

SHORTER CONTRIBUTIONS

CONSTANT ABSOLUTE VORTICITY TRAJECTORY TABLES*

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(Manuscript Received November 29, 1945)

1. **Introduction.** For the past several years a relatively mechanical technique has been known and used for prognosticating trajectories of air parcels. This scheme, known as the constant absolute vorticity trajectory method, has been used primarily to predict the position and direction of a given air parcel at some fixed time in the future. This paper presents tabulations† of these two items for various trajectories and time intervals. This information can be of considerable aid in preparation of upper-air prognostic charts. It is believed that these tables will materially shorten the time necessary to obtain the desired data, thus increasing the usefulness of the method to the forecaster, who is generally quite pressed for time.

In any application of the results of constant absolute vorticity trajectories it is necessary to keep in mind the limitations that are imposed by the assumptions that put the mathematics in a tractable form. These assumptions are divisible into two major classes: Those arising from the vorticity theorem of Rossby (4) and those necessary to evolve the theory of the trajectories from the vorticity theorem (1).

2. Discussion of assumptions.

The assumptions leading to the vorticity theorem. The theorem states that $(f + \zeta)/D$ is a constant, where f is the Coriolis parameter, ζ is the vorticity relative to the earth, and D is the depth of the air column under consideration.

1. It is presumed that the equations of motion are expressible as

$$\frac{du}{dt} = fv - \frac{1}{\rho} \frac{\partial P}{\partial x},$$

$$\frac{dv}{dt} = -fu - \frac{1}{\rho} \frac{\partial P}{\partial y},$$

where u and v are the eastward and northward components of the horizontal velocity, t is time, ρ is the density, P is pressure, and x and y are horizontal axes pointing eastward and northward respectively.

These equations represent the motion of air which

* This paper was prepared under the auspices of the United States Army Air Forces.

† A set of tables similar to these but not so extensive have been prepared by Lieutenant Harry S. Hall and Technical Sergeant William G. Cook at the Weather Station at Andrews Field, Maryland.

is not subject to viscous, frictional and turbulent forces. It is difficult to estimate the effect of this particular simplification on the dimensions of a trajectory, but in general these stresses will be somewhat smaller than those due to the pressure gradient and the Coriolis effect so that neglect of these terms is probably justified (see assumption 3).

Another implication of these equations is that the motion takes place in a flat plane tangent to the earth's surface at the original point considered. The results of a trajectory computation, however, are universally applied to quasi-horizontal surfaces, that is, to curved surfaces that are roughly geopotential. It has been demonstrated by Haurwitz (2) that expressions for stationary wave lengths derivable from the vorticity theorem for flat planes are probably good approximations to the corresponding expressions for a spherical earth. This is to say that using the given form of the equations of motion has no appreciable effect upon the wave length, at least, of the trajectory.

2. The second major assumption is that the atmosphere is horizontally homogeneous, that is,

$$\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = 0.$$

This does not preclude the variation of density with height, so that we are effectively considering an atmosphere which is horizontally uniform but which possesses stable, vertical stratification. It will be recognized that this is a good description of the actual atmosphere.

3. It is further assumed that the motion is horizontal only, that is, no vertical motion occurs. The actual vertical velocities in the atmosphere are, except in such phenomena as thunderstorms, much smaller than the horizontal velocities, being some 1-2 cm/sec. Consequently this too is a good approximation to the real situation. It might be pointed out here that although, strictly speaking, this assumption requires motion along surfaces of fixed geopotential, the slope of isobaric surfaces is generally so small that no considerable error is introduced by applying this technique in constant pressure work.

The simplifications leading to the theory of constant absolute vorticity trajectories from the vorticity theorem.

1. The vorticity theorem is modified by presuming

that there are no depth changes or that

$$f + \zeta = \text{constant.}$$

This is to say that no divergence exists. This is perhaps one of the most restrictive assumptions made. The effect of divergence on the vorticity of an air column is similar to the effect of northward motion, that is, a decrease in vorticity is produced. Conversely, the effect of convergence is to increase the vorticity, or to produce the same effect as southward motion. Two possibilities can be recognized, therefore. If northward motions are combined with divergence (and southward motions with convergence), vorticity changes will take place faster than the theory predicts and the amplitudes of the actual trajectories will be smaller than those of the theoretical paths, since the vorticity changes will manifest themselves as changes in the curvature of the air motion. On the other hand, if northward motions occur with convergence (and southward motions with divergence), the two effects will tend to balance each other and the vorticity changes will be slower than the theory predicts. This may result in larger actual amplitudes. As an example of this latter effect, it is not uncommon for a strong southerly current to persist over a distance of a thousand miles without turning eastward, as the vorticity theorem demands, due merely to the change in latitude. This phenomenon is due to the existence of convergence in the air columns. We recognize, therefore, that sufficient divergence may actually exist in the atmosphere to cause serious deviations from theoretically predicted trajectories.

2. It is assumed that no pressure-gradient forces or other forces act parallel to the streamlines. This is to say that the magnitude of the velocity remains constant. It is not to be expected, consequently, that marked changes in flow patterns and velocities due to accelerations of parcels of air can be detected by means of constant absolute vorticity trajectories. Frictional stresses against the ground will tend to slow the parcel under consideration and cause decreased amplitude and wave length of the trajectory, as will horizontal diffusion of momentum. Accelerations will produce increased dimensions of the trajectory.

