Diurnal Variation of Wind, Pressure, and Temperature in the Troposphere and Stratosphere over the Azores

MILES F. HARRIS, FREDERICK G. FINGER AND SIDNEY TEWELES

U. S. Weather Bureau, Washington, D. C.

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ABSTRACT

The diurnal and semidiurnal variations of wind, pressure, and temperature at Lajes Field, Terceira, Azores, were computed for each month of the year for 30 levels between the surface and 10 mb. The semidiurnal variations were found to agree fairly closely with those at Washington, D. C., where data for the troposphere during the summer months are available for comparison. However, the diurnal variations of pressure and wind at the two stations are quite dissimilar. In order to check the consistency of the wind and pressure variations at Lajes Field, the diurnal and semidiurnal height variations were computed from the wind variation at each isobaric surface by the use of a model based on a linearized form of the equations of motion, frictionless flow, and the assumption that the oscillations are simple progressive waves. Results of the analysis indicate that the radiosonde observations contain diurnal, and smaller semidiurnal, temperature errors which superimpose fictitious pressure variations on the true pressure oscillations in the stratosphere, causing the phase of the latter to be moved toward noon. These results are supported by a similar analysis of rawinsonde data for Ft. Worth, Texas. The diurnal and semidiurnal temperature variations implied by the wind-derived height changes in the stratosphere are in general agreement with the results of determinations based on radiation theory. The diurnal error of the radiosonde temperature observations (observed minus computed temperature change) shows a maximum near noon at levels between 12 and 27 km, the diurnal range increasing from about 1C at the lower to about 2.5C at the upper level.

1. Introduction

In an earlier paper, one of the authors (Harris, 1959) presented data on the diurnal and semidiurnal variations of wind, pressure, and temperature in the troposphere at Washington, D. C., for the summer months. In that study it was found that the diurnal variation of the wind in the troposphere was, in general, consistent with the observed pressure variation. Meanwhile, the other two authors (Teweles and Finger, 1960) confirmed earlier evidence that large diurnal variations of pressure observed in the stratosphere by certain types of radiosonde instruments are fictitious. These erroneous pressure values are caused by temperature errors resulting from solar radiation on the instrument and the inability of the temperature sensing element to come into equilibrium with the surrounding air. In order, if possible, to determine the true diurnal variation of pressure in the stratosphere, as well as to obtain information needed to further our empirical knowledge of the diurnal and semidiurnal waves, it was decided to apply the method used in the Washington study to another station, employing observations in the stratosphere as well as in the troposphere for all months of the year. It is the purpose of this paper to present the results of the latter investigation.

Rawinsonde data from the United States Air Force Weather Station at Lajes Field, Terceira, Azores (38°44′N, 27°4′W), were selected for the study. These data were chosen because observations were obtained at Lajes Field to very high elevations during much of the year, and because this station used an externally-mounted thermistor which reportedly shows only a small error. As explained in the earlier article by Harris (1959), it is possible to compute the semidiurnal variation of the various elements by combining data for two years during which four observations per day were obtained at different scheduled times, resulting effectively in a frequency of eight instead of four observations per day. Justification for combining the observations in this way was discussed in the earlier paper.

Although the semidiurnal variation of surface pressure shows nearly the same phase everywhere over the earth, the phase of the diurnal wave varies considerably, and over the North Atlantic in particular differs markedly from that observed at most land stations. Rosenthal and Baum (1956) computed the diurnal and semidiurnal pressure variations from observations made at nine Ocean Vessel Stations in the North Atlantic. Their study revealed that at all nine ocean stations the diurnal wave characteristically reaches a maximum in the late afternoon, in contrast to the early morning maximum observed at land stations. This interesting phase difference, amounting to about 180 deg between land and ocean stations, presented an additional incentive for studying the Azores data. The annual variation of the diurnal wave at Lajes Field is quite similar to that at Ocean Vessel Station K.

In the following sections we shall first present the results of the harmonic analysis of wind, pressure and temperature at Lajes Field, and then discuss the evidence for consistency between wind and pressure variations.

2. Data and computational details

Prior to 1 April 1957, rawinsonde observations at Lajes Field were made at 0300, 0900, 1500, and 2100 GMT, and beginning with that date, at 0000, 0600, 1200, and 1800 GMT. Accordingly, the Lajes Field upper air data for the period 1 April 1956, to 31 March 1958, were obtained in punched card form from the National Weather Records Center at Asheville, N. C. The eastward and northward wind components were then computed for each observation from the surface to 10 mb, and six-hourly differences in the wind components were averaged for each month. The linear trend was removed from the resulting values, and a harmonic analysis was performed on the eight observations following the standard technique described by Conrad and Pollak (1950). At elevations above 60 mb, where the data were sparse, both forward and backward differences were taken in order to obtain as many pairs as possible. The same process was repeated for height and temperature observations and for pressure observations at the surface, but no temperature computations were made above the 30-mb level. All of the computations were performed on a Bendix G-15 digital computer. The amplitude, Ψ, and phase, α, were computed for representation according to the convention

$$\psi' = \Psi_n \sin(n\theta + \alpha_n),$$

where the prime indicates the deviation of the element ψ from the daily mean, n is the order of the harmonic, and θ is the hour angle measured from midnight local time. Since the observations were begun at Lajes Field one hour before the scheduled time, and since the balloon rises at a rate of approximately 1000 ft min^{-1}, a phase correction was applied at each level to make the data refer to actual local time. The relation between the phase and the time of maximum is given by $t_{\max} = \frac{450° - \alpha_n}{15n}$. Thus the time of maximum and the phase angle vary inversely.

Table 1. Diurnal and semidiurnal variations of the eastward and northward components of the wind at Terceira, Azores. Annual mean values of the amplitude, A, in cm sec^{-1}, and phase, α, in degrees. P.E. is the radius of the probable-error circle of the annual means.

Mean pressure, mb	Variation of eastward wind component						Variation of northward wind component					
	Diurnal			Semidiurnal			Diurnal			Semidiurnal		
	A	α	P.E.	A	α	P.E.	A	α	P.E.	A	α	P.E.
SFC	2	75	6	8	324	7	7	341	8	21	52	6
1000	4	115	9	12	298	6	6	337	6	17	52	6
950	2	154	8	14	317	8	21	271	11	22	35	6
900	2	248	3	19	292	6	32	272	9	23	31	8
850	8	257	11	14	266	7	25	256	13	23	14	8
800	4	322	10	22	313	8	18	265	11	31	359	8
750	22	145	14	22	278	10	16	251	15	29	12	14
700	5	304	11	18	304	8	9	4	11	22	33	9
650	4	255	11	20	292	5	13	318	9	23	50	8
600	8	63	12	20	272	6	12	281	11	16	1	7
550	20	159	10	31	327	9	18	249	13	16	7	10
500	20	124	9	25	276	10	15	317	15	15	63	9
450	17	76	11	26	295	10	22	295	16	20	346	10
400	18	1	8	28	291	10	14	342	17	10	317	12
350	19	258	12	42	291	9	13	257	19	16	319	9
300	24	193	18	51	292	14	8	247	21	14	4	13
250	52	177	16	26	245	12	52	267	17	8	285	12
200	56	153	15	46	267	11	18	238	14	15	338	12
175	13	164	13	37	278	14	34	241	13	25	339	10
150	16	186	11	39	300	11	18	185	14	52	18	9
125	14	127	8	29	242	9	5	241	7	23	4	8
100	27	112	10	55	280	10	34	153	11	28	19	6
80	31	111	11	37	280	9	21	194	9	28	21	7
60	34	109	11	36	262	8	40	196	9	27	5	10
50	19	132	9	41	256	8	34	236	10	42	356	7
40	6	96	11	44	263	12	27	235	11	49	4	8
30	23	181	11	67	280	13	21	221	10	65	17	11
20	30	147	12	62	295	12	64	235	13	60	36	14
15	25	114	23	91	303	20	66	238	14	61	30	10
10	—	—	—	—	—	—	—	—	—	—	—	—

As an indication of the reliability of the results, the radius of the probable-error circle of the mean of each set of twelve monthly values (hereafter, for brevity, referred to as the probable error) was obtained. The probable errors are included in Tables 1–3 presenting the results of the harmonic analyses. According to Chapman (1951) a determination may be considered reasonably good if the amplitude is at least three times as large as its probable error. It should be noted that a true annual variation is probably present, especially prominent in the first harmonic, and that for this reason the computed probable error is almost certainly an overestimate of the true error. An analysis of variance was performed on the data to obtain further information on the reliability of the annual mean values.

