A FEW FOG DROP-SIZE DISTRIBUTIONS [1]

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(Manuscript received 19 September 1960)

ABSTRACT

An independent calibration of an instrument called a Particle Counter and the resulting fog drop-size distributions are discussed. The distributions exhibit an increasing number of droplets with decreasing size. The magnitude of the sampling error inherent in the instrument is inferred from a speculative argument based on the optical properties of measured drop-size distributions.

1. Introduction

The interest in field apparatus for the measurement of cloud or fog drop-size distributions has brought forth a parade of instruments through the years. A few years ago, a photoelectric instrument called a Particle Counter was developed by the Armour Research Foundation. The purpose of this paper is (1) to describe briefly an empirical calibration of the Particle Counter and (2) to present a few measured fog drop-size distributions.

The Particle Counter is a device to count the number of aerosols per unit volume of air in the size range of 1 to 64 microns diameter. This is accomplished by drawing the air containing the aerosols through a sampling chamber which is illuminated by an intense light. The stream of aerosols is collimated by an air sheath such that each particle being measured passes through the center of the sampling chamber. Light is scattered at 90 deg from the incident beam into a photomultiplier tube which converts the pulse of light caused by the aerosol into an electric pulse, the height of which is measured and counted in the appropriate channel of a scaler. A full discussion of the instrument is contained in the final report [1] written by Armour Research Foundation.

Incorporated in the design of the Particle Counter are two calibration systems. The first of these calibrates the electronic components; the second calibrates the optics. These two calibrations provide adequate standardization of the instrument. However, it is still necessary to determine the relationship between the flux scattered at 90 deg by a particular size aerosol and that scattered by the gold wire used in the optical calibration. To this end, an experimental approach was used to determine the flux scattered by glass microspheres in such a manner that the results could be reasonably extrapolated to include water aerosols.

2. Optical considerations

The detector and optical components of the Particle Counter are designed to operate at wavelengths principally in the visible region of the spectrum. A complete and terse treatment of the flux removed from the incident beam would consider the flux scattered according to Mie theory. In this practical situation, the index of refraction of water and of glass microspheres consists only of a real part since the particles are transparent to the incident radiation. Because absorption is negligible, there is no imaginary part. When the wavelength of the incident light is small (*i.e.*, one-tenth or less of the diameter of the scattering particle), the scattered light may be described by use of geometric optical principles. Only the smaller sizes, those less than approximately 5 microns in diameter, will not satisfy this general criterion. The deviation of the theoretical scattered intensity from the geometric optical intensity may be estimated by considering a reasonable instrumental field of view and extrapolating the data of Gumprecht, *et al* [2]. The results indicate that the geometric optical intensities underestimate the flux by about a factor of three in the 1- to 2-micron size range, by about a factor of 2.4 in the 2- to 4-micron size range, and overestimate it by less than 2 per cent in the 4- to 8-micron size range. However, experimental verification of these estimates would be extremely difficult.

The theory of light scattered by aerosols which are large compared to the wavelength of the incident light is fully treated by Van de Hulst [3]. Most of the flux scattered at 90 deg can be accounted for by considering only the first external reflection. By using Fresnel's

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[1] The research reported in this article was sponsored in part by USAF Cambridge Research Center under Contract No. AF19(604)-3483.

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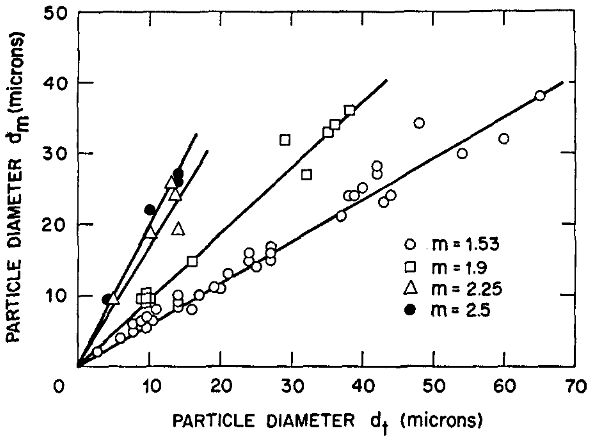

FIG. 1. Experimental data illustrating the relationship among the diameters of glass microspheres of four different indices of refraction as measured with the Particle Counter (d_m) and with an optical microscope (d_t).