3. The curvature of the trajectory is assumed to be the same as the curvature of the streamlines. It is the former quantity which is desired, but it is the latter quantity which is measurable. Actually,

$$K_T = K_S + \frac{1}{c} \frac{\partial \psi}{\partial t},$$

where K_T is the curvature of the trajectory, K_S is the curvature of the streamline, c is the velocity, and $\partial \psi / \partial t$ the local rate of turning of the wind. We see, therefore, that we have assumed $\partial \psi / \partial t$ to be zero.

4. The initial point chosen for computation must be one without horizontal shear or curvature of trajectory (streamline), that is, an inflection point without shear. This amounts merely to a restriction of the number of available points that can be treated by this method. However, Professor J. C. Bellamy has prepared a series of graphs and slide rules that enable one to apply this technique to any point without horizontal shear, provided that the curvature is known. Only the results under the former condition are considered here.

5. It is assumed that all the vorticity generated relative to the surface of the earth manifests itself as curvature of the trajectory and none as horizontal shear. This has its justification in the empirical observation that marked shears in the atmosphere tend to be rather unstable and to break up into vortices. This is the same as saying that there is a pronounced tendency for vorticity to express itself primarily as curvature.

6. The approximation is made of substituting a quotient of finite differences for derivatives, for example

$$f_0 - f = -\beta y,$$

where f_0 is the Coriolis parameter at the initial point, f is the Coriolis parameter at any given point, β is the rate of change of f northward, and y is the distance northward from the original point. This approximation is more accurate the smaller the displacements north and south and less accurate the greater these displacements.

7. The quantity β is assumed to be constant, that is, not a function of y for a given trajectory. Actually β is a cosine function of latitude, being at a maximum at the equator and at a minimum at the pole. Some idea of its variability can be gained from the fact that β decreases approximately 29 per cent from 45°N to 60°N (3).

3. Construction of Tables. In all, three tables have been computed, i.e., for 24-, 36- and 48-hour movements. It has been assumed that all trajectories originated at the inflection points. The semigraphical method of computation has been used. The procedure of computation is given below:

1. Using the Slide Rule for Constant Vorticity Trajectories by J. C. Bellamy, a system of 192 trajectories have been computed and grouped in the following manner: for each initial latitude of 30°, 40°, 50°, and 60°, eight families of trajectories for eight initial inflection angles of 20°, 30°, 40°, 50°, 60°, 70°, 80°, and 90° have been computed with six trajectories in each family for six initial wind speeds of 10, 15, 20, 25, 30 and 35 meters per second.

2. The computed trajectories have been accurately