Inspection of the original data tabulations revealed that in many instances the wind direction had obviously been rounded off to the nearest 10 deg, undoubtedly contributing to the large probable errors in the determination of the wind components.

3. Diurnal and semidiurnal variation of wind

In Table 1, the mean annual amplitude and phase of the diurnal and semidiurnal variations of the two components of the wind are shown for each level, together with the probable errors of the means. The individual monthly values (not reproduced here) exhibit irregular changes of phase and amplitude, from level to level and from month to month, which probably reflect errors in the determination of the harmonic coefficients and tend to obscure an underlying systematic variation.

It is of interest to compare the wind variation at Terceira with that obtained at Washington. In Figs. 1a and 1b, the amplitude and phase of the diurnal variation of the eastward and northward components of the wind, for the summer months, are shown as functions of pressure for Washington and Terceira. In each case the values are averages for the months June, July, and August. The figures show that both the eastward and northward components of the wind undergo diurnal variations over the Azores which are about 180 deg out of phase with the variations at Washington. This phase difference probably reflects the 180 deg phase difference in the surface diurnal pressure variation at the two stations. The amplitude of each component is considerably smaller at Lajes Field than at Washington, particularly in the upper troposphere.

Figs. 1c and 1d show, on the contrary, that there is little difference between the semidiurnal variation of

TABLE 2. Diurnal and semidiurnal variations of pressure at Terceira, Azores. Annual mean values of the amplitude, A, in mb, and phase, α, in degrees. P.E. is the radius of the probable-error circle of the annual means.

Mean pressure, mb	Pressure variation					
	Diurnal			Semidiurnal		
	A	α	P.E.	A	α	P.E.
SFC	0.10	135	0.04	0.50	156	0.03
1000	0.10	164	0.03	0.53	155	0.03
950	0.12	174	0.06	0.46	152	0.04
900	0.16	207	0.03	0.49	149	0.03
850	0.18	209	0.03	0.47	149	0.03
800	0.20	207	0.03	0.44	149	0.03
750	0.20	206	0.03	0.38	145	0.04
700	0.25	213	0.04	0.37	149	0.04
650	0.18	208	0.04	0.40	135	0.03
600	0.25	208	0.04	0.33	140	0.03
550	0.27	223	0.05	0.33	133	0.04
500	0.28	221	0.04	0.29	134	0.03
450	0.27	234	0.04	0.24	132	0.04
400	0.31	224	0.04	0.24	127	0.03
350	0.31	224	0.04	0.20	127	0.03
300	0.32	229	0.04	0.18	116	0.03
250	0.33	235	0.04	0.16	119	0.02
200	0.32	238	0.03	0.14	115	0.02
175	0.32	242	0.03	0.13	111	0.02
150	0.30	243	0.02	0.11	120	0.02
125	0.28	248	0.02	0.11	108	0.02
100	0.26	254	0.01	0.09	106	0.01
80	0.24	255	0.01	0.09	110	0.01
60	0.23	256	0.01	0.07	108	0.01
50	0.21	256	0.01	0.07	113	0.01
40	0.20	259	0.01	0.06	112	0.01
30	0.18	256	0.007	0.05	115	0.01
20	0.16	256	0.007	0.04	106	0.007
15	0.15	257	0.006	0.03	102	0.007
10	0.12	254	0.008	0.01	88	0.010

TABLE 3. Diurnal and semidiurnal variation of temperature at Terceira, Azores. Annual mean values of the amplitude, A, in degrees C, and phase, α, in degrees. P.E. is the radius of the probable-error circle of the annual means.

Mean pressure, mb	Temperature variation					
	Diurnal			Semidiurnal		
	A	α	P.E.	A	α	P.E.
SFC	1.12	260	0.10	0.30	90	0.03
1000	0.74	256	0.05	0.16	77	0.03
950	0.28	243	0.04	0.09	71	0.03
900	0.16	221	0.05	0.04	55	0.03
850	0.20	209	0.05	0.02	18	0.03
800	0.16	205	0.06	0.05	2	0.04
750	0.18	220	0.04	0.04	26	0.04
700	0.14	245	0.04	0.02	42	0.03
650	0.14	240	0.04	0.02	14	0.02
600	0.18	239	0.03	0.06	59	0.02
550	0.18	238	0.03	0.06	37	0.02
500	0.19	239	0.04	0.04	75	0.02
450	0.20	234	0.03	0.02	10	0.02
400	0.20	247	0.04	0.01	72	0.02
350	0.25	262	0.05	0.01	161	0.02
300	0.27	264	0.03	0.04	55	0.02
250	0.27	266	0.04	0.02	94	0.03
200	0.29	263	0.04	0.04	92	0.03
175	0.30	267	0.05	0.08	130	0.03
150	0.39	279	0.03	0.10	120	0.03
125	0.38	285	0.05	0.04	74	0.02
100	0.40	276	0.05	0.09	128	0.04
80	0.46	273	0.04	0.15	125	0.03
60	0.60	270	0.04	0.11	120	0.04
50	0.68	269	0.04	0.16	129	0.04
40	0.77	265	0.04	0.19	141	0.04
30	0.98	259	0.06	0.09	156	0.05

