

Theoretical Estimates of the Average Surface Temperature on Mars¹

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ABSTRACT

Estimates of the average surface temperature on Mars are derived from radiative equilibrium considerations. A minimum possible surface temperature is estimated by computing the radiative equilibrium temperature that the Martian surface would have if the planet had no atmosphere. An estimate of the maximum possible value of the average surface temperature is obtained by computing the surface temperature that would result from a maximum greenhouse model. The computations indicate that the average surface temperature is in the range 219K to 233K. Comparisons of the theoretical computations with indications of surface temperature obtained from thermal emission observations are found to be in reasonable agreement.

1. Introduction

Of extreme interest to meteorologists and astrophysicists who are involved in planetary atmospheres research is the average surface temperature of the planets. Planetary surface temperature can be derived from both experimental observation and theoretical reasoning. The observational determinations, which work well in the case of a rather tenuous atmosphere such as Mars', are based upon measurements of the planet's emission of infrared radiation; the theoretical determinations are based largely upon some sort of radiative equilibrium considerations. In the present study we derive estimates, based upon radiative equilibrium considerations, of the minimum and maximum possible value of the average temperature of the Martian surface. These estimates are then compared to some of the thermal emission observations.

2. Theory

The average surface temperature of a planet which has an atmosphere that is transparent to long wave radiation can be computed by equating the energy received from the sun to the energy emitted by the planet's surface. The following formula for the temperature T of such a planet can be easily derived:

$$T^4 = \frac{(s.c.)(1-A)\mu}{4\sigma} \quad (1)$$

In this formula, $s.c.$ is the solar constant at the earth's distance from the sun, A is the planetary albedo, σ is the

Stefan-Boltzmann constant, and μ is a dilution factor equal to r_e^2/r_p^2 , where r_e is the mean distance from the earth to the sun and r_p is the mean distance from the planet to the sun.

If one now introduces into such an atmosphere gases that absorb long wave radiation, a "green-house" effect is created and the average surface temperature will increase above that given by (1). Thus for an atmosphere that is not completely transparent to long wave radiation (1) can be used to derive an estimate of the lowest possible value of the average planetary surface temperature. For the planet Mars the following numerical values can be substituted in (1):

$$A = 0.15,$$

$$\mu = 0.44,$$

$$s.c. = 2.0 \text{ cal cm}^{-2} \text{ min}^{-1},$$

$$\sigma = 8.13 \times 10^{-11} \text{ cal cm}^{-2} \text{ deg}^{-4} \text{ min}^{-1}.$$

With these substitutions the lowest possible value for the average planetary surface temperature for Mars turns out to be 219K.

The problem of estimating the maximum possible value of the average Martian surface temperature is more complex. We wish to maximize the green-house effect, within reason, in order to obtain a realistic estimate of the maximum surface temperature. The green-house effect depends essentially upon the amount and vertical distribution of absorbing gases, and on the vertical distribution of temperature. The greater the amount of absorbing gases—the more opaque the atmosphere—the greater will be the green-house effect, and thus the higher will be the surface temperature. In our model (discussed later) we attempt to make realistic estimates of the maximum amounts of carbon dioxide, water vapor and ozone in the Martian atmos-

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here, and their distribution with height. Once the maximum amount of absorbing gas is fixed we have only to choose a realistic temperature distribution with height, which would maximize the surface temperature. For simplicity, and for lack of knowledge, we shall assume that the temperature variation with height is linear. This is probably a good approximation in the troposphere of Mars, as it is for the troposphere of the earth, and it is the troposphere rather than the upper atmosphere that contributes most to the radiation fluxes. The question we must now answer is, "What choice of constant lapse rate will produce a maximum green-house effect and thus a maximum surface temperature?" We can answer this question by going through the following qualitative reasoning.

The upward flux of long wave radiation at the top of the Martian atmosphere must balance the incoming solar radiation that is not reflected back to space. This can be written as

$$I_0(1-A) = W, \quad (2)$$

where I_0 is the average incoming flux of radiation at the top of the Martian atmosphere, and W is the upward flux of long wave radiation at the top of the atmosphere. The upward flux W is composed of radiation coming directly from the planet's surface and of radiation emanating from the planet's atmosphere. If the atmosphere is isothermal, the radiation originating in the atmosphere will be at the same temperature as the radiation originating from the surface. Thus, the upward flux will remain constant with height and have a value equal to the black body flux emitted by the surface. The surface temperature of an isothermal atmosphere would then be the same as that computed from (1), i.e., 219K. If the temperature decreases with height, however, the radiation emitted by the atmosphere would be at a lower temperature than that emitted by the ground. In order that W now balance the incoming radiation, the surface temperature must be higher than 219K. The greater the lapse rate the greater is the surface temperature in order for balance to occur at the top of the atmosphere. In a planetary atmosphere the limiting lapse rate is the adiabatic lapse rate and we choose this as the lapse rate that will maximize the green-house effect and hence the surface temperature.