TABLE 1
24-Hour Positions and Wind Directions of Parcel of Air

Initial latitude, deg.	Initial wind speed, m.p.s.	Inflection angle, deg.							
		20	30	40	50	60	70	80	90
30	10	0902 1	0803 1	0704 2	0705 2	0606 3	0506 3	0407 4	0207 5
	15	1303 0	1204 0	1205 1	1107 1	1008 1	0809 2	0710 2	0510 3
	20	1703 0	1705 0	1606-1	1508-1	1309 0	1110 0	1011 0	0813 1
	25	2203-1	2105-1	2007-1	1908-2	1710-1	1511-1	1313-1	1114 0
	30	2603-1	2505-1	2407-2	2308-2	2110-2	1912-2	1713-2	1415-2
	35	3003-1	2905-2	2807-2	2708-3	2510-3	2312-3	2014-3	1716-3
32	10	0902 1	0803 1	0804 2	0705 2	0606 3	0506 3	0407 4	0207 5
	15	1303 0	1304 0	1205 1	1107 1	1008 1	0809 2	0710 2	0510 3
	20	1803 0	1705 0	1606-1	1508-1	1409 0	1211 0	1012 1	0813 1
	25	2203-1	2105-1	2007-1	1908-1	1810-1	1512-1	1313-1	1114 0
	30	2703-1	2605-1	2507-2	2308-2	2210-2	1912-2	1714-2	1415-2
	35	3103-1	3005-2	2907-2	2708-3	2510-3	2312-3	2114-3	1716-3
34	10	0902 1	0903 1	0804 2	0705 2	0606 3	0506 3	0407 4	0207 5
	15	1403 0	1304 0	1205 1	1107 1	1008 1	0809 2	0710 2	0510 3
	20	1803 0	1705 0	1606-1	1508 0	1409 0	1211 0	1012 1	0813 2
	25	2303-1	2205-1	2107-1	1909-1	1810-1	1612-1	1413-1	1115 0
	30	2703-1	2605-1	2507-2	2409-2	2211-2	2012-2	1714-2	1416-2
	35	3203-1	3105-2	3007-2	2809-3	2611-3	2412-3	2114-3	1816-3
36	10	1002 1	0903 1	0804 2	0705 2	0606 3	0506 3	0407 4	0208 5
	15	1403 0	1304 0	1205 1	1107 1	1008 1	0809 2	0710 3	0510 3
	20	1903 0	1805 0	1706 0	1608 0	1410 0	1211 0	1012 1	0813 2
	25	2304-1	2205-1	2107-1	2009-1	1810-1	1612-1	1413-1	1115 0
	30	2804-1	2705-1	2607-2	2509-2	2211-2	2012-2	1814-2	1516-2
	35	3303-1	3205-2	3007-2	2909-3	2711-3	2412-3	2214-3	1816-3
38	10	1002 1	0903 1	0804 2	0705 2	0606 3	0506 3	0407 4	0208 5
	15	1403 0	1404 0	1305 1	1207 1	1008 1	0809 2	0710 3	0510 3
	20	1903 0	1805 0	1706 0	1608 0	1410 0	1211 0	1012 1	0813 2
	25	2404-1	2305-1	2207-1	2109-1	1911-1	1612-1	1414 0	1115 0
	30	2904-1	2805-1	2607-2	2509-2	2311-2	2113-2	1814-1	1516-1
	35	3404-1	3305-2	3107-2	2909-2	2711-3	2513-3	2215-3	1817-2
40	10	1002 1	0903 1	0804 2	0805 2	0706 3	0506 4	0407 4	0208 5
	15	1503 0	1404 1	1305 1	1207 1	1108 1	0909 2	0710 3	0511 4
	20	2003 0	1905 0	1806 0	1608 0	1510 0	1211 1	1112 1	0813 2
	25	2504-1	2405-1	2207-1	2109-1	1911-1	1712-1	1414 0	1215 0
	30	3004-1	2905-1	2707-2	2609-2	2411-2	2113-2	1815-1	1516-1
	35	3404-1	3405-2	3207-2	3009-2	2811-3	2513-3	2215-2	1917-2
42	10	1002 1	1003 1	0904 2	0805 2	0706 3	0506 4	0407 4	0208 5
	15	1503 0	1404 1	1306 1	1206 1	1108 1	0909 2	0710 3	0511 4
	20	2003 0	1905 0	1807 0	1708 0	1510 0	1311 1	1112 1	0813 2
	25	2504-1	2406-1	2308-1	2209-1	2011-1	1713 0	1414 0	1215 1
	30	3104-1	3006-1	2808-1	2710-2	2411-2	2213-1	1915-1	1517-1
	35	3604-1	3506-2	3308-2	3110-2	2912-3	2614-3	2316-2	1918-2
44	10	1002 1	1003 1	0904 2	0805 2	0706 3	0507 4	0407 4	0208 5
	15	1503 0	1504 1	1406 1	1207 1	1108 2	0909 2	0710 3	0511 4
	20	2103 0	2005 0	1907 0	1708 0	1510 0	1311 1	1112 1	0813 2
	25	2604 0	2506-1	2408-1	2209-1	2011 0	1713 0	1514 0	1216 1
	30	3104-1	3006-1	2908-1	2710-1	2512-2	2214-1	1915-1	1517-1
	35	3704-1	3606-2	3408-2	3210-2	3012-2	2714-2	2316-2	2018-2