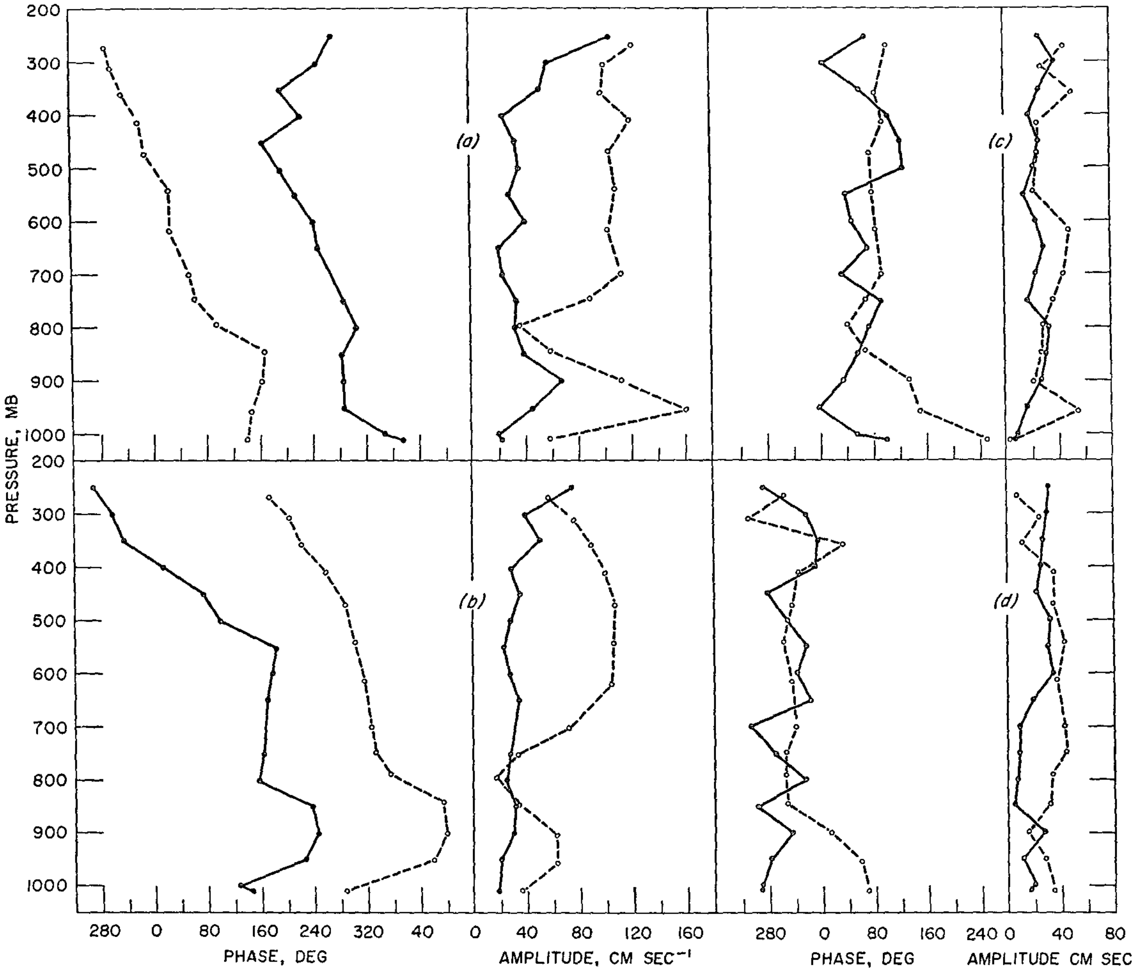

FIG. 1. Daily variation of wind in the troposphere during the summer months (June–August, 1956–58) at Terceira, Azores (solid lines) and Washington, D. C. (dashed lines). Diurnal variation, (a) northward component, (b) eastward component; semidiurnal variation, (c) northward component, (d) eastward component.

wind at the two stations, except in the lowest 5000 ft. This agreement is consistent with the conformity of the semidiurnal variation of surface pressure at the two stations.

4. Diurnal and semidiurnal variations of pressure

The height variations of the isobaric surfaces were converted to pressure variations at the equivalent standard atmosphere heights. In Table 2, the amplitude and phase of the annual mean diurnal and semidiurnal variations of pressure are shown for each level, together with the probable errors of the annual means. The semidiurnal variation of pressure shows a relatively slight, steady decrease of phase with height, indicating a delay in the time of maximum pressure of about two hours at the higher elevations. The diurnal pressure variation shows a greater phase change, the maximum pressure occurring about 8 hr earlier in the stratosphere than at the surface. The relatively slight annual variation of the diurnal wave in the upper troposphere is in marked contrast to the large annual variation of this component of the wave near and at the surface, suggesting that the phase of the surface diurnal wave is controlled largely by factors near the earth's surface. Fig. 2 is included to bring out, more effectively than the tabular data, the seasonal changes in the diurnal surface pressure wave at the Azores, as well as at Ocean Vessel Station K, in contrast to the uniformity of the wave at Washington.

In Fig. 3, the diurnal and semidiurnal pressure oscillations at Lajes Field are compared with those at Washington for the summer season—June, July, and August. The phase of the semidiurnal wave is very nearly the same at both stations, and shows agreement also with the phase of the semidiurnal pressure oscillation found by Haurwitz (1947) for stations in the Eastern Caribbean. However, at the Azores the amplitude of the semidiurnal wave decreases with height approximately as the pressure itself, as shown by the line of constant slope in Fig. 3b, in agreement with the findings of Hann (1926) from mountain station data and in contrast to the results for Washington and the Eastern Caribbean. Near the surface, the phase of the diurnal wave at the Azores differs by approximately 180 deg from that at Washington, but this difference is gradually eliminated with height, the phase decreasing at Terceira and increasing at Washington, until, at the 300-mb level, the phase is the same at both stations. The amplitude of the diurnal wave increases with height at the Azores, and above 800 mb is very nearly the same as at Washington. An increase in amplitude of the diurnal wave with height was found also in the Caribbean data analyzed by Haurwitz (1947), suggesting that this feature may be typical of maritime locations.

Perhaps the most striking results of the analysis of the Azores pressure data are the gradual eradication of the 180 deg diurnal phase anomaly at upper levels, and the regular decrease in amplitude of the semidiurnal wave with height.

5. Diurnal and semidurnal variations of temperature

The annual mean values of the diurnal and semidiurnal variations of temperature are shown in Table 3, together with the probable errors. The amplitude of the diurnal temperature wave decreases throughout the troposphere, but begins to increase in the stratosphere, until at the 30-mb level the amplitude is approximately the same as at the surface. The diurnal wave reaches a maximum near noon, or a few hours after noon, at all levels. The amplitude of the semidiurnal temperature wave is much smaller, and the many irregularities in phase suggest that this component of the temperature wave is not very accurately determined. In Fig. 4, the diurnal and semidiurnal temperature waves for the period June-August at Lajes Field are compared with the results found for Washington. The phase of the diurnal wave is nearly the same at both stations, the maximum at Lajes Field occurring on the average about one hour earlier than at Washington. The larger diurnal amplitude in the surface layers at Washington reflects the difference between a continental and an oceanic station. The amplitude of the semidiurnal temperature wave is larger at Washington than at Lajes Field throughout the troposphere. The marked difference in the phase of the semidiurnal temperature wave at the two stations is rather surprising, in view of

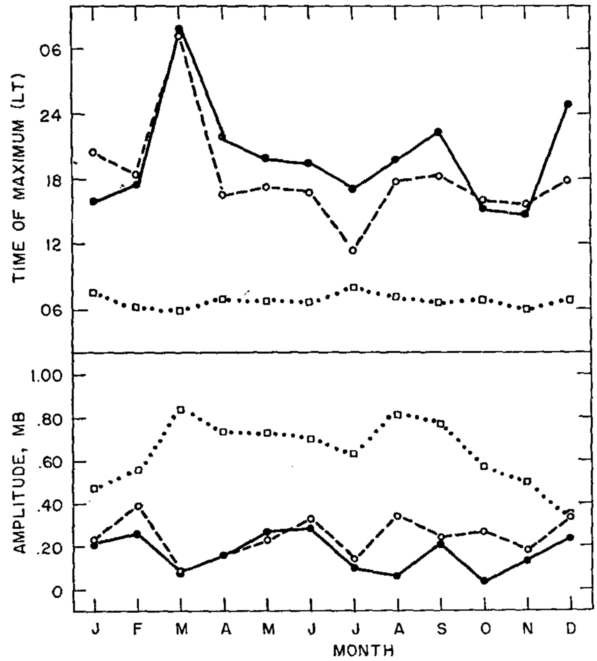

FIG. 2. Amplitude and time of maximum of the diurnal variation of surface pressure, Terceira, Azores, solid lines; Ocean Vessel Station K (45N, 16W), dashed lines; Washington, D. C., dotted lines.