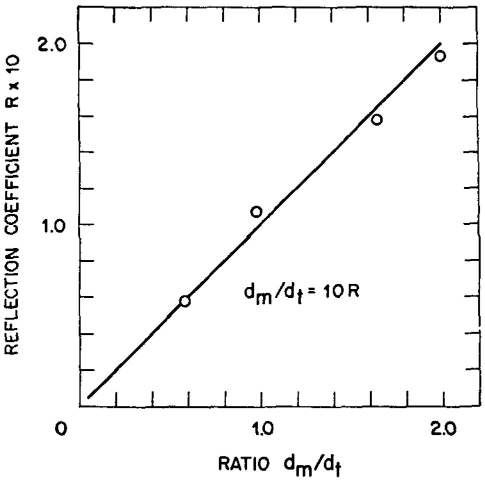

FIG. 2. The empirical relationship between the ratios of d_m/d_t and the reflection coefficients for the four indices of refraction.

reflection coefficient formula, which contains both planes of polarization as well as an index-of-refraction term, computation of the flux deflected from the incident beam may be determined for transparent spheres having a real index of refraction. An experimental procedure was then designed to ascertain the relationship between the flux reflected by glass microspheres of several indices of refraction.

3. Experimental calibration

Any student of aerosol sampling techniques knows the difficulties of measuring small particles (order of a few microns diameter) in a moving air stream. He also is aware of the pitfalls to be encountered with the generation of monodispersed, or even nearly monodispersed, distributions of aerosols. Because of these two problems, the experimental approach used avoided sampling aerosols less than 5 microns diameter and employed the major identifiable characteristics of the glass microsphere particle-size distributions to correlate the particle sizes measured with the Particle Counter to their true sizes.

Briefly, the experiment consisted of sampling glass microspheres of a particular index of refraction with the Particle Counter (after it had been internally calibrated) and comparing the size distribution with that measured with a microscope. This technique required samples of microspheres whose distributions had definite maxima and minima, thus affording easily identifiable features for comparisons.

A series of experiments was performed with microspheres of four different indices of refraction. Corresponding diameters of particles measured with the Particle Counter (d_m) and observed with the microscope (d_t) are shown in fig. 1 for four different indices of refraction, m, of the glass microspheres.

The values of R, the Fresnel reflection coefficients, for various glasses were computed with Fresnel's formula at 45 deg incidence and are shown in table 1 together with the experimental ratios of d_m/d_t taken from fig. 1. When the observed ratios of d_m/d_t were plotted against the reflection coefficients for each index of refraction, a functional relationship was found such that

$$d_m/d_t = 10R. \tag{1}$$

This empirical relationship, shown in fig. 2, forms the basis for the calibration of the Particle Counter and permits the extrapolation to water ($m = 1.33$; $R = 0.027$). The values of R computed with Fresnel's formula compare with those presented by Van de Hulst [3] for use with spherical particles which are large compared to the wavelength of the incident light. The relationship between the refractive index, m, and the reflective coefficient expressed as Fresnel's formula at 45-deg incident angle can be closely approximated over the range $1.3 < m < 2.5$ by

$$m = 1.14 + 7.03R. \tag{2}$$

Then, by equating eq (1) and (2), the ratio d_m/d_t may be expressed as

$$\frac{d_m}{d_t} = \frac{m - 1.14}{0.703} \tag{3}$$

within the arbitrary limits of $1.30 < m < 2.5$. Eq. (3)

TABLE 1. Properties of glass microspheres.