plotted on rectangular coordinate graph paper with both axes in degrees latitude.

3. The points for 24-, 36- and 48-hour movements have been located on every trajectory, and the final wind directions have been measured by means of a protractor.

4. The above results have been tabulated, resulting in three tables for 24-, 36- and 48-hour movements, showing in each table, for the above initial conditions,

the displacements due east in degrees longitude, the displacements due north or south in degrees latitude and final wind directions in degrees.

5. In order to obtain the same data for trajectories originated at every two degrees of initial latitude (from 30° to 60°), the values for displacements due east, displacements due north or south, and final wind directions, have been independently plotted against the initial latitudes. The desired values have then

TABLE 1 (cont.)

Initial latitude, deg.	Initial wind speed, m.p.s.	Inflection angle, deg.							
		20	30	40	50	60	70	80	90
46	10	1102 1	1003 2	0904 2	0805 2	0706 3	0507 4	0407 5	0208 5
	15	1603 0	1504 1	1406 1	1307 1	1108 2	0909 2	0710 3	0511 4
	20	2204 0	2105 0	2007 0	1809 1	1610 1	1312 1	1113 2	0814 2
	25	2704 0	2606 0	2408 0	2310 0	2111 0	1813 0	1514 0	1216 1
	30	3304-1	3206-1	3008-1	2810-1	2612-1	2314-1	2016-1	1617 0
	35	3804-1	3706-2	3508-2	3310-2	3112-2	2714-2	2416-2	2018-1
48	10	1102 1	1103 2	1004 2	0905 2	0706 3	0607 4	0407 5	0208 5
	15	1703 0	1604 1	1506 1	1307 1	1208 2	0909 3	0710 3	0511 4
	20	2204 0	2105 0	2007 0	1809 1	1610 1	1412 1	1113 2	0814 3
	25	2804 0	2706 0	2508 0	2410 0	2212 0	1913 0	1515 0	1216 1
	30	3404-1	3306-1	3109-1	2910-1	2612-1	2414-1	2016 0	1618 0
	35	3904-1	3806-1	3708-2	3411-2	3213-2	2815-2	2517-1	2019-1
50	10	1202 1	1103 2	1004 2	0905 2	0706 3	0607 4	0407 5	0208 6
	15	1703 1	1704 1	1506 1	1407 2	1208 2	1009 3	0710 3	0511 4
	20	2304 0	2205 0	2107 1	1909 1	1710 1	1412 1	1113 2	0814 3
	25	2904 0	2806 0	2608 0	2410 0	2212 0	1914 0	1615 1	1216 1
	30	3504-1	3406-1	3209-1	3011-1	2713-1	2415 0	2116 0	1618 0
	35	4104-1	4007-1	3809-1	3611-2	3313-2	2915-2	2517-1	2119-1
52	10	1202 1	1103 2	1004 2	0905 3	0806 3	0607 4	0407 5	0208 6
	15	1803 1	1704 1	1606 1	1407 2	1209 2	1010 3	0810 3	0511 4
	20	2404 0	2306 0	2207 1	2009 1	1811 1	1512 2	1213 2	0814 3
	25	3004 0	2906 0	2708 0	2510 0	2312 0	2014 1	1615 1	1216 2
	30	3705 0	3507-1	3309-1	3211-1	2813-1	2515 0	2117 0	1619 0
	35	4204-1	4107-1	4009-1	3711-1	3413-2	3016-1	2618-1	2120 0
54	10	1302 1	1203 2	1104 2	1005 3	0806 3	0607 4	0407 5	0208 6
	15	1903 1	1805 1	1706 1	1507 2	1309 2	1010 3	0810 4	0411 5
	20	2604 0	2406 1	2307 1	2109 1	1811 1	1512 2	1213 2	0814 3
	25	3204 0	3006 0	2908 0	2710 0	2412 0	2014 1	1616 1	1217 2
	30	3805 0	3707 0	3509-1	3311-1	3013-1	2616 0	2217 0	1719 1
	35	4405-1	4307-1	4109-1	3912-1	3614-1	3116-1	2618-1	2120 0
56	10	1302 1	1303 2	1104 2	1005 3	0806 4	0607 4	0407 5	0208 6
	15	2003 1	1905 1	1806 2	1607 2	1309 2	1110 3	0811 4	0411 5
	20	2704 0	2506 1	2407 1	2209 1	1911 2	1612 2	1213 3	0814 3
	25	3305 0	3207 0	3009 0	2811 0	2513 1	2115 1	1716 1	1217 2
	30	4005 0	3907 0	3710 0	3412 0	3114 0	2716 0	2218 0	1719 1
	35	4605-1	4507-1	4310-1	4112-1	3715-1	3317-1	2719-1	2221 0
58	10	1402 1	1303 2	1204 2	1105 3	0906 4	0707 5	0407 5	0208 6
	15	2103 1	2005 1	1906 2	1707 2	1409 3	1110 3	0811 4	0411 5
	20	2804 0	2706 1	2508 1	2309 2	2011 2	1613 2	1214 3	0814 4
	25	3505 0	3407 0	3209 0	2911 1	2613 1	2216 1	1716 2	1217 2
	30	4205 0	4107 0	3910 0	3612 0	3214 0	2816 1	2318 1	1720 1
	35	4905-1	4808-1	4510-1	4313-1	3915-1	3417 0	2819 0	2221 0
60	10	1502 1	1403 2	1304 2	1105 3	0906 4	0707 5	0407 5	0208 6
	15	2203 1	2105 1	2006 2	1807 2	1405 3	1110 3	0811 4	0411 5
	20	3004 0	2806 1	2708 1	2409 2	2111 2	1713 2	1214 3	0715 4
	25	3705 0	3607 0	3409 1	3111 1	2713 1	2316 1	1716 2	1217 3
	30	4505 0	4307 0	4110 0	3812 0	3415 0	2917 1	2319 1	1820 2
	35	5305 0	5108-1	4811-1	4513 0	4116 0	3518 0	2820 0	2222 1

been obtained by means of graphical interpolation, and the results have been tabulated into the final form of Tables 1, 2 and 3.

4. Directions for use of constant absolute vorticity trajectory tables.

Definition of symbols.

- ϕ_0 initial latitude: latitude of the inflection point (in degrees).
 ϕ_i inflection angle: angle between the initial wind

vector and eastward vector (in degrees). ϕ_i is positive for all winds having a southerly component and is negative for all winds having a northerly component.

- v initial wind speed (in meters per second).
 xx displacement of the parcel due east (in whole degrees longitude).
 yy displacement of the parcel due north, when y is positive, due south when y is negative (in whole degrees latitude).

