LETTERS TO THE EDITOR

DIVERGENCE OF THE GRADIENT WIND

November 2, 1945

Dear Sir:

In view of the widespread current interest in the mechanism of pressure changes, it is believed that the following note on the solenoid term in the divergence of the gradient wind might be of interest to readers of the *Journal*. These ideas occurred to me as a result of observations of the behavior of the "Advective Pressure Change Chart" which was introduced at the Weather Bureau Analysis Center by Mr. Bice. It was noted that advection of warm air below 20,000 feet was associated more closely with rising pressure at 20,000 feet than with falling pressure at the surface. It appeared probable that there was some mechanism whereby advection of warm air in the lower levels produces upward motion and rising pressure aloft. The search for such a mechanism led to an evaluation of the contribution to the surface pressure tendency produced by the horizontal divergence of the velocity field due to horizontal temperature gradients.

The expression for the horizontal divergence of the gradient wind velocity as given by Petterssen (*Weather Analysis and Forecasting*, page 228) contains the following term:

$$\frac{N}{\lambda \pm \dfrac{v_{gr}}{r}},$$

where N is the number of isobaric-isosteric solenoids per unit horizontal area. This term, which we shall denote by $(\mathrm{div}_2\, \boldsymbol{v})_N$, represents the part of the horizontal divergence due to temperature gradients and is called the solenoid term. The contribution of this term to the surface pressure tendency will be calculated. N may be represented by

$$N = (\nabla p \times \nabla \alpha)\cdot \boldsymbol{k},$$

where $\boldsymbol{k}$ is a vertical unit vector. This reduces to

$$N = \frac{\partial p}{\partial x}\frac{\partial \alpha}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial \alpha}{\partial x}.$$

When expressed in terms of density, this becomes

$$N = \frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x} - \frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right).$$

The solenoid term in the divergence may now be written

$$(\mathrm{div}_2\, \boldsymbol{v})_N = \frac{1}{\rho^2\left(\lambda \pm \dfrac{v}{r}\right)}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x} - \frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right).$$

Introduction of the gradient wind equation,

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = v_y\left(\lambda \pm \frac{v}{r}\right) \quad \text{and} \quad \frac{1}{\rho}\frac{\partial p}{\partial y} = -v_x\left(\lambda \pm \frac{v}{r}\right)$$

gives the following:

$$(\mathrm{div}_2\, \boldsymbol{v})_N = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial x}v_x + \frac{\partial \rho}{\partial y}v_y\right).$$

The pressure change at the surface due to this part of the divergence is given by

$$\left(\frac{\partial p}{\partial t}\right)_N = -\int_0^\infty g\rho(\mathrm{div}_2\, \boldsymbol{v})_N dz = \\ = +\int_0^\infty g\left(v_x\frac{\partial \rho}{\partial x} + v_y\frac{\partial \rho}{\partial y}\right)dz.$$

It will be noted that this is the negative of the advection term in the tendency equation developed by J. Bjerknes (reproduced on page 326 of Petterssen's *Weather Analysis and Forecasting*). This shows that, as far as the pressure variations at the ground are concerned, the solenoid term in the horizontal divergence of the gradient wind exactly cancels the advection term in the tendency equacion. In other words the fall in pressure at the ground which would naturally be expected as a result of advection of warm air is exactly counterbalanced by horizontal convergence in the velocity field.

The fact that falling pressure at the ground usually does accompany the advection of warmer air may be explained as follows: Advection of warm air produces convergence in the velocity field resulting in upward motion and rising pressure aloft. The divergence of the isallobaric wind component (as given by the theory of Brunt and Douglas), which is associated with these high level pressure changes, causes the surface pressure to fall. Similar reasoning applies to the increase of surface pressure associated with the advection of colder air.

It should be noted that in a recent paper by Bjerknes and Holmboe (*Journal of Meteorology*, Vol. 1, page 1), the tendency equation is expressed in the following form:

$$\left(\frac{\partial p}{\partial t}\right)_0 = -\int_0^\infty \mathrm{div}_2\,(\rho\boldsymbol{v})\delta\phi\,.$$

The divergence of the specific momentum, ρv, is used instead of the velocity since it represents mass flow. In this form the advection term does not appear in the tendency equation and there is no solenoid term in the divergence. The equation in this form must be interpreted with care, since divergence in the velocity field can exist and cause vertical motion and high level pressure changes even when *the divergence of the* mass flow is zero. These high level pressure changes may cause significant divergence in the mass flow aloft through the isallobaric wind component.

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