m	R	d_m/d_t
1.53	0.056	0.59
1.9	0.109	0.98
2.25	0.158	1.67
2.5	0.192	2.00

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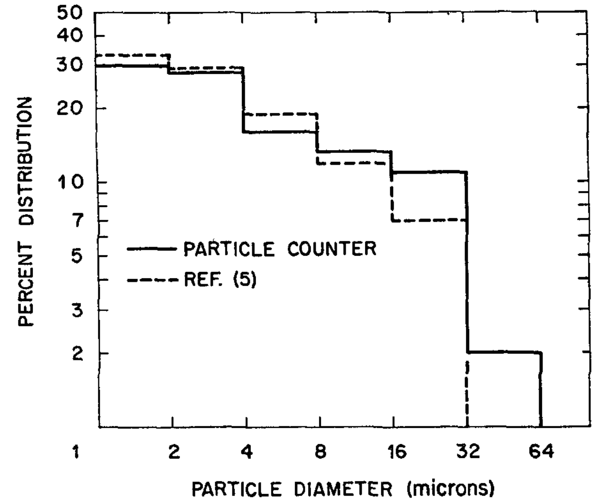

Fig. 3. A comparison of artificial fog drop-size distributions as inferred by an optical technique (dashed histogram) and observed with the Particle Counter (solid histogram).

permits calculation of the true drop size (d_t) from a drop size measured with the Particle Counter, (d_m), whose index of refraction is known and real. The remaining step in the calibration is that of relating the above empirical relations to the optical calibration of the Particle Counter.

The circuit of the Particle Counter is so arranged that the information represented in eq (3) can be incorporated by manual adjustment of a signal attenuator potentiometer, permitting the instrument to yield particle sizes directly (for a given index m) without need for further computation. Details of this procedure are given by reference [4] and footnote 3.

4. Measured drop-size distributions

Some years ago the drop-size distribution produced by a pneumatic atomizing spray nozzle[4] was determined by an optical technique [5, 6]. The same nozzle was used to produce an artificial fog whose drop-size distribution was measured with the Particle Counter and is shown in fig. 3 (solid histogram). The second distribution on the diagram (dashed histogram) is that obtained with the above-referenced optical technique. In view of the differences in instrumentation and class intervals of measurement, as well as the differences of the environment of the spray, the comparison seems quite satisfactory. As a result of this comparison, some observations of fog drop-size distributions were undertaken.

On the evening of 23 October 1958, a series of fog drop-size distributions was observed. Each observation of the particle-size distribution was made over a twenty-second interval. The results of these observations are presented in table 2. The number of drops per cubic centimeter is obtained by considering the rate of flow of air containing fog through the sampling chamber and the number of particles counted during the interval of time.

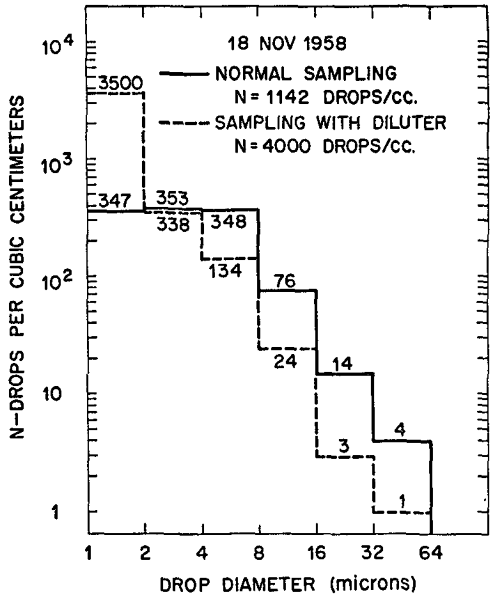

Fig. 4. A summary of the fog drop-size distributions observed on 18 November 1958 with the Particle Counter using (1) the normal entrance tube (solid histogram) and (2) the diluting mechanism on the entrance tube (dashed histogram).