TABLE 2
36-Hour Positions and Wind Directions of Parcel of Air

Initial latitude, deg.	Initial wind speed, m.p.s.	Inflection angle, deg.							
		20	30	40	50	60	70	80	90
30	10	1302 0	1303-1	1204-1	1105-1	1006-1	0907-1	0808-1	0609 0
	15	2002-1	1903-2	1804-2	1706-2	1607-3	1508-3	1309-3	1110-3
	20	2602-2	2503-2	2404-3	2305-4	2206-4	2007-5	1809-5	1610-5
	25	3301-2	3102-3	3002-4	2803-4	2604-6	2406-6	2207-7	1908-7
	30	3900-2	3800-3	3601-4	3401-5	3102-6	2803-7	2504-8	2206-9
	35	45-1-2	44-1-3	41-1-4	38-1-5	3500-6	3101-7	2702-8	2304-9
32	10	1302 0	1303-1	1204-1	1105-1	1006-1	0907-1	0808 0	0609 0
	15	2002-1	1903-2	1804-2	1706-2	1607-3	1508-3	1309-3	1110-3
	20	2702-2	2603-2	2504-3	2405-4	2206-4	2007-5	1809-5	1610-5
	25	3301-2	3202-3	3103-4	2903-4	2704-6	2506-6	2207-7	1909-7
	30	4000-2	3800-3	3601-4	3401-5	3102-6	2903-7	2605-8	2207-9
	35	46-1-2	45-1-3	42-1-4	39-1-5	3500-6	3201-7	2802-8	2404-9
34	10	1402 0	1303-1	1204-1	1205-1	1106-1	1008-1	0809 0	0610 0
	15	2002-1	2004-2	1905-2	1806-2	1607-2	1508-3	1309-3	1111-3
	20	2702-2	2603-2	2504-3	2405-4	2206-4	2008-4	1809-5	1610-5
	25	3401-2	3302-3	3203-4	3004-4	2805-5	2506-6	2308-7	2009-7
	30	4100-2	3901-3	3701-4	3502-5	3203-6	2904-7	2605-8	2307-8
	35	47-1-2	46-1-3	4300-4	4000-5	3600-6	3301-7	2903-8	2505-9
36	10	1402 0	1403-1	1304-1	1206-1	1107-1	1008 0	0809 0	0610 1
	15	2102-1	2004-2	1905-2	1806-2	1707-2	1509-3	1310-3	1111-3
	20	2802-2	2703-2	2604-3	2505-4	2307-4	2108-4	1910-4	1711-5
	25	3501-2	3402-3	3203-4	3004-4	2805-5	2607-6	2308-6	2010-6
	30	4200-2	4001-3	3801-4	3602-5	3303-6	3004-7	2706-8	2408-8
	35	49-1-2	4700-3	4400-4	4100-5	3801-6	3402-7	3003-8	2605-9
38	10	1402 0	1403-1	1304-1	1206-1	1107-1	1008 0	0809 0	0610 1
	15	2203-1	2104-1	2005-2	1906-2	1707-2	1609-2	1410-2	1211-2
	20	2902-1	2803-2	2705-3	2506-3	2407-4	2209-4	1910-4	1711-4
	25	3601-2	3502-3	3303-4	3104-4	2905-5	2707-6	2408-6	2110-6
	30	4300-2	4101-3	3902-4	3702-5	3403-6	3105-7	2806-8	2508-8
	35	5000-2	4800-3	4500-4	4200-5	3901-6	3502-7	3104-8	2706-9
40	10	1502 0	1403-1	1404 0	1306 0	1107 0	1008 0	0809 0	0610 1
	15	2303-1	2104-1	2005-2	1906-2	1808-2	1609-2	1410-2	1211-2
	20	3002-1	2903-2	2705-3	2606-3	2407-4	2209-4	2010-4	1712-4
	25	3701-2	3603-3	3403-3	3205-4	3006-5	2707-5	2509-6	2211-6
	30	4501-2	4301-3	4102-4	3803-5	3504-6	3205-7	2907-7	2609-8
	35	5200-2	5000-3	4701-4	4401-5	4002-6	3603-7	3205-8	2806-9
42	10	1502 0	1503 0	1405 0	1306 0	1207 0	1008 0	0809 1	0610 1
	15	2303-1	2204-1	2105-2	2007-2	1808-2	1609-2	1410-2	1212-2
	20	3002-1	2904-2	2805-3	2706-3	2508-3	2309-4	2011-4	1812-4
	25	3802-2	3703-3	3504-3	3305-4	3106-5	2808-5	2609-6	2211-6
	30	4601-2	4402-3	4202-4	3903-5	3604-6	3306-7	3008-7	2609-7
	35	5300-2	5100-3	4801-4	4501-5	4102-6	3803-7	3405-8	3007-9
44	10	1602 0	1504 0	1405 0	1306 0	1207 0	1008 0	0909 1	0710 2
	15	2403-1	2304-1	2205-2	2007-2	1908-2	1710-2	1511-2	1212-2
	20	3103-1	3004-2	2905-3	2806-3	2608-3	2410-3	2111-4	1813-4
	25	4002-2	3803-3	3604-3	3405-4	3207-5	2908-5	2610-5	2312-5
	30	4701-2	4602-3	4303-4	4104-5	3805-6	3506-6	3208-7	2810-7
	35	5500-2	5301-3	5001-4	4702-5	4303-6	3904-7	3506-8	3108-9

d final wind direction, given as an angle between the wind vector and the eastward vector (in tens of degrees).

Description of tables. Each table is comprised of 8 vertical columns for inflection angles, ranging from 20° to 90° inclusive, and 16 horizontal groups for initial latitudes, ranging from 30° to 60° inclusive. Each latitude group is divided into 6 lines for wind speeds, ranging from 10 to 35 m.p.s. inclusive.

For any combination of the three initial conditions ϕ_0 , ϕ_i , and v , the table provides the three values xx , yy , and d . These are given by a six-space group, represented by $xxyysd$. xx is always positive. yy and d may be positive or negative. When d is negative a minus sign is entered in position s . For positive d , s is left blank. When yy is negative, a minus sign is entered in the first y position. When negative yy is in excess of -9 , -10 is entered as $-T$, and -11 is entered

TABLE 2 (cont.)