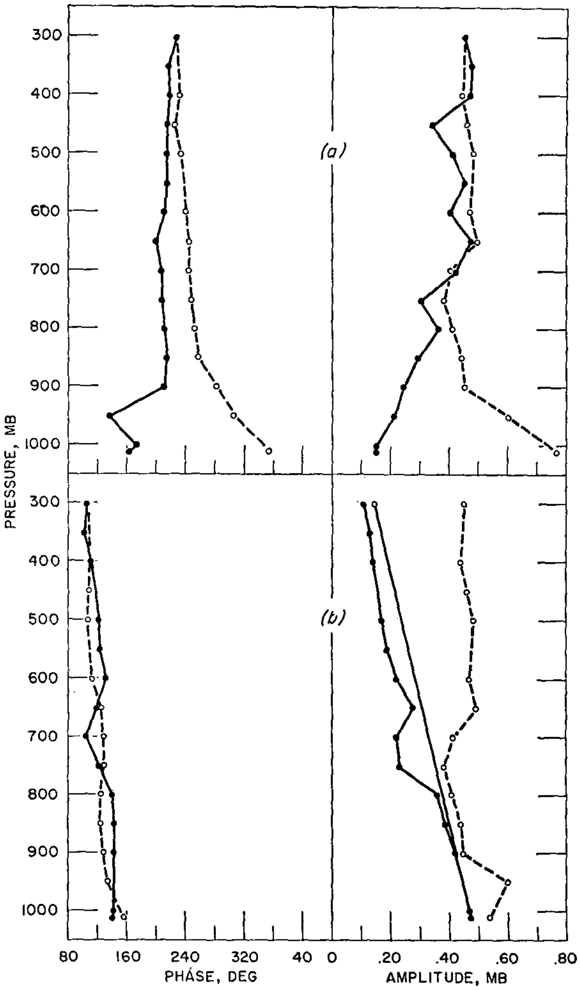

FIG. 3. Daily variation of pressure in the troposphere during the summer months (June-August, 1956–58) at Terceira, Azores (solid lines) and Washington, D. C. (dashed lines). Diurnal variation (a); semidiurnal variation, (b).

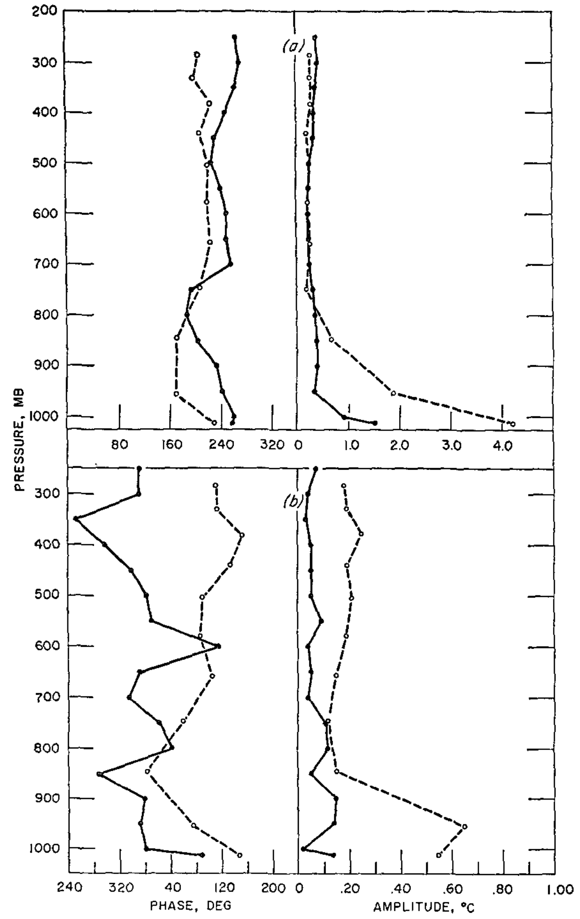

FIG. 4. Daily variation of temperature in the troposphere during the summer months (June–August, 1956–58) at Terceira, Azores (solid lines) and Washington, D. C. (dashed lines). Diurnal variation, (a); semidiurnal variation, (b).

the agreement in wind and pressure variations at Washington and the Azores, but may be no more than a reflection of errors in determination of the wave.

6. Discussion of results

A comparison of the probable errors of the annual means with the means themselves (Tables 1–3) reveals that the diurnal and semidiurnal variations of wind are not very accurately determined. In order to investigate further the reliability of the results, an analysis of variance was performed on the 12 monthly harmonic components of each variable at each level. As shown in Table 4, an F-test bears out the conclusions suggested by an inspection of the probable errors. The semidiurnal pressure wave is the most accurately determined element, and the diurnal pressure and temperature waves also are quite closely determined. The semidiurnal variation of wind is much more reliably determined than the diurnal wind variation, which, together with the semidiurnal temperature variation, is the least accurate of the means. The accuracy of the wind analysis increases markedly in the stratosphere, a result no doubt of the damping of interdiurnal fluctuations at progressively higher levels. The statistical analysis, of course, sheds no light on the possible existence of systematic diurnal errors in the wind, pressure, and temperature observations.

While systematic errors may be present in the pressure and temperature results at stratospheric levels, there is no evidence for such a bias in the wind observations. It is sometimes argued that a fictitious diurnal variation of wind may be observed in regions of pronounced vertical wind shear as a result of the diurnal variation of height. Since the diurnal height variation is of the order of 10 m, a wind shear of 50 m sec^{-1} over a height interval of 1 km would be required to produce a diurnal wind change of 50 cm sec^{-1}. A mean vertical shear of this magnitude is quite unlikely at the heights with which we are concerned. It is therefore justifiable to analyze the wind observations in an effort to determine whether or not diurnal biases are present in the pressure and temperature observations. The following discussion is confined to elevations above the surface layer, so that friction may be neglected.

Since the scale of the diurnal wave is large and the gradients are small, the linearized equations of motion,

$$\frac{\partial u}{\partial t}-2\omega v\sin\phi+\frac{1}{\rho_0 a\cos\phi}\frac{\partial p}{\partial\theta}=0, \tag{1}$$

$$\frac{\partial v}{\partial t}+2\omega u\sin\phi+\frac{1}{\rho_0 a}\frac{\partial p}{\partial\phi}=0, \tag{2}$$

are appropriate for such an analysis (Harris, 1959). In Eq (1) and (2), u and v are the eastward and northward components of the perturbation velocity, p is the disturbed pressure, and ρ_0 is the undisturbed density. The coordinates are t, time; ϕ, latitude; θ, longitude; and z, height. The earth's radius and angular velocity are indicated by a and ω. Choosing a time factor $e^{2i\omega ft}$, where f is one-half the ratio of the frequency of the perturbation to the frequency of the earth's rotation, so that $\pi/\omega f$ is the period of oscillation, we can solve (1) and (2) for u and v (Wilkes, 1949):

$$u=\frac{1}{2a\rho_0\omega(\sin^2\phi-f^2)}\left[\frac{if}{\cos\phi}\frac{\partial p}{\partial\theta}-\sin\phi\frac{\partial p}{\partial\phi}\right], \tag{3}$$

$$v=\frac{1}{2a\rho_0\omega(\sin^2\phi-f^2)}\left[-if\frac{\partial p}{\partial\phi}+\tan\phi\frac{\partial p}{\partial\theta}\right]. \tag{4}$$

Since $f=\frac{1}{2}$ for a diurnal wave and $f=1$ for a semidiurnal wave, the coefficient of the bracketed terms on the right-hand side of Eq (3) and (4) is always negative for the semidiurnal wave, and is positive for the diurnal wave at latitudes higher than 30 deg. It follows that, for pressure waves which progress westward uniformly

TABLE 4. Significance of the annual means of the diurnal and semidiurnal variations of wind, pressure, and temperature at Terceira, Azores, showing the probability that the means do not differ from zero. The subscripts (1) and (2) refer to the diurnal and semidiurnal terms, respectively. Eastward component of the wind, u; northward component, v; pressure, p; temperature, T.
The summary at the bottom of the table shows the percentage of cases occurring within each probability interval.