With a given distribution of absorbers and temperature lapse rate, the outgoing radiation, W , is a function only of the surface temperature, and can be written as (Elsasser, 1960)

$$W = \sigma T_s^4 - \int_{T_t}^{T_s} R(\text{CO}_2) dT - \int_{T_t}^{T_s} R(\text{H}_2\text{O}) dT - \int_{T_t}^{T_s} R(\text{O}_3) dT. \quad (3)$$

The function R in this expression stands for

$$R = \int_{\nu_1}^{\nu_2} \frac{dB_\nu}{dT} (1 - \tau_f) d\nu, \quad (4)$$

T_s is the surface temperature, T_t is the temperature at the top of the atmosphere, B is the black body flux, τ_f is the flux transmissivity, and ν is frequency.

R is a function of path length, u , temperature, and the particular gas in question. Tables of R for carbon dioxide, water vapor and ozone have been presented by Elsasser (1960). R for carbon dioxide covers the 15- μ band, R for H_2O covers the 6.3- μ band, the window region, and the rotational band, and R for ozone covers the 9.6- μ band. The integrals in (3) can be evaluated numerically, given the temperature and absorber distribution in the atmosphere. It should be noted that both water vapor and carbon dioxide absorb radiation in the 15- μ band. However, since we are attempting to maximize the greenhouse effect we shall treat these absorptions independently and make no correction for the overlap.

Equation (2) can now be written as:

$$I_0(1-A) + \int_{T_t}^{T_s} R(\text{CO}_2) dT + \int_{T_t}^{T_s} R(\text{H}_2\text{O}) dT + \int_{T_t}^{T_s} R(\text{O}_3) dT = \sigma T_s^4. \quad (5)$$

To solve this equation for the surface temperature, T_s , one can assume a value for T_s and perform the integrations on the left side of the equation. Upon adding the incoming radiation to the evaluated integrals, we can compute a second approximation of T_s from (5). This new value is then used to recalculate the integrals and obtain a third approximation, and so on, until a value of T_s is obtained that balances (5). The surface temperature obtained from this model represents an estimate of the maximum possible value of the average surface temperature on Mars.

3. Green-house model and computational techniques

The green-house model that we shall adopt is based upon reasonable estimates of the maximum amounts of carbon dioxide, water vapor and ozone in the atmosphere of Mars. Of these three constituents, carbon dioxide is the only one that has definitely been detected in the Martian atmosphere. Grandjean and Goody (1955), in a re-analysis of Kuiper's (1952) near infrared measurements, derive a carbon dioxide content of about 2 per cent for the commonly accepted surface pressure of 85 mb. We shall simply double this value for our estimate of maximum carbon dioxide content. We shall further assume that all three gases are uniformly mixed

with height so that the path lengths are a function only of pressure, and that a linear pressure correction can be applied directly to the path lengths as suggested by Elsasser (1960). With these assumptions and a carbon dioxide content of 4 per cent the carbon dioxide path length for any pressure thickness is

$$\Delta u_{\text{CO}_2} = 86.8 \left(\frac{\bar{p}}{1000} \right) \Delta p, \quad (6)$$

where Δu_{CO_2} is the pressure corrected path length (cm NTP) in a layer with mean pressure \bar{p} mb and thickness Δp mb.

Although water vapor has not been detected spectroscopically in the Martian atmosphere, the nature of the polar caps (hoar frost) indicates that there must be some water vapor present. Dunham (1952), on the basis of spectroscopic observations, derived an upper limit of 0.0015 times the terrestrial amount over Mt. Wilson on an average clear night. This is of the order of 10^{-3} cm of precipitable water with the terrestrial amount of about 0.7 cm prevailing during Dunham's observations. DeVaucouleurs (1954) states that the amount of precipitable water is very likely much less than 4×10^{-2} cm and probably less than 1×10^{-2} cm. We take as our estimate of the maximum water vapor content the value 10^{-2} cm of precipitable water. Assuming a constant mixing ratio with height, the pressure corrected water vapor path length (precipitable cm) for any pressure thickness can be written as

$$\Delta u_{\text{H}_2\text{O}} = 1.18 \times 10^{-4} \left(\frac{\bar{p}}{1000} \right) \Delta p. \quad (7)$$