On the evening of 18 November 1958, a fog occurred which may best be described as moderately dense, wet and approaching what is sometimes called a "Scotch Mist." The fog on this evening appeared more stable than that on 23 October. The drop-size distributions measured during the early part of the evening are included in table 2.

Subsequent observations of the drop-size distributions measured on the night of 18 November 1958 indicated that the instrument was being saturated in the 1- to 2-micron channel and at times in the 2- to 4-micron size channel. Each particle is recorded on a glow-tube counter which in turn trips a mechanical

Table 2. Fog drop-size distributions (drops/cc).

Date Time	Drop-size classes (microns) 1–2	2–4	4–8	8–16	16–32	32–64	1–64
23 Oct '58							
2117h	401	384	208	12	1	0	1006
2120h	407	272	138	12	1	0	830
2123h	415	457	207	13	1	0	1093
18 Nov '58							
2111h	497	340	53	8	8	5	901
2114h	440	298	46	7	6	5	812
2131h	492	424	110	10	7	4	1020
2134h	471	350	91	6	4	4	947

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[3] Personal communication: to B. Silverman, GRD, AFCRL, L. G. Hanscom Field, Bedford, Mass.

[4] Spray Nozzle type ¼ JN, manufactured by Spray System Company, Bellwood, Illinois.

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counter for each ten events. The dead time of the glow-tube counter is of the order of four microseconds; the maximum counting rate of the mechanical counter is of the order of 7 to 10 counts per second.[5] The maximum counting rate per channel is then 100 counts per second. When this counting rate is exceeded, the mechanical counter will "freeze" with a digit partially showing while the glow tube continues to record. During the measurements on the latter part of the evening of 18 November 1958, the concentrations of aerosols in the moderately dense fog periodically exceeded the counting rate of the mechanical counter, causing it to "freeze" momentarily several times during a 20-sec sample.

When this situation became evident, another series of measurements was made employing the diluter mechanism. This mechanism causes a reduction in the particles entering the sampling chamber by a factor of eleven. By sampling the fog alternately with and without the diluter, significantly different drop-size distributions resulted. The data are tabulated in table 3 and shown graphically in fig. 4. The ordinate in fig. 4 is the number of drops per cubic centimeter; the abscissa is the particle diameter in microns. The total number of droplets, N, in each distribution is listed with each histogram. The number at the top of each column indicates the number of drops in the respective class interval represented by the width of the column. All the data obtained with the diluting mechanism have been corrected with the dilution factor and converted into drops per cubic centimeter. It is quite obvious that significantly different drop-size distributions resulted.

TABLE 3. Fog drop-size distributions — 18 November 1958 (drops/cc).

	Drop-size classes (microns)						
Time	1–2	2–4	4–8	8–16	16–32	32–64	1–64
2152h	407	435	345	67	14	5	1293
2154h*	3210	284	53	4	3	1	3555
2156h	436	427	151	8	5	2	1029
2158h*	3330	364	58	14	3	1	3770
2200h	405	343	288	49	30	6	1121
2205h*	3960	365	292	53	2	0	4671
2207h	116	207	570	181	6	1	1081

* Distributions measured with the diluting mechanism.

5. Comments

Cloud and fog droplets are a rather delicate and perishable commodity and must be treated as such; any disturbance of their environment can affect their characteristics. For this reason, there is concern over the possible effect of drawing water droplets into the sampling chamber through an entrance tube about 10 millimeters in diameter. One would suspect that the act of drawing samples into a confining tube would subject the aerosols and their environment to changes in pressure and temperature which could substantially change the distribution of small droplets. Because of the difficulty of theoretically assessing the effect of temperature and pressure on the small droplets, an empirical estimate was made.