Initial latitude, deg.	Initial wind speed, m.p.s.	Inflection angle, deg.							
		20	30	40	50	60	70	80	90
46	10	1603 0	1604 0	1505 0	1406 0	1207 0	1108 1	0909 1	0710 2
	15	2503-1	2404-1	2206-1	2107-2	1908-2	1710-2	1511-2	1212-2
	20	3303-1	3204-2	3006-2	2907-3	2708-3	2410-3	2112-3	1913-3
	25	4102-2	4003-2	3805-3	3606-4	3307-4	3009-5	2711-5	2412-5
	30	4901-2	4702-3	4503-4	4204-5	3905-6	3607-6	3309-7	2911-7
35	5701-2	5501-3	5202-4	4902-5	4503-6	4105-7	3607-8	3209-8	
48	10	1703 0	1604 0	1505 0	1406 0	1307 0	1108 1	0909 1	0710 2
	15	2603-1	2504-1	2306-1	2207-2	2009-1	1810-1	1511-1	1212-1
	20	3403-1	3304-2	3106-2	3007-3	2809-3	2510-3	2212-3	1913-3
	25	4202-2	4104-2	3905-3	3706-4	3508-4	3209-5	2911-5	2513-5
	30	5102-2	4903-3	4704-4	4405-5	4106-5	3708-6	3410-7	3012-6
35	5901-2	5701-3	5402-4	5103-5	4704-6	4206-7	3808-7	3410-8	
50	10	1803 0	1704 0	1605 0	1506 0	1307 0	1108 1	0909 2	0710 2
	15	2703-1	2605-1	2406-1	2307-2	2109-1	1910-1	1611-1	1313-1
	20	3503-1	3404-2	3306-2	3107-3	2909-3	2611-3	2312-3	2014-3
	25	4502-2	4304-2	4105-3	3907-4	3608-4	3310-4	3012-4	2613-4
	30	5302-2	5103-3	4904-4	4605-4	4307-5	3908-6	3610-6	3112-6
35	6101-2	5902-3	5603-4	5303-5	4905-6	4506-6	4008-7	3611-7	
52	10	1803 0	1804 0	1705 0	1506 0	1407 0	1209 1	0909 2	0711 3
	15	2803-1	2705-1	2506-1	2408-1	2209-1	1911-1	1612-1	1313-1
	20	3703-1	3605-2	3406-2	3208-2	3009-3	2711-3	2413-3	2014-3
	25	4603-2	4404-2	4206-3	4007-3	3709-4	3411-4	3112-4	2714-4
	30	5602-2	5403-3	5105-4	4806-4	4507-5	4109-6	3711-6	3313-6
35	6401-2	6202-3	5903-4	5604-5	5106-5	4707-6	4209-7	3712-7	
54	10	1903 0	1904 0	1805 0	1607 0	1408 1	1209 1	1010 2	0711 3
	15	2903-1	2805-1	2706-1	2508-1	2310-1	2011-1	1712-1	1313 0
	20	3803-1	3705-2	3507-2	3408-2	3110-2	2812-2	2513-2	2015-2
	25	4803-1	4605-2	4406-3	4208-3	3909-4	3611-4	3213-4	2815-4
	30	5802-2	5604-3	5305-3	5006-4	4708-5	4310-5	3912-6	3414-5
35	6702-2	6503-3	6204-4	5805-5	5407-5	4908-6	4410-7	3913-7	
56	10	2003 0	2004 0	1805 0	1707 1	1508 1	1209 2	1010 2	0711 3
	15	3103 0	2905-1	2807-1	2608-1	2410-1	2111-1	1712 0	1314 0
	20	4003-1	3905-1	3707-2	3509-2	3310-2	3012-2	2614-2	2115-2
	25	5103-1	4905-2	4607-3	4408-3	4110-3	3812-4	3414-3	2916-3
	30	6103-2	5904-2	5606-3	5307-4	4909-5	4511-5	4113-5	3615-5
35	7102-2	6803-3	6504-4	6206-4	5708-5	5209-6	4712-6	4214-6	
58	10	2103 0	2104 0	2006 1	1807 1	1508 1	1309 2	1010 3	0711 3
	15	3304 0	3105-1	2907-1	2708-1	2510 0	2212 0	1813 0	1414 1
	20	4304-1	4105-1	3907-2	3709-2	3511-2	3113-2	2715-2	2216-1
	25	5403-1	5105-2	4907-2	4709-3	4311-3	4013-3	3515-3	3017-3
	30	6503-2	6205-2	5906-3	5608-4	5210-4	4812-5	4314-5	3816-4
35	7502-2	7204-3	6905-3	6507-4	6009-5	5511-6	5013-6	4416-6	
60	10	2303 0	2204 0	2106 1	1907 1	1608 1	1309 2	1010 3	0711 4
	15	3404 0	3305 0	3107-1	2909-1	2610 0	2212 0	1913 0	1414 1
	20	4604-1	4406-1	4208-1	4009-2	3711-2	3313-1	2815-1	2216-1
	25	5704-1	5506-2	5307-2	5009-3	4611-3	4213-3	3716-3	3118-3
	30	6803-2	6605-2	6307-3	6009-4	5611-4	5113-4	4515-4	4018-4
35	7903-2	7704-2	7406-3	7008-4	6510-5	6012-5	5314-5	4717-5	

as -E. Since no negative values of yy in excess of -11 are encountered, no difficulty in interpretation arises. A few values for xx in excess of 99 are entered as T0, T1, and T2 for 100, 101, and 102, respectively.

When ϕ_i is positive, the signs for values of yy and d should be used as indicated in the table. When, however, ϕ_i is negative, the signs for values of yy and d should be reversed.

Examples.

1. Given: $\phi_i = 40^\circ$, $\phi_0 = 35^\circ\text{N}$, $v = 25$ m.p.s.

Find: xx , yy and d after 24 hours.