Pressure, mb	$0<P\leq 0.001$	$0.001<P\leq 0.01$	$0.01<P\leq 0.05$	$0.05<P\leq 0.20$	$0.20<P<0.80$	$0.80\leq P<0.95$	$0.95\leq P<0.99$	$0.99\leq P$
SFC	$v_2p_2T_1T_2$	p_1			v_1u_2			u_1
1000	$p_2T_1T_2$	p_1	v_2	u_2	v_1		u_1	
950	p_2T_1	T_2	v_2	$v_1u_2p_1$				u_1
900	p_1p_2	$u_2v_2T_1$	v_1		u_1T_2			
850	p_1p_2	T_1	v_2	v_1u_2	u_1T_2			
800	p_1p_2	v_2	u_2T_1		v_1T_2	u_1		
750	$p_1p_2T_1$			u_2v_2	$u_1v_1T_2$			
700	p_1p_2	T_1		u_2v_2	v_1T_2	u_1		
650	$p_1p_2T_1$	u_2	v_2		v_1T_2	u_1		
600	$p_1p_2T_1$	u_2T_2		v_2	u_1v_1			
550	$p_1p_2T_1$	u_2	T_2	u_1	v_1v_2			
500	$p_1p_2T_1$			$u_1u_2T_2$	v_1v_2			
450	$p_1p_2T_1$		u_2	v_2	$u_1v_1T_2$			
400	$p_1p_2T_1$		u_2	u_1	v_1	v_2T_2		
350	$u_2p_1p_2T_1$		T_2	v_2	u_1v_1			
300	$p_1p_2T_1$	u_2			$u_1v_1v_2T_2$			
250	$p_1p_2T_1$	u_1v_1		u_2	v_2T_2			
200	$u_1u_2p_1p_2T_1$	v_1		T_2	u_2			
175	$p_1p_2T_1$		$v_1u_2v_2T_2$		u_1			
150	$p_1p_2T_1$	u_2v_2	T_2		u_1v_1			
125	$p_1p_2T_1$	v_2	u_2		u_1T_2	v_1		
100	$u_2v_2p_1p_2T_1$		$u_1v_1T_2$					
80	$v_2p_1p_2T_1T_2$	u_2	u_1v_1					
60	$u_2p_1p_2T_1$	v_1	$u_1v_2T_2$					
50	$u_2v_2p_1p_2T_1T_2$	v_1		u_1				
40	$v_2p_1p_2T_1T_2$	u_2	v_1			u_1		
30	$u_2v_2p_1p_2T_1$			u_1v_1	T_2			
20	$u_2v_2p_1p_2$	v_1	u_1					
15	$v_1u_2v_2p_1$	p_2			u_1			
10	p_1			p_2				
Pressure	92%	5%	—	3%	—	—	—	—
Temp.	52%	9%	13%	4%	20%	2%	—	—
Wind	16%	15%	17%	17%	27%	5%	1%	2%

around the earth, with amplitudes decreasing toward the poles, the semidiurnal variation u_2 is 180 deg out of phase with the semidiurnal pressure wave, while at the latitude of the Azores, the diurnal variation u_1 is in phase with the diurnal pressure wave. Under similar conditions, the corresponding variations of the northward components v_2 and v_1 are 90 deg out of phase with the respective pressure waves, the maximum of v_2 preceding the semidiurnal pressure maximum, and the peak of v_1 occurring after the diurnal pressure maximum. With these relationships, we can determine the phase of the pressure waves indicated by the wind observations.

In Fig. 5a, the phase of the diurnal pressure wave indicated by the v_1-component of the wind is compared with the observed phase. Fig. 5b shows a similar comparison for the u_1-component. The lowest elevation shown is 700 mb, since the probable existence of eddy friction invalidates the assumptions of the model in a deep layer next to the ground. The hatched areas represent the probable error limits of the wind components. At elevations below 350 mb (8 km), there is little reason to suspect any appreciable error in the observed pressure wave. Unfortunately, in this region the wind components are inconsistent with the model, for they are obviously not exactly 90 deg out of phase. If we assume that frictional effects are not propagated higher than the 700-mb level, this inconsistency in the wind component must mean that the diurnal wave in the troposphere is not uniform in the region of the Azores. Above the 8-km level, evidence for uniformity of the wave is to be found in the fact that u_1 and v_1 are in quadrature. The wind components consistently indicate that the pressure wave should reach a maximum progressively later as one ascends into the stratosphere until, at a height of 100 mb (16 km), this trend is reversed.

Referring now to Figs. 5c and 5d, we see that, although the wind components are not entirely consistent throughout the troposphere, at elevations above the 200-mb level there is little evidence that the phase of the observed semidiurnal pressure wave is appreciably in error. Since radiational heating of the thermistor should produce only a slight semidiurnal temperature error, this result is in line with what one would expect.

By making use of Eq (1), and of the assumption that the waves move westward with uniform phase and

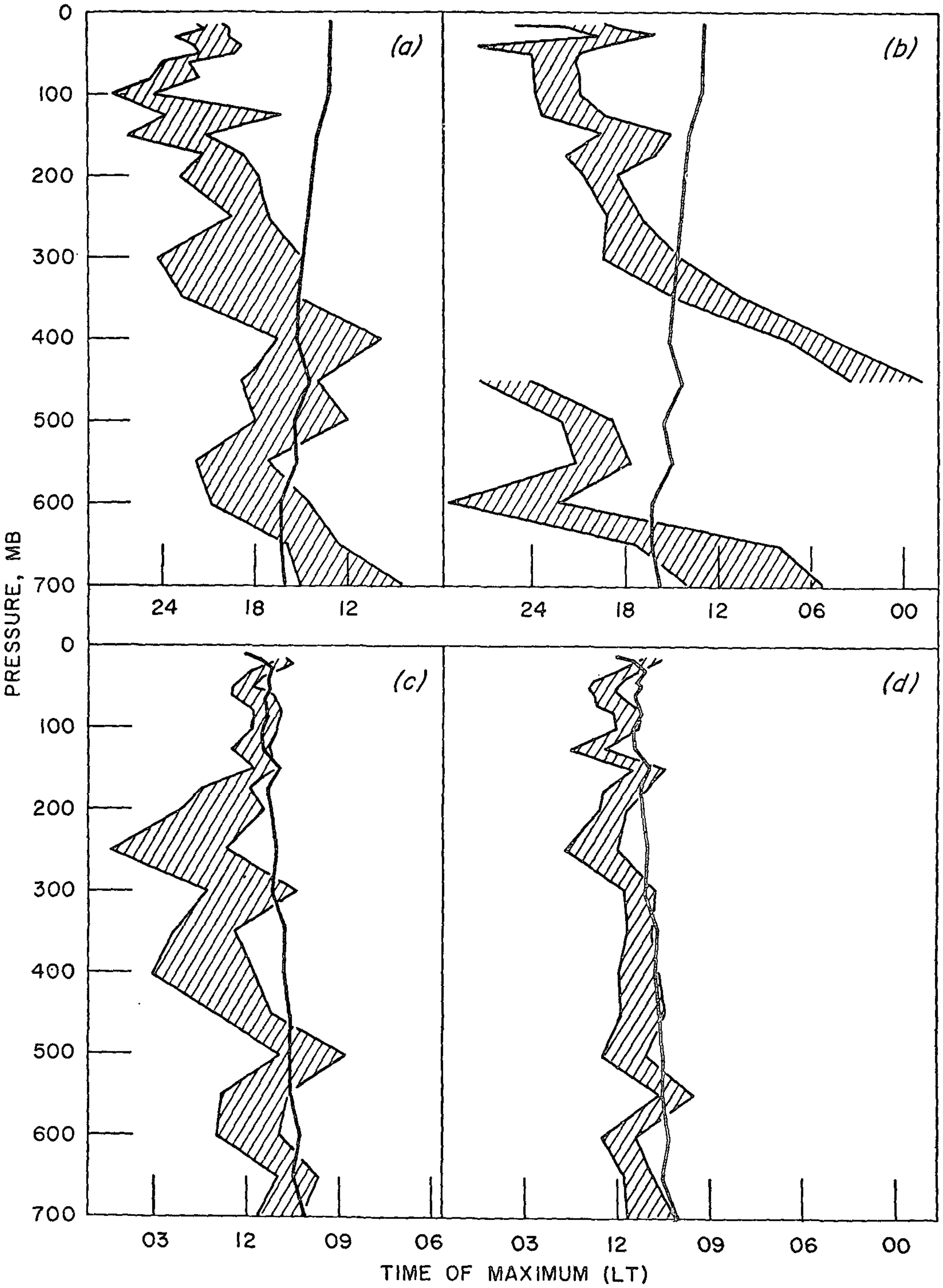

FIG. 5. Observed phase of the daily pressure variation at Terceira, Azores (heavy line) and phase indicated by wind components: diurnal variation, (a) northward component, (b) eastward component; semidiurnal variation, (c) northward component, (d) eastward component. The hatched area indicates the uncertainty in phase reflecting the probable errors of the wind components.