Ozone also has not been detected in the Martian atmosphere. An upper limit of 0.05 cm has been given by Kuiper (1952), who based his estimate on the failure of attempts to detect ozone by means of absorption spectra. Another estimate of the maximum amount of ozone has been derived by Marmo and Warneck (see Appendix), who computed the total amount of photochemically produced ozone for an atmosphere that had a maximum possible amount of oxygen. The maximum amount is about 2.5 m (Dunham, 1952) and for this oxygen content Marmo and Warneck computed a value of about 0.15 cm for the total amount of ozone. Using this latter value as our estimate of the maximum amount of ozone on Mars we can express the pressure corrected ozone path lengths (cm NTP) as

$$\Delta u_{\text{O}_3} = 1.8 \times 10^{-3} \left(\frac{\bar{p}}{1000} \right) \Delta p. \quad (8)$$

As discussed previously, the adiabatic lapse rate (approximately 3.7 deg km^{-1}) is to be used in the radiation computations. The surface pressure is taken

as 85 mb. In order to evaluate numerically the integrals appearing in (5), the atmosphere is divided into nine layers of thickness 10 mb, except for the topmost layer, whose thickness is 5 mb. For example the integral for carbon dioxide is written as

$$\sum_{i=1}^9 R_i (\log \bar{u}_i, \bar{T}_i) \Delta T_i,$$

where R_i is the R value of the i^{th} layer, which is a function of \bar{u}_i , the pressure-corrected carbon dioxide path length from top of the atmosphere to the middle of the layer, and \bar{T}_i , the average temperature of the i^{th} layer, and ΔT_i is the temperature difference between the bottom and top of the i^{th} layer.

Mean temperatures and path lengths for each layer are used to obtain values of the R function from Elsasser's (1960) tables, and the computation is similar in all respects to that suggested by Elsasser except that numerical rather than graphical integration is used. In our model the temperature at the top of the atmosphere is zero K and the temperatures in the upper part of the atmosphere are below 193K. Since Elsasser's tables of R cover only the range $193 \leq T \leq 313$, they were extrapolated from 193K to a temperature of zero K, for selected path lengths, in our computations. The extrapolation to zero degrees K was performed with the knowledge that R at zero degrees is equal to zero [See equation (4).] As a check on the extrapolation, integrals of the form $\int_0^{193} R dT$ at constant path length were evaluated graphically and compared to similar integrals given by Elsasser. Although there still may be some error in the values of R used at temperatures less than 193K, these will not significantly affect the computations since the contributions of the upper layers to the outgoing radiation are much less than the contributions from the lower layers. Graphs showing the extrapolated R curves are contained in Figs. 1, 2 and 3.

4. Results

For a solar constant of $2.0 \text{ cal cm}^{-2} \text{ min}^{-1}$ and a dilution factor of 0.44, the average incoming radiation at the top of the Martian atmosphere, I_0 , is $317 \text{ cal cm}^{-2} \text{ day}^{-1}$. With the commonly accepted value of 0.15 for the Martian planetary albedo the average amount of solar radiation remaining after reflection is

$$I_0(1-A) = 269 \text{ cal cm}^{-2} \text{ day}^{-1}.$$

Using this value and the maximum green-house model discussed above, we obtain from (5) a value of T_s equal to 233K. This temperature is our estimate of the maximum possible value of the average surface temperature on Mars. Comparing 233K with our previous estimate of 219K for the minimum possible value, we see that even a maximum green-house effect would raise the surface temperature only about 14 deg. This maximum green-house effect can be compared to the average

