LETTERS TO THE EDITOR

USE OF GEOSTROPHIC WIND AND THE VORTICITY EQUATION IN CYCLONE STUDY

July 12, 1945

Dear Sir:

In meteorology we are satisfied if the theories help us to understand the atmospheric phenomena which combine to make the weather. With this modest goal, the restrictions of theoretical meteorology often constitute a first step which is both permissible and desirable, provided they do not affect the essential character of the problem. With this in mind, and also recognizing that I lack the knowledge and competence a criticism usually requires, it is with hesitancy that I question some of the assumptions that Professors Bjerknes and Holmboe made in their paper "On the Theory of Cyclones" that was published in the first issue of *The Journal of Meteorology*. However, due to the importance of the subject, I wish to submit the following comments:

In section 9 the authors use the geostrophic wind to show that the strength of the zonal circulation and the shape of the streamline pattern do not vary with height. Since only the nongeostrophic components of the wind produce divergence, it appears necessary to demonstrate that the streamlines of the nongeostrophic wind do not vary with height.

In section 12, the authors use the geostrophic wind to show that the isotherms, along which the relative wind blows, are in phase and have the same shape at all levels. In nongeostrophic winds this rule would not hold, and it is the nongeostrophic winds that produce the divergence that the authors are investigating.

In section 8, the authors use the equation $f + \zeta = \text{constant}$, derived by Rossby. Rossby (in Q. J. R. M. S. Supplement, Vol. 66, 1940) refers to $f + \zeta$ as the absolute vorticity. In the absence of depth variations, the equation $f + \zeta = \text{constant}$, he says, expresses the principle of the conservation of absolute vorticity. Figure 1 shows schematically the operation of the principle of the conservation of absolute vorticity. If the absolute vorticity remains constant, $f + \zeta \neq \text{constant}$ as the earth rotates or the fluid element moves about the surface of the earth. It would therefore appear that the conclusions given by Bjerknes and Holmboe in sections 6 and 8 are open to question because they agree with those derived by Rossby from a principle that does not work in the atmosphere. The agreement is probably due to the fact that each of the investigators assumed the earth to be flat. It appears that the assumption that the earth is flat may simplify the mathematics, but it affects the essential character of the problem.

In section 5 it is assumed that the shape of the isobaric pattern remains unchanged. Since the divergence is associated with the acceleration terms of the equation of motion, it appears that the assumption of constant shape of the isobars puts a restriction upon the acceleration of the fluid elements. It would appear, however, that this restriction is necessary if their equation (5.1) is to be satisfied strictly at the wedge and trough lines.

EDWARD V. ASHBURN
Washington, D. C.

[Since some of the comments made in the letter above relate to the vorticity principle, which I have myself used in various ways, I am taking the liberty to point out that the questions raised are in part due to inconsistencies in nomenclature in the literature. Thus when Dr. Rossby speaks about the absolute vorticity, he means simply the component about the vertical of the total absolute vorticity, and hence his statement regarding the conservation of absolute vorticity relates only to this component about the vertical. That this component (denoted by $f + \zeta$) must remain very nearly constant can be demonstrated without reference to a cartesian coordinate system as follows:

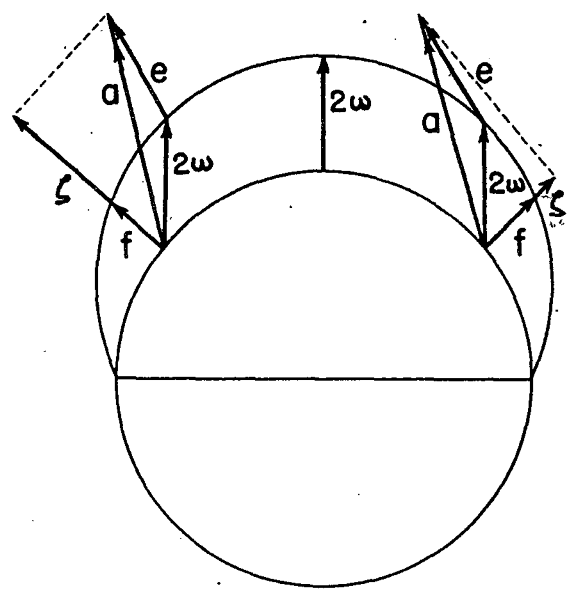

FIG. 1. Conservation of absolute vorticity.

Assuming that there is no friction, the Bjerknes circulation theorem can be written as

$$\frac{dC}{dt} = N - 2\Omega \frac{d\Sigma}{dt}, \qquad (1)$$

where C is the circulation around a moving closed chain of particles, N is the number of solenoids, Ω is the angular velocity of the earth's rotation about the polar axis, Σ is the area of the projection of this closed chain upon the equatorial plane and t is time. Let us restrict attention to a chain of particles which is at all times horizontal and is of very small size, so that the relative vorticity about a vertical (ζ) does not vary over its area. Also assuming that there are no solenoids enclosed by it, we can write equation (1) as

$$\frac{d}{dt}(C + 2\Omega\Sigma) = 0 \qquad (2)$$

or

$$C + 2\Omega\Sigma = \text{constant}. \qquad (3)$$

Since now $C = A\zeta$, where A is the actual area of the chain, and since from geometrical considerations $\Sigma = A \sin \phi$, where ϕ is the latitude, it follows that

$$A\zeta + 2\Omega A \sin \phi = \text{constant}. \qquad (4)$$

If there is no horizontal divergence so that A remains constant, we get that

$$\zeta + 2\Omega \sin \phi = \zeta + f = \text{constant}, \qquad (5)$$

which is the equation which Rossby has used. It is to be noted that this equation only relates to the component of vorticity about the vertical and does not imply that the total absolute vorticity is conserved.

In the atmosphere the requirement that the circuit should remain horizontal as it changes latitude can be fulfilled sufficiently well by choosing it so as to lie in a given isentropic surface. Under these circumstances, since the isentropic sheets remain practically horizontal and consist of essentially the same particles as time progresses, the effect of the stratification will be to keep the circuit practically horizontal. At the same time this choice of the circuit ensures that no solenoids are present, since the isolines of density and pressure drawn on an isentropic surface must coincide.—Ed.]