velocity, it is possible to compute the pressure variation indicated by the winds. In one sense this approach is less restrictive than that used above, since it does not require that the phase of the wave be invariant with latitude. On the other hand, the method is highly sensitive to errors in the wind components, for the two wind components are added. Since $\partial/\partial t=\omega\partial/\partial\theta$, Eq (1) can be expressed in the form

$$\frac{\partial u}{\partial t}-2\omega v\sin\phi+\frac{g}{a\omega\cos\phi}\frac{\partial z}{\partial t}=0, \tag{5}$$

where g is the acceleration of gravity and z is the variation in the height of an isobaric surface. Upon applying the time factor $e^{2i\omega ft}$ to Eq (5), it is a simple matter to compute the height variation by adding the vectors representing the observed wind variation, and if desired, to convert the height variation to one of pressure. If P and Q are, respectively, the coefficients of the cosine and sine terms of the Fourier series, then

$$\begin{aligned} P_{z,f}&=-\frac{a\omega\cos\phi}{g}P_{u,f}-\frac{a\omega\sin\phi\cos\phi}{gf}Q_{v,f},\\ Q_{z,f}&=-\frac{a\omega\cos\phi}{g}Q_{u,f}+\frac{a\omega\sin\phi\cos\phi}{gf}P_{v,f}, \end{aligned} \tag{6}$$

where f has been defined earlier and the subscripts z, u, and v refer to the height, the eastward wind component, and the northward wind component, respectively. At the latitude of the Azores, the coefficients of the first and second terms on the right-hand side of Eq (6) are approximately 37 sec and $23/f$ sec, respectively.

Before undertaking the computation of the pressure variation, the harmonic components of the wind ($P_{u,f}$, $P_{v,f}$, $Q_{u,f}$, $Q_{v,f}$) were smoothed separately with respect to both height and month to eliminate, insofar as possible, random errors in phase and amplitude. To accomplish this smoothing, the eight components surrounding a central value, together with the central value itself, were averaged to obtain a first approximation of the central value. If the central value differed from the mean by more than 40 cm sec^{-1}, it was adjusted to the mean, and the correction distributed among the surrounding values. The criterion of 40 cm sec^{-1} was selected by inspection, in order to eliminate erratic values possibly reflecting errors in the punched card data. The resulting smoothed values show a certain continuity which in the unsmoothed values is obscured by numerous random errors.

In Figs. 6 and 7, the smoothed diurnal and semidiurnal variations of the wind components for selected levels and months, as well as for the year, are represented on harmonic dials. There is evidence in these two figures that the diurnal variation of wind shows greater seasonal variability than does the semidiurnal variation. This result is consistent with the marked annual variation in the surface diurnal pressure wave at the Azores. A radiation error would have the effect of reducing the seasonal variability of the pressure wave at high elevations, introducing a spurious consistency in the observed pressure wave.

The results of the computation of the pressure variation from the wind are shown in Figs. 8 and 9. Computations were made for the four seasons centered on March, June, September and December, and also for the year, but only the annual results are presented here. The seasonal results follow in general the patterns illustrated in Figs. 8 and 9. The computations suggest that the diurnal pressure wave, above 8 km, reaches a maximum considerably later than the radiosonde observations indicated, and that the amplitude of the wave is smaller than that observed. To verify this result, computations were made for another station, Ft. Worth, Texas; the phase of the computed diurnal pressure variation is plotted on Fig. 8 to facilitate comparison. The Ft. Worth wind data, which represent the mean of 10 months' observations, were not smoothed as were the Azores data, but it can be seen that they also support the indication that the maximum of the true diurnal pressure wave occurs later in the stratosphere than the observed pressures suggest.

The temperature variation implied by the wind-derived height changes was computed for several layers between 250 and 15 mb. From the integrated form of the hydrostatic equation, it follows that the harmonic components of the temperature wave, $P_{T,f}$ and $Q_{T,f}$, are proportional to the like components of the thickness wave, $P_{\Delta Z,f}$ and $Q_{\Delta Z,f}$, in the layer between two isobaric surfaces. If p_1 and p_2 are the pressures at the base and the top of the layer, respectively,

$$\begin{aligned} P_{T,f}&=\frac{g}{R\ln(p_1/p_2)}P_{\Delta Z,f},\\ Q_{T,f}&=\frac{g}{R\ln(p_1/p_2)}Q_{\Delta Z,f}, \end{aligned} \tag{7}$$

where R is the gas constant. Since this computation involves an additional differentiation, the temperature variation was computed only for annual values of the thickness change, at layers well above the levels where friction could conceivably be of importance. In Table 5, the diurnal and semidiurnal wind-derived temperature oscillations are compared with those derived from the observed height changes for five layers whose mean elevations range from 12 to 27 km.

The difference between the observed and computed temperature waves presumably represents the diurnal temperature error due to radiation. At a maritime location where there is no mid-afternoon peak in long-wave radiation from heated ground surfaces to influence the radiosonde temperature, the maximum radiational