The first appraisal of the effect of the entrance tube walls consisted of visually inspecting the tube to note if any glass microspheres were adhering to the walls. No microspheres were found in the entrance tube, but a few did cling to the walls of the inner collimating tube. Inspection of the entrance tube after water aerosols had passed through indicated that a significant amount of moisture had collected on the walls of the entrance tube as well. An attempt was made to see if size discrimination was taking place by comparing the drop-size distribution of an artificial fog produced by the same spray nozzle and measured with the Particle Counter and with the optical technique (fig. 3).

Inspection of fig. 3 shows that there is good relative agreement between the two independent measurements of the drop-size distribution. Unfortunately, though, these measurements were made in different environments. Observations with the Particle Counter were made in an environment whose volume was some 50 times larger than that used for the optical measurement. The possible mixing of the aerosols with the environment may account for the absolute number of droplets counted with the Particle Counter being about a factor of 10 less than with the optical technique. If there were size discrimination, one would expect the smaller droplets to follow the air stream and the large droplets (*i.e.*, over 15 to 20 microns in diameter) to tend to coalesce on the walls of the tube. In spite of the difference in concentration, the two distributions are practically alike. However, this comparison would have been more conclusive if both measuring devices could have sampled the same artificial fog (a desirable situation seldom experienced in particle-size studies).

Evaluation of the drop-size distributions presented in tables 2 and 3 and in fig. 4 must be made subjectively because no complementary measurements were available. To aid in making this judgment, the liquid-water contents and visual range, V_m, in terms of scattering-theory parameters are computed. The latter, the visual range, may be computed as

$$V_m = \frac{1}{k_s} \ln \frac{1}{\epsilon} = \frac{3.912}{k_s} \qquad (4)$$

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[5] Personal communication: with Baird Atomic, Inc. (Atomic Instrument, Inc.), Cambridge, Mass.

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TABLE 4. Meteorological visual ranges and liquid-water contents.

Distribution	Reference	N (drops/cc)	V_m (km)	L.W.C. (gm/m^3)
23 Oct 1958	Table 2	976	0.159	0.039
18 Nov 1958	Table 2	921	0.112	0.287
18 Nov 1958	Table 3	1126	0.056	0.365
18 Nov 1958*	Table 3	4000	0.086	0.121

* Distribution measured with the diluting mechanism.

where the standard value of 0.02 is used for ϵ, the threshold of brightness contrast. The scattering coefficient, k_s, is determined by utilizing Mie scattering theory as previously reported by Eldridge [7] using the scattering area coefficients published by Houghton and Chalker [8].

Examination of table 4 reveals some interesting characteristics of these few measured drop-size distributions. In general, the atmosphere inferred from the drop-size distributions measured with the Particle Counter appears to have visual ranges and liquid-water contents representative of fog.

Because the drop-size distribution is a significant parameter describing the fog, one further pertinent point might be considered. The drop-size distributions measured on the 18th with the diluting mechanism on the entrance tube of the Particle Counter are quite different from those measured with the normal sampling tube. With reference to fig. 4, the most noticeable difference between the two distributions is the larger population of smaller drops which results in a significantly larger total number of drops per cubic centimeter. However, the number of drops in the size ranges between 4 and 64 microns diameter is about one-third that counted with the normal sampling entrance tube. As a result, the computed visual range indicated in table 4 is greater and the liquid-water content much less for the distributions measured with the diluting mechanism. This is true because the numerous small droplets, though important to the opacity of the atmosphere, add a negligible amount of liquid water.

The observed difference between the distributions shown in fig. 4 is a factor of about three. Increasing the number of particles by this factor in all size ranges results in good correlation between the two distributions in the size range between 4 and 64 microns diameter.