Using Table 1, in the 40° vertical column and 35° horizontal group, locate the 25 m.p.s. line, which is the fourth line in the group. The six-space group 2107-1 means $xx = 21^\circ$ longitude due east, $yy = 7^\circ$

TABLE 3
48-Hour Positions and Wind Directions of Parcel of Air

Initial latitude, deg.	Initial wind speed, m.p.s.	Inflection angle, deg.							
		20	30	40	50	60	70	80	90
30	10	1802-1	1702-2	1603-3	1504-3	1405-4	1306-4	1207-4	1008-4
	15	2601-2	2501-3	2401-4	2302-5	2103-6	1904-7	1705-7	1506-8
	20	35-1-2	33-1-3	31-1-4	29-1-5	2700-6	2301-7	2101-8	1802-9
	25	44-2-2	41-3-2	39-3-3	35-4-4	32-4-5	28-4-7	24-3-8	2001-9
	30	52-3-1	50-4-2	47-6-2	43-7-3	38-8-4	33-8-5	27-7-7	22-5-9
	35	61-4-1	58-6-1	55-8-1	51-9-2	45-T-3	39-E-4	32-E-5	24-T-7
32	10	1802-1	1702-2	1703-3	1604-3	1505-4	1306-4	1207-4	1008-4
	15	2701-2	2601-3	2402-4	2302-5	2103-6	1904-6	1805-7	1506-7
	20	36-1-2	34-1-3	32-1-4	30-1-5	2700-6	2401-7	2102-8	1803-9
	25	45-2-2	42-3-2	40-3-3	36-4-4	33-4-6	29-3-7	25-2-8	2101-9
	30	53-3-1	51-4-2	48-6-2	44-7-3	39-7-5	34-8-5	28-7-7	22-5-9
	35	62-4-1	59-6-1	56-7-1	52-9-2	46-T-3	39-E-4	33-E-5	25-T-7
34	10	1902-1	1802-2	1703-3	1604-3	1505-3	1406-4	1207-4	1008-4
	15	2801-2	2601-3	2502-4	2402-5	2203-6	2004-6	1805-7	1606-7
	20	36-1-2	35-1-3	33-1-4	31-1-5	2800-6	2501-7	2202-8	1903-9
	25	46-2-2	43-3-3	41-3-3	37-4-4	34-4-6	30-3-7	25-2-8	2100-9
	30	55-3-1	52-4-2	49-6-3	45-7-3	40-7-5	35-7-6	29-6-8	23-4-9
	35	63-4-1	61-6-1	57-7-2	53-9-2	47-T-3	40-E-5	33-T-6	25-9-8
36	10	1902-1	1803-2	1703-2	1604-3	1505-3	1406-4	1207-4	1108-4
	15	2801-2	2701-3	2602-4	2403-5	2303-6	2004-6	1905-7	1607-7
	20	3700-2	36-1-3	3400-4	3200-5	2900-6	2602-7	2303-8	2004-9
	25	47-2-2	44-2-3	42-3-4	38-3-5	35-3-6	31-3-7	26-1-8	2200-9
	30	56-3-1	53-4-2	50-5-3	46-6-3	41-7-5	36-7-6	30-6-8	24-4-9
	35	65-4-1	62-6-1	58-7-2	54-9-3	48-T-3	41-T-5	34-T-6	26-9-8
38	10	2002-1	1903-2	1803-2	1704-3	1605-3	1406-3	1307-4	1109-4
	15	2901-2	2801-3	2602-4	2503-5	2304-6	2105-6	1906-7	1707-7
	20	3800-2	3700-3	3500-4	3300-5	3001-6	2702-7	2403-8	2104-9
	25	48-2-2	45-2-3	43-3-4	40-3-5	36-3-6	32-2-7	27-1-8	2301-9
	30	57-3-1	55-4-2	51-5-3	48-6-4	42-6-5	37-6-6	31-6-8	25-4-9
	35	66-4-1	64-6-1	60-7-2	55-9-3	49-T-4	42-T-5	35-T-6	27-8-8
40	10	2002-1	1903-2	1803-2	1704-3	1606-3	1507-3	1308-4	1109-3
	15	3001-2	2902-3	2702-4	2603-5	2404-5	2205-6	2006-6	1707-6
	20	4000-2	3800-3	3600-4	3400-5	3101-6	2802-7	2504-8	2105-9
	25	49-2-2	47-2-3	44-3-4	41-3-5	37-3-6	33-2-7	2800-8	2401-9
	30	59-3-1	56-4-2	53-5-3	49-6-4	44-6-5	38-6-6	32-5-8	26-3-9
	35	68-4-1	65-5-2	62-7-2	57-8-3	51-9-4	44-9-5	36-9-7	28-7-8
42	10	2102-1	2003-2	1904-2	1805-3	1706-3	1507-3	1308-3	1109-3
	15	3101-2	2902-3	2803-4	2703-5	2504-5	2306-6	2107-6	1808-6
	20	4100-2	3900-3	3700-4	3501-5	3202-6	2903-7	2604-8	2205-9
	25	51-1-2	48-2-3	45-2-4	42-2-5	38-2-6	34-1-7	2900-8	2502-9
	30	61-2-1	58-4-2	54-4-3	50-5-4	45-5-5	39-5-7	33-4-8	27-2-9
	35	70-4-1	67-5-2	63-7-2	59-8-3	52-9-4	45-9-6	37-8-7	30-7-9
44	10	2202-1	2103-2	2004-2	1905-2	1706-3	1607-3	1408-3	1209-3
	15	3201-2	3002-3	2903-4	2804-4	2605-5	2306-6	2107-6	1808-6
	20	4200-2	4100-3	3801-4	3601-5	3302-6	3004-7	2705-8	2306-9
	25	52-1-2	50-2-3	47-2-4	44-2-5	40-2-6	35-1-7	3101-8	2603-9
	30	63-2-1	60-3-2	56-4-3	52-5-4	46-5-6	41-4-7	34-4-8	29-2-9
	35	72-3-1	67-5-2	65-7-3	60-8-3	54-8-4	46-8-6	38-8-7	31-6-9

latitude due north, and $d = -10^\circ$, or wind direction 28 on the 36-point wind scale.