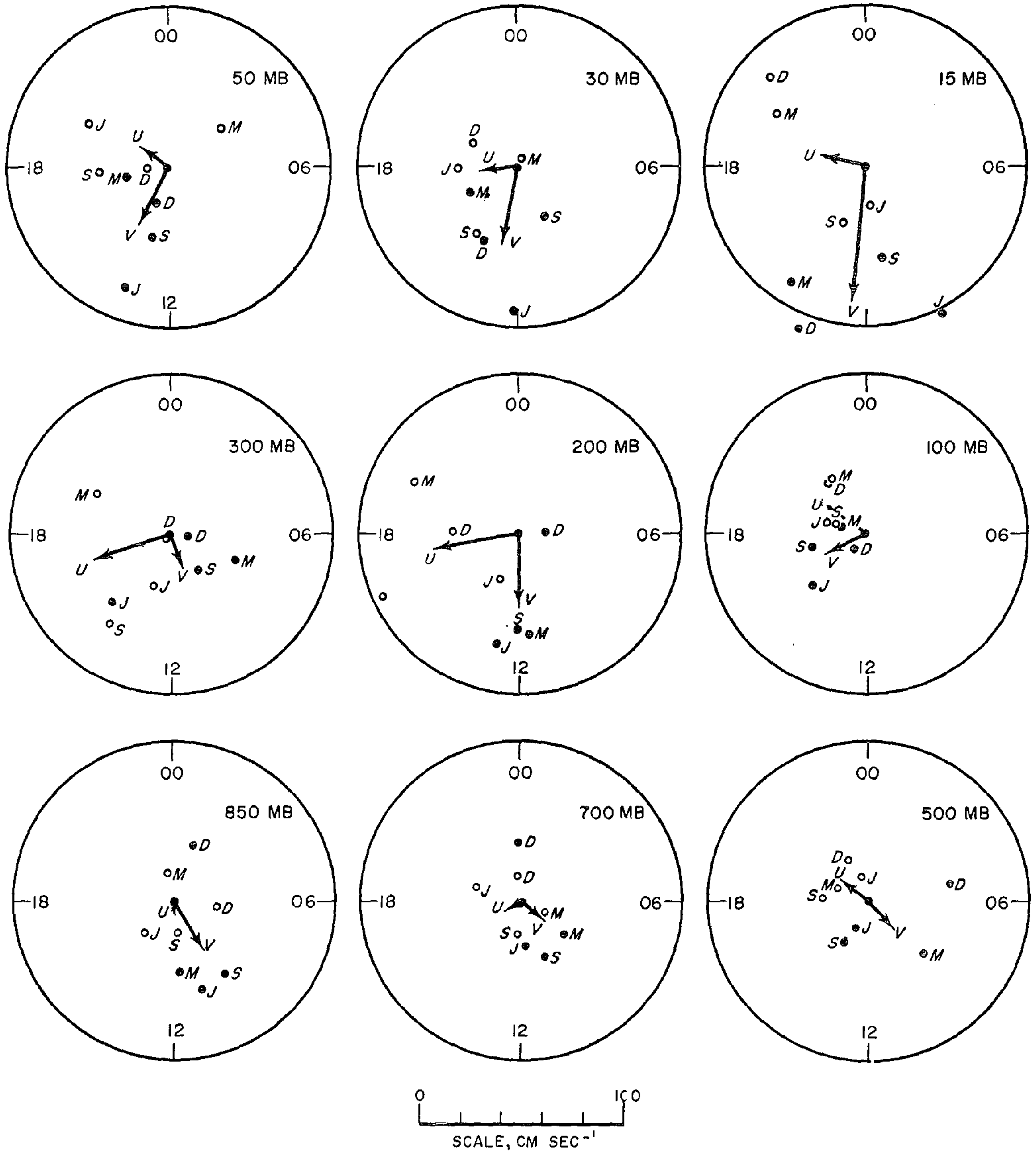

Fig. 6. Harmonic dials showing diurnal variation of wind at Terceira, Azores, after smoothing. U, annual value, eastward component; V, annual value, northward component. Open circles indicate eastward component for individual months, (M) March, (J) June, (S) September, (D) December; closed circles indicate northward component for individual months.

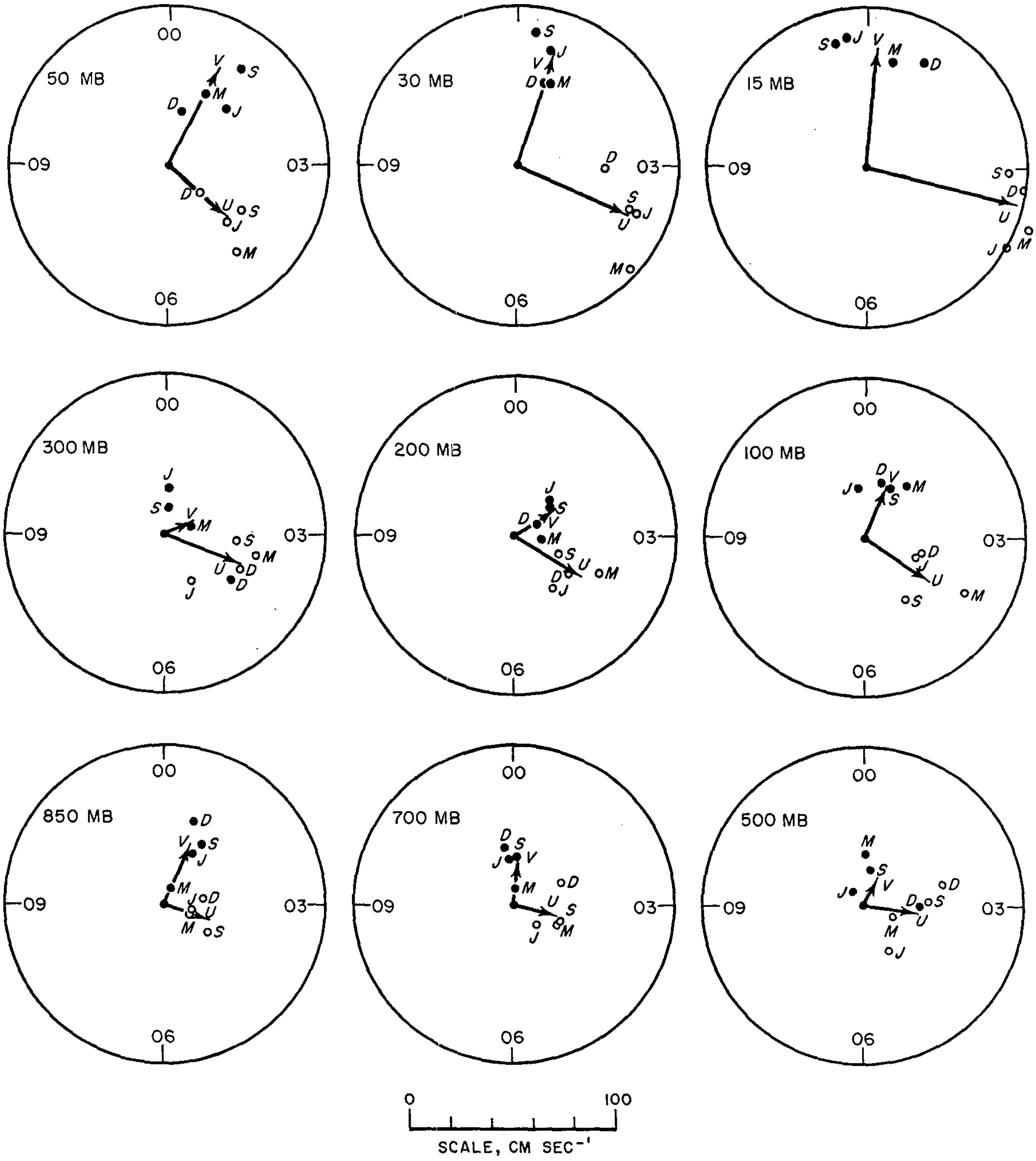

FIG. 7. Harmonic dials showing semidiurnal variation of wind at Terceira, Azores, after smoothing. *U*, annual value, eastward component; *V*, annual value, northward component. Open circles indicate eastward component for individual months, (M) March, (J) June, (S) September, (D) December; closed circles indicate northward component for individual months.

