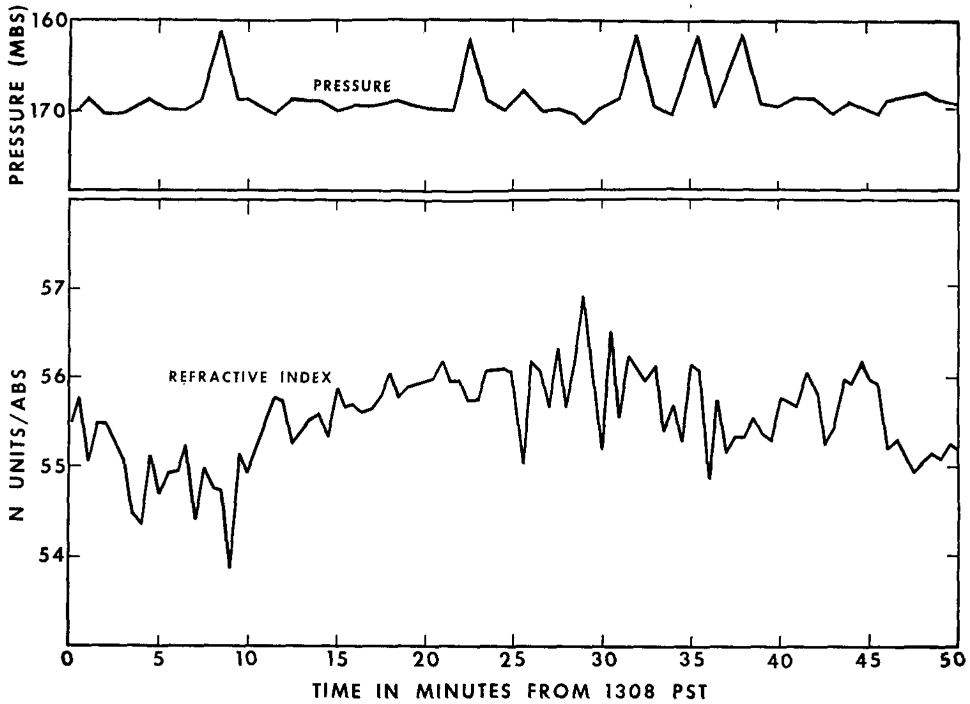

FIG. 7. Index of refraction *vs* time. Portion of the data gathered on 18 August 1959 plotted on expanded scale at 30-sec intervals. Pressure is plotted as a reference.

During the 150-mb portion of the flight, the index values on the two curves are never separated by more than 3.5 *N* units. This agreement is considered to be excellent. In an effort to determine the variability of the index at this altitude, a plot was made at 30-sec time intervals. Fig. 7 shows this plot for the 50-min period between 1308 and 1358 PST. It can be seen that the refractive index has many small-scale oscillations within the accuracy limits of the instrument which may or may not be real variations. No variations greater than the ±1 *N* unit accuracy of the instrument occur in the 50-min period. If a smooth curve were drawn for the values shown, this smoothed curve would only vary about 1 to 1.5 *N* units over the 50-min period. It appears that the variability of the refractive index at this level is extremely small, which substantiates earlier theories. Also, the measured refractive modulus is in good agreement with the values reported by Dubin (1954) and shown in table 1 and by Bean and Thayer (1958) shown in table 2. Some of the small-scale variations shown in fig. 7 are related to the perturbations of the pressure caused by the aircraft rising and falling in its attempt to maintain constant altitude, but the lack of a perfect matching of inflections leaves some variability still to be explained.

Short portions of the climb-out and descent profiles are also shown in fig. 6 along with a local flight portion near the 300-mb level.

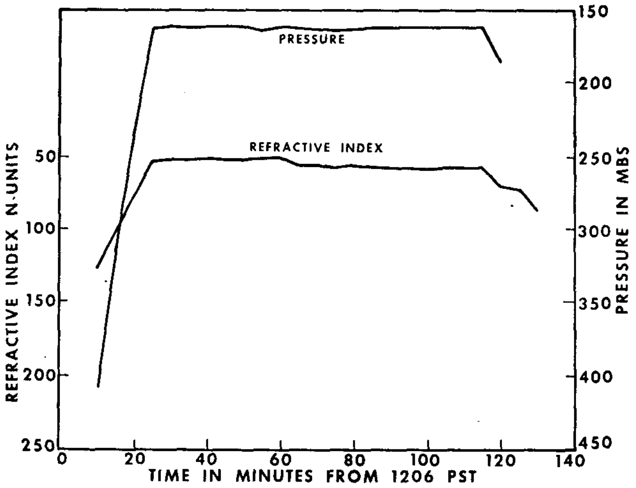

FIG. 8. Index of refraction *vs* time. Data gathered on 21 August 1959 starting at 1206 PST. Refractive index shown was measured with a Crain microwave refractometer. Pressure is plotted as a reference.

The second flight occurred on 21 August 1959, and the 150-mb leg of this flight was about 91 min in length, running from about 1231 to about 1420 PST. Fig. 8 shows the flight profile for the first two hours of flight. For this figure, the measured values of the refractive index were plotted every five minutes, along with the static pressure observations made at the same time. A small variation is noted starting at time 60 min or 1306 PST. The index values seem to