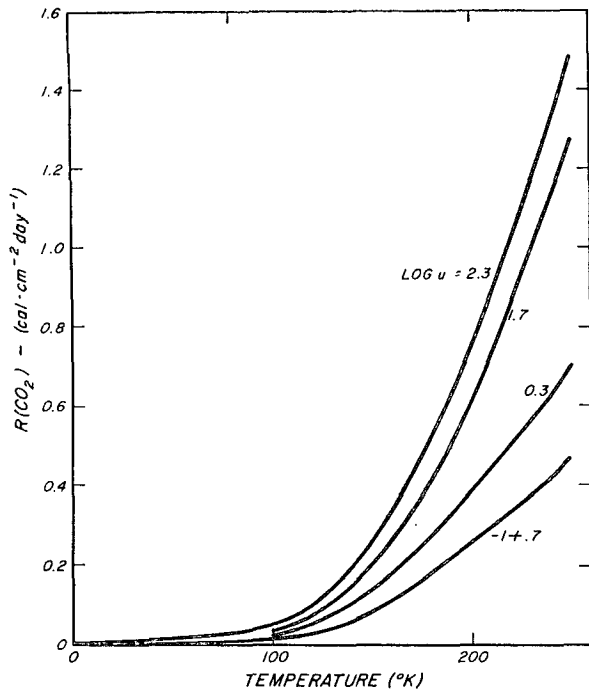


FIG. 1. Extrapolated values of $R(\text{CO}_2)$.

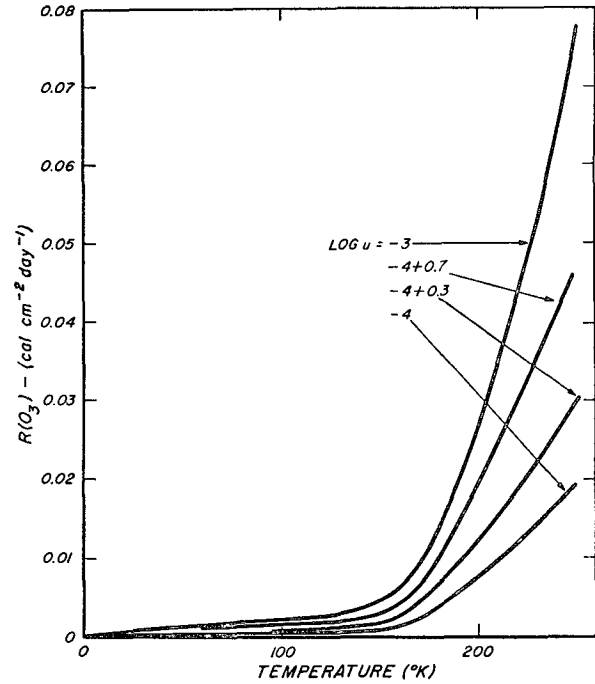


FIG. 3. Extrapolated values of $R(\text{O}_3)$.

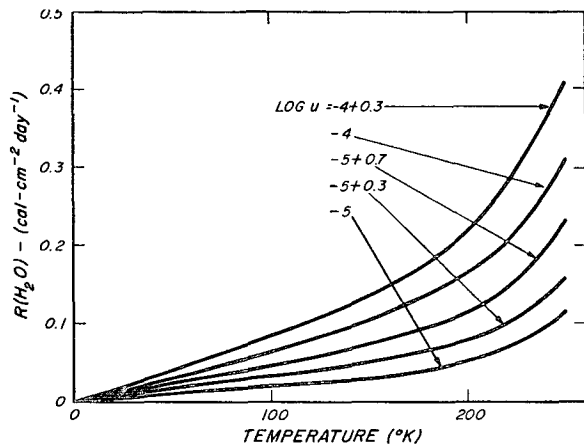


FIG. 2. Extrapolated values of $R(\text{H}_2\text{O})$.

can be seen from the following values of the integrals appearing in (5):

$$\int_{T_t}^{T_s} R(\text{CO}_2) dT = 57 \text{ cal cm}^{-2} \text{ day}^{-1},$$

$$\int_{T_t}^{T_s} R(\text{H}_2\text{O}) dT = 18 \text{ cal cm}^{-2} \text{ day}^{-1},$$

$$\int_{T_t}^{T_s} R(\text{O}_3) dT = 4 \text{ cal cm}^{-2} \text{ day}^{-1}.$$

This is in contrast to the earth's atmosphere where water vapor is the most important absorber of long-wave radiation.

Another measure of the effectiveness of the greenhouse is the ratio of outgoing long-wave radiation at the top of the atmosphere to the long-wave radiation emitted by the surface, which may be termed the infrared transparency of the atmosphere. The lower the infrared transparency, the more effective is the greenhouse. For average conditions on earth the infrared transparency (based on radiation fluxes computed by London, 1957) is about 57 per cent; for our maximum greenhouse on Mars, the infrared transparency is 77 per cent.