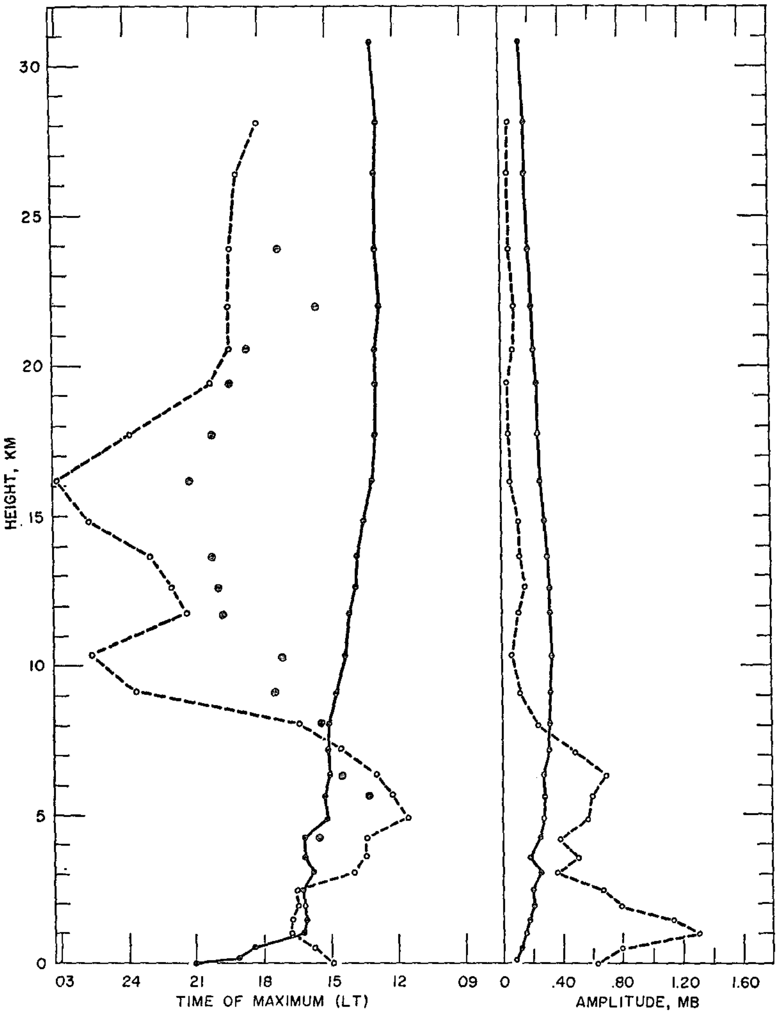

Fig. 8. Observed diurnal variation of pressure (solid lines) and variation computed from the winds (dashed lines), Terceira, Azores. The closed circles indicate the time of maximum of the diurnal pressure wave computed from wind data at Ft. Worth, Tex.

temperature error should occur near noon. The composite temperature error, found by combining the diurnal and semidiurnal components, shows that the maximum positive error at Lajes Field occurs within about an hour of noon, varying from 1100LT at the base to 1300LT at the top of the layer. The observed temperature wave has a peak near noon at each level, while the computed wave, on the average, reaches a maximum about 6 hr later.

The real diurnal temperature wave due to direct solar heating of the ozone layer has been shown by Pressman (1955) to have a peak just before sunset, with an amplitude of "a few tenths of a degree" at 20 km and about 0.95C at 30 km (12 mb), at the spring equinox. The remarkable degree to which the amplitude and phase based on wind observations agree with those based on radiation theory supports the idea that the difference figures shown in Table 5 represent the average temperature error of the 1680-mc outrigger radiosonde used at Lajes Field during the two-year period.

7. Summary and conclusions

The diurnal and semidiurnal variations of wind, temperature, and pressure were computed at 30 levels between the surface and 10 mb for Lajes Field, Terceira, Azores. Statistical analysis showed that the semidiurnal pressure variation and the diurnal pressure and temperature variations were closely determined, while the accuracy of the other oscillations is rather limited.

The consistency between wind and pressure observations was investigated by substituting the wind variation in a linearized form of the equation of motion and computing the pressure variation, which was then compared with the observed. This analysis, based on

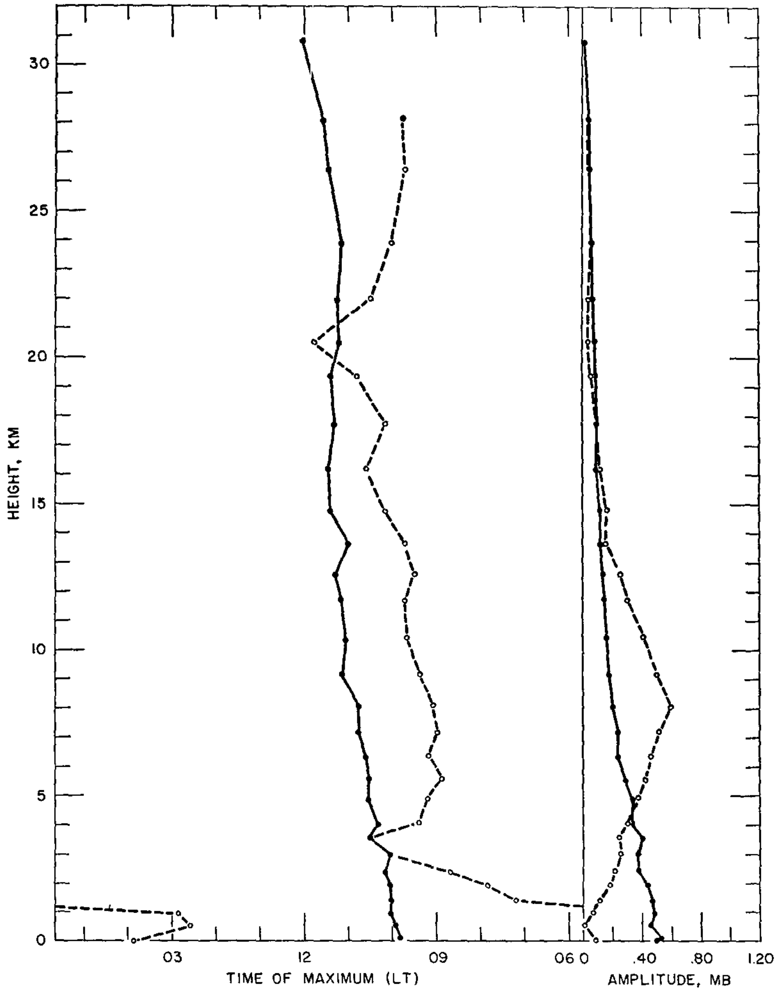

FIG. 9. Observed semidiurnal variation of pressure (solid lines) and variation computed from the winds (dashed lines), Terceira, Azores.

TABLE 5. Computed and observed temperature variation in the stratosphere at Lajes Field, Terceira, Azores, and the difference observed minus computed. *A* is the amplitude in deg C and *t* is the time of maximum. (For the semidiurnal wave, another maximum occurs at $t \pm 1200$.)

	Diurnal variation						Semidiurnal variation					
Layer (mb)	Computed		Observed		Difference		Computed		Observed		Difference	
	A	*t*	*A*	*t*	*A*	*t*	*A*	*t*	*A*	*t*	*A*	*t*
250–125	0.16	0148	0.31	1216	0.46	1248	0.18	1436	0.08	1214	0.06	0932
125–60	0.31	1604	0.52	1148	0.48	0924	0.14	1448	0.09	1120	0.19	0944
60–30	0.34	1848	0.80	1220	0.91	0952	0.24	2140	0.11	1150	0.25	1414
30–20	0.31	1644	1.11	1256	0.98	1152	0.27	2052	0.12	1156	0.31	1408
20–15	0.97	1604	1.66	1256	1.23	1036	0.31	2154	0.16	1440	0.45	1433

the assumptions that friction is absent and that the pressure waves move uniformly from east to west, without change of amplitude or phase, suggests that the diurnal wave above 8 km reaches a maximum after 1800LT instead of shortly after noon as the radiosonde observations indicate. The wind data for another station, Ft. Worth, Texas, support this result. The computations also indicate that the semidiurnal pressure wave at higher elevations reaches a maximum at about 1000LT instead of at 1100LT as the radiosonde reports show.

From these results it may be tentatively concluded that the radiosonde observations at Lajes Field, in the stratosphere and upper troposphere, contained a diurnal temperature bias. The temperature error, as computed from the height variations obtained from the winds and from the radiosonde observations, shows on the average a maximum at noon, but the phase of the error varies slightly from the base to the top of the layer between 12 and 27 km. The fact that the amplitude and phase of the computed temperature wave show general agreement with the amplitude and phase determined from radiation theory supports the assumption that the difference between observed and computed temperature variations is a good approximation to the diurnal error.

Other results of the study are of interest primarily to specialists in the field of atmospheric tides and oscillations. For example, the semidiurnal wind variation suggests that the semidiurnal pressure wave in the free atmosphere is very nearly though not quite in phase with the surface pressure wave. The diurnal temperature waves are quite similar at the Azores and Washington, but the surface diurnal pressure waves are 180 deg out of phase. This result suggests that the phase of the surface diurnal pressure wave is largely controlled by some other influence than temperature. It seems probable that this controlling feature may be found in the vertical exchange of momentum at the earth's surface, which has a marked diurnal period.

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