The importance of defining the characteristics of the drop-size distribution has been discussed by aufm Kampe and Weickman [9], Eldridge, [7], Fritz [10] and others. The correlation between liquid water content and visual range, known as Trabert's formula [11], is rather weak. These few drop-size distributions indicate quite forcibly the problem of determining visual range based on the drop-size distribution and on the liquid-water content. The 1- to 2-micron-diameter particles numbering 10,500 per cubic centimeter seem unusually large though not entirely without precedent. The author has inferred the existence of small droplets, about 50,000 drops per cubic centimeter, in an earlier paper [7]. Recent studies of the growth of cloud drops by Neiburger and Chien [12] make the existence of these small droplets for any length of time seem untenable. Yet the growth from nuclei to cloud droplets must exist for some time, no matter how brief, in this size range. Few measurements, if any, have been made of drop-size distributions as a function of time, let alone during the dymanic growth process. Small droplets (the order of a few microns) contribute only a negligible amount of liquid water; yet, because their size is comparable to the wavelength of visible light, they are significant in attenuating and/or scattering radiant energy. A few large droplets contribute substantially to the liquid-water content of fog but not appreciably to the reduction of the visual range.

6. Conclusions

The drop-size distributions presented in this paper have been subjected to some speculation suggesting sampling errors which are inherent in the apparatus. (One specific problem which requires attention is the determination of the effect of entrainment at the top of the entrance tube.) It is the author's opinion that suggested magnitude of these errors may be high, but this analysis serves to illustrate the necessity of measuring simultaneously all three parameters of fog — *i.e.*, drop-size distribution, liquid-water content and visual range.

The general characteristics of these few drop-size distributions, their number and in some cases the liquid-water content, correspond to those observed by other techniques. These distributions were measured with an instrument which is capable of observing droplets a few microns in diameter and, as a result, exhibit a large population of small droplets when present, especially when the diluting mechanism was used. The trend toward an increasing number of aerosols with decreasing size should be of interest to those concerned with radiant-energy transfer through atmospheric media.

Acknowledgments. The author wishes to express his appreciation to David K. Beaver and Nancy R. York for their assistance during the experiments and to Dr. Eric T. Clarke of Technical Operations, Inc. and Dr. John C. Johnson of Worcester Polytechnical Institute for their constructive criticisms and encouragement.

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REFERENCES

1. Armour Research Foundation, 1956: *Automatic counting and sizing of cloud particles.* Final Rept., Contract No. 19(604)1212.
2. Gumprecht, R. O., N. Sung, J. H. Chin, C. M. Sliepcevich, 1952: Angular distribution of intensity of light scattered by large droplets of water. *J. Opt. Soc. Amer.*, **42**, 226–231.
3. Van de Hulst, H. C., 1957: *Light scattering by small particles.* New York, N. Y., John Wiley & Sons, Inc., 470 pp.
4. Eldridge, R. G., and E. T. Clarke, 1959: *An independent calibration of the Armour Research Foundation Particle Counter.* Contract AF19(604)-3483 Tech. Ops., Inc.
5. Eldridge, R. G., 1953: Unpubl. M.S. Thesis, Mass Inst. Tech.
6. Johnson, J. C., 1952: *Analysis of size distributions of sulphur hydrosols by optical transmission methods.* Sci. Rep. No. 1, Contract AF19(122)-245, M.I.T.
7. Eldridge, R. G., 1957: Measurements of cloud drop-size distributions. *J. Meteor.*, **14**, 55–59.
8. Houghton, H. G., and W. R. Chalker, 1949: The scattering cross section of water droplets in air for visible light. *J. Opt. Soc. Amer.*, **39**, 955–957.
9. aufm Kampe, H. J., and H. K. Weickmann, 1952: Traberts' formula and the determination of water content in clouds. *J. Meteor.*, **9**, 167–171.
10. Fritz, S., 1954: Small drops, liquid-water content and transmission in clouds. *J. Meteor.*, **11**, 428–429. (See also "Replies" immediately following.)
11. Trabert, W., 1901: Die Extinction des Hichtes in Einem Truben Medium (Sehweite in Wolhen). *Meteor*, **18**, 518–525.
12. Neiburger, M. and C. W. Chien, 1960: Computations of the growth of cloud drops by condensation using an electronic digital computer, in *Physics of precipitation.* Geophys. Monogr. No. 5. Amer. Geophys. Union, 191–209.

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