Based upon our theoretical estimates the true average surface temperature of Mars should lie between 219K and 233K. In order to fix the average temperature more accurately, it would be necessary to know the actual

green-house effect for the earth. The temperature of the earth's surface, if it had an atmosphere transparent to long wave radiation, can also be computed from (1). With an albedo of 0.35, $\mu = 1$ and a solar constant of $2.0 \text{ cal cm}^{-2} \text{ min}^{-1}$, (1) yields a temperature of 252K for the earth. The observed average temperature of the earth is about 288K, indicating that the average green-house effect for the earth is about 36 deg, or about $2\frac{1}{2}$ times as large as the maximum green-house effect for Mars.

Of the three gases contributing to the green-house effect, carbon dioxide is by far the most important as

amounts and distribution of absorbing gases, and the actual vertical temperature distribution. A sample calculation indicated that the use of the estimated average carbon dioxide concentration—2 per cent by volume—rather than a maximum concentration—4 per cent by volume—did not affect the outgoing radiation too much. Also the vertical temperature distribution we used cannot be too far from the actual temperature distribution. We therefore suggest that the actual mean temperature is closer to 233K than to 219K. In the next section we compare our estimate with some of the observations of Martian temperatures.

5. Comparison with observations

The observations of surface temperature on Mars are generally based upon measurements of the planet's thermal emission; these measurements are usually made during oppositions and therefore refer largely to the sunlit side of the planet. Gifford (1956) has analyzed systematically large numbers of such observations and has derived average latitudinal surface temperature distributions on the noon meridian for each of the four seasons for the latitude range 60N to 80S. On the basis of continuity and solar insolation considerations we have extrapolated these curves to the poles; they are shown in Fig. 4, with extrapolations indicated by dashed lines. In order to compute the average planetary surface temperature from these observations we must perform the following operations:

- (1) Average the seasonal curves to obtain an average annual curve;
- (2) Subtract from these noon-time temperatures the amplitude of the average diurnal variation of temperature as a function of latitude in order to obtain a mean daily temperature;
- (3) Compute the average planetary surface temperature by weighting the mean temperature at each latitude belt according to the area of the latitude belt.

When the first of these operations is performed the average annual noon curve shown in Fig. 5 results. The correction of this curve for diurnal variation is important because such variations are apparently appreciable on Mars. There have been a number of measurements, mostly in tropical regions, of the variations of Martian surface temperature during the day. Gifford's (1956) analysis of 214 daytime surface temperatures near the Martian equator indicates a diurnal range of about 70 deg, which is equivalent to an amplitude of 35 deg. DeVaucouleurs (1954), in his book on Mars, suggests an average amplitude of 25 deg in the equatorial regions. In the recent observations of Sinton and Strong (1960), however, the indicated amplitude is about 45 deg or higher. Since the analyses of Gifford and DeVaucouleurs are based upon more data we shall rely on their estimates for the amplitude of the diurnal variation at the equator; we therefore

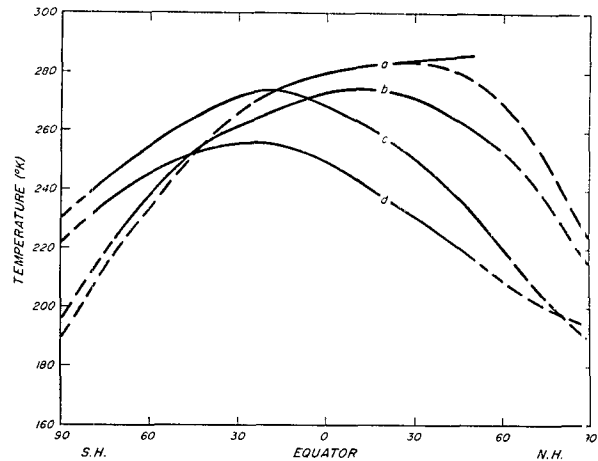


FIG. 4. Average surface temperature variation along the Martian noon meridian for southern hemisphere seasons (after Gifford, 1956) (a) winter, (b) fall, (c) summer, (d) spring.