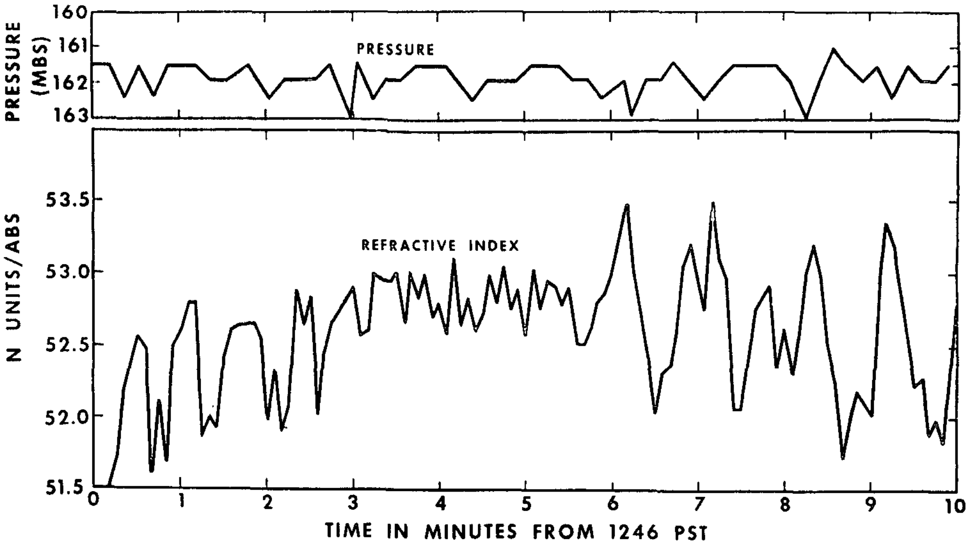

FIG. 9. Index of refraction *vs* time. Portion of the data gathered on 21 August 1959 plotted on an expanded scale at 5-sec intervals. Pressure is plotted as a reference.

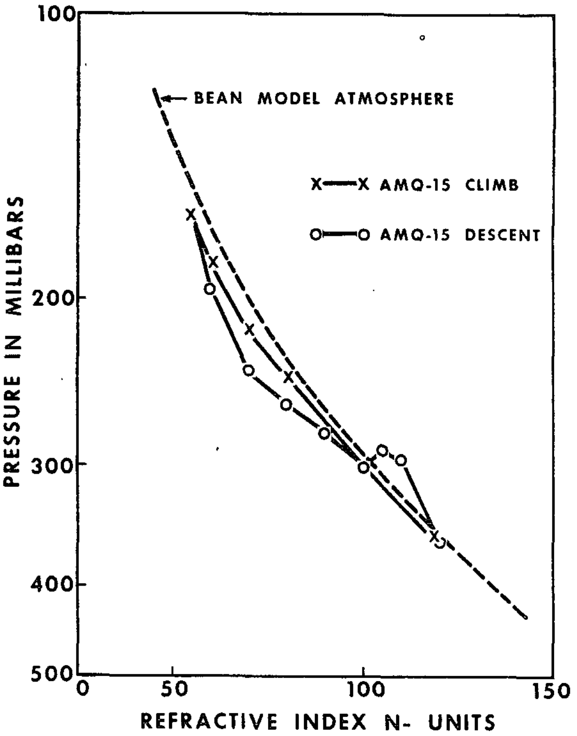

FIG. 10. Section of vertical index of refraction profile for 18 August 1959. Data gathered on aircraft climb and descent from about 350 mb to 150 mb. Bean's model index is shown for comparison.

increase approximately 2 N units over a period of about 4 to 5 min and then trend to slightly higher values over the next 5-min period. After time 80 min the index again remained relatively constant. The pressure varied slightly during this time (about 4 mb), and this pressure (or height of the aircraft) variation would account for about 1.5 N units of variation (from eq (1)). The remainder of the 2- to 4-N-unit variation must be explained by other causes such as sensor instability, the variability of the index, some turbulence or other external effect.

To further magnify the variations in the index of refraction profiles, a segment of the flight of the 21st of August was also plotted at extremely short-time intervals in an effort to detect the variability of the horizontal profile. Fig. 9 is a 10-min portion of the 150-mb segment plotted at 5-sec intervals. The extremely small-scale variations are again noted, but, as in fig. 7 for the flight on the 18th of August, the small-scale variations are all within the ± 1 N unit accuracy of the instrument and therefore may or may not be real variations. It can be readily noted that a smoothed curve drawn for these observations (fig. 9) would vary only a small amount with time. At about 2 min on the chart, the value of a smoothed curve would be about 52 N units, while, at about 3 to 4 min, the value would be about 52.8 N units. At 9 to 10 min, the smoothed curve would again have a value of approximately 52 N units (52.1 N units). Most of this small variability (of approximately 1 N unit) can be explained by the small variations in the altitude of the aircraft as shown by the pressure variations on the upper part of fig. 9. This pressure curve is plotted at 10-sec time intervals. A variation of about 2 mb would account for about 0.7 N unit of variation.

The profile of index of refraction, as it was measured during the aircraft ascent and descent on the 18th of August, is shown in fig. 10 together with the model refractive-index profile proposed by Bean. The agreement is quite good, although the strange behavior during the latter period of the descent might prove disconcerting to someone looking for ducting layers. This difficulty is related to the failure in achieving steady descent.

4. Conclusions

The values of the refractive modulus, and hence the profiles of the index of refraction measured on these high-altitude flights in the lower stratosphere, agree well with the values computed by Bean and Thayer (1959) and M. Dubin (1954). No significant horizontal variations of the index were detected in the areas investigated at the 150-mb level.

The measured data tend to justify the use of the theoretical mean profiles suggested by Bean and Thayer (1959) based on the assumption of an exponential atmosphere.

Bean and Thayer's data (table 2) indicates a total range of data for the 14-km level of about 15.5 N units. Their data are based on a variety of profiles from carefully selected stations representing the range of N profile conditions during summer and winter at 13 climatically diverse locations. It is interesting to note that the N unit variation from the 18th of August to the 21st of August was about 3 N units. This appears to be a relatively large day-to-day variation in light of the total 15.5 N unit variation in the sample of data reported by Bean and Thayer.

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