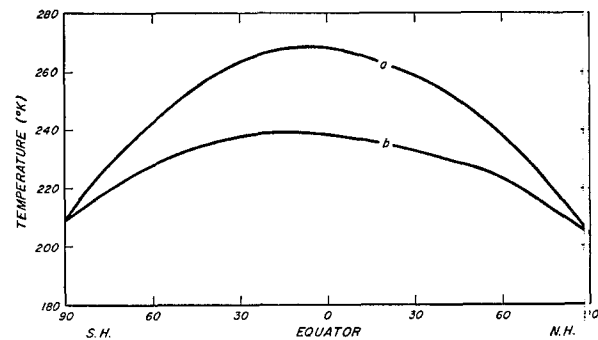


FIG. 5. Average annual surface temperature as a function of latitude (a) noon temperatures, (b) mean daily temperatures.

adopt an amplitude of 30 deg in the equatorial regions of Mars. Assuming that the diurnal variation depends only on insolation, we can approximate the latitudinal variation of the amplitude by a simple cosine function

$$a_{\varphi} = 30 \cos \varphi, \quad (9)$$

where a_{φ} is the amplitude at any latitude φ . When these amplitudes are subtracted from the noon curve, the average annual curve shown in Fig. 5 is obtained. The curve indicates an average equatorial temperature close to 240K and an average polar temperature of between 200K and 210K. The average planetary surface temperature can be obtained from a numerical integration of this curve from pole to pole, weighting the temperatures according to surface area as follows:

$$\bar{T} = \frac{1}{2} \int_{-\pi/2}^{\pi/2} \bar{T}(\varphi) \cos \varphi d\varphi. \quad (10)$$

\bar{T} is the average planetary surface temperature and $\bar{T}(\varphi)$ is the average surface temperature at latitude φ . Upon performing this integration we obtain a value of

233K for the average Martian surface temperature. Within the scope of the uncertainties involved in obtaining this value, it can be considered to be in reasonable agreement with the computed theoretical range of 219K to 233K.

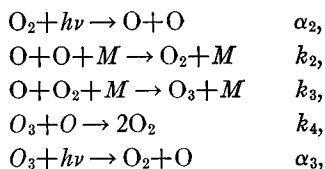
6. Summary

From theoretical computations based upon radiative equilibrium considerations, the average surface temperature of the planet Mars is found to be in the range of 219K to 233K.

This is in reasonable agreement with the thermal emission observations which suggest a mean temperature of about 233K. The computations also indicate that the *maximum* green-house effect on Mars is about 40 per cent of the *average* green-house effect in the earth's atmosphere, and that carbon dioxide is the most important contributor to the Martian greenhouse.

APPENDIX

The results of Marmo and Warneck² were obtained by computing the photochemical equilibrium ozone distribution for the following set of reactions:



where the k 's are rate constants for the collision processes, the α 's represent the total number of quanta per cc per sec which produce dissociation, h is Planck's constant, ν is frequency of incident radiation, and M is an arbitrary third body (primarily nitrogen for Mars).

Under conditions of equilibrium and with the oxygen only slightly dissociated, the equation for the ozone density can be obtained as a cubic equation which is solved by Newton's method of approximation:

$$\begin{aligned} n_{\text{O}_3}^3 + \left(\frac{\alpha_2}{\alpha_3} n_{\text{O}_2} - \frac{k_2}{k_4^2} \alpha_3 n_m - \frac{k_3}{k_4} n_m n_{\text{O}_2} \right) n_{\text{O}_3}^2 \\ - \left(2 \frac{\alpha_2 k_3}{\alpha_3 k_4} n_m n_{\text{O}_2}^2 \right) n_{\text{O}_3} + \frac{\alpha_2}{\alpha_3} \left(\frac{k_3}{k_4} \right)^2 n_m^2 n_{\text{O}_2}^3 = 0. \end{aligned}$$

The following physical model and constants were employed.

² A more detailed description is cited in the list of references.

Solar spectrum at top of Martian atmosphere: After Watanabe (1958), but corrected for Mars' greater distance from sun.

Wavelength region for O₂ dissociation: 1750 Å to 2400 Å.

Wavelength region for O₃ dissociation: 2000 Å to 3500 Å.

Absorption coefficient for O₂: After Craig (1950).

Absorption coefficients for O₃: After Inn and Tanaka (1953).

$k_2 = 2.7 \times 10^{-33}$ cm⁶ molecule⁻² sec⁻¹.

$k_3 = 5 \times 10^{-34}$ cm⁶ molecule⁻² sec⁻¹.

$k_4 = 6.8 \times 10^{-17}$ cm³ molecule⁻¹ sec⁻¹ (for $T = 220$ K, the temperature of the isothermal atmosphere used).

Scale height $H = 17.4$ km.

Computations were conducted for three different oxygen amounts: 250 cm NTP, 100 cm NTP, and 20 cm NTP. For an oxygen amount of 250 cm NTP, the total amount of ozone turns out to be about 0.15 cm NTP.

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