2. Given: $\phi_i = -70^\circ$, $\phi_0 = 33^\circ N$, $v = 35$ m.p.s.

Find: xx , yy and d after 48 hours.

Using Table 3 and the same method as above, the six-space group 40-E-4 is located. This means

$xx = 40^\circ$, $yy = 11^\circ$, and $d = 40^\circ$ (signs for yy and d have been reversed because ϕ_i is negative).

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TABLE 3 (cont.)

Initial latitude, deg.	Initial wind speed, m.p.s.	Inflection angle, deg.							
		20	30	40	50	60	70	80	90
46	10	2202-1	2103-2	2004-2	1905-2	1806-3	1607-3	1408-3	1210-3
	15	3301-2	3202-3	3003-4	2904-4	2705-5	2406-5	2207-6	1909-6
	20	4400-2	4201-3	4001-4	3802-5	3402-6	3104-7	2805-8	2406-8
	25	54-1-2	51-1-3	49-1-4	45-1-5	41-1-6	3700-7	3201-8	2803-9
	30	65-2-2	62-3-2	58-4-3	54-4-4	48-4-6	42-4-7	36-3-8	30-1-9
	35	75-3-1	71-5-2	68-6-3	62-7-4	56-8-5	48-7-6	40-7-7	33-5-9
48	10	2302-1	2203-1	2104-2	2005-2	1907-2	1708-3	1509-3	1210-2
	15	3402-2	3302-3	3103-3	3004-4	2806-5	2507-5	2308-5	2009-5
	20	4501-2	4401-3	4101-4	3902-5	3603-6	3204-7	2906-8	2607-8
	25	56-1-2	53-1-3	51-1-4	47-1-5	4300-6	3901-7	3402-8	2904-9
	30	67-2-2	64-3-3	60-3-3	56-4-4	50-4-6	44-3-7	38-2-8	3200-9
	35	78-3-1	74-5-2	70-6-3	65-7-4	58-7-5	50-7-6	42-6-8	35-4-9
50	10	2402-1	2303-1	2204-2	2106-2	1907-2	1708-2	1509-3	1310-2
	15	3502-2	3403-2	3304-3	3105-4	2906-5	2607-5	2408-5	2110-5
	20	4701-2	4501-3	4302-4	4102-5	3704-6	3405-7	3106-8	2708-8
	25	59-1-2	56-1-3	53-1-4	5000-5	4500-6	4101-7	3603-8	3105-9
	30	70-2-2	67-2-3	63-3-4	59-4-4	53-3-6	46-2-7	40-1-8	3401-9
	35	81-3-2	78-4-2	73-5-3	67-6-4	60-7-5	53-6-6	44-5-8	37-3-9
52	10	2502-1	2404-1	2305-2	2106-2	2007-2	1808-2	1609-2	1311-2
	15	3702-1	3503-2	3404-3	3205-4	3007-4	2808-5	2509-5	2110-5
	20	4901-2	4702-3	4502-4	4303-5	3904-6	3606-7	3207-8	2809-8
	25	6100-2	5800-3	5500-4	5200-5	4701-6	4302-7	3804-8	3306-9
	30	73-1-2	70-2-3	66-2-4	61-3-5	55-2-6	49-1-7	4200-8	3602-9
	35	85-3-2	81-4-2	77-5-3	70-6-4	63-6-5	55-5-6	47-4-8	39-2-9
54	10	2603-1	2504-1	2405-2	2206-2	2107-2	1809-2	1610-2	1311-1
	15	3902-1	3703-2	3604-3	3406-4	3207-4	2909-4	2610-4	2211-4
	20	5101-2	4902-3	4703-4	4504-5	4105-6	3706-7	3408-7	3009-7
	25	6400-2	6100-3	5800-4	5501-5	5002-6	4503-7	4005-8	3507-9
	30	77-1-2	73-2-3	69-2-4	64-3-5	58-1-6	5100-7	4501-8	3903-9
	35	89-2-2	85-4-2	80-4-3	73-5-5	66-5-5	58-4-7	50-3-8	42-1-9
56	10	2803-1	2704-1	2505-1	2306-1	2208-1	1909-1	1710-2	1411-1
	15	4102-1	3904-2	3805-3	3606-3	3308-4	3009-4	2710-4	2312-4
	20	5402-2	5203-3	5003-4	4704-4	4306-5	4007-6	3609-7	3210-7
	25	6700-2	6401-3	6101-4	5802-5	5303-6	4804-7	4206-8	3808-9
	30	81-1-2	76-1-3	73-1-4	68-2-5	6100-6	5502-7	4802-8	4204-9
	35	94-2-2	89-3-3	84-4-4	77-4-5	70-4-6	62-3-7	53-2-8	4500-9
58	10	2903-1	2804-1	2705-1	2407-1	2308-1	2009-1	1710-1	1412 0
	15	4302-1	4104-2	4005-3	3807-3	3508-3	3210-4	2911-3	2513-3
	20	5702-2	5503-3	5204-4	4905-4	4606-5	4208-6	3810-7	3411-7
	25	7101-2	6801-3	6502-4	6103-5	5604-6	5105-7	4507-7	4109-8
	30	8500-2	81-1-3	77-1-4	72-2-5	6501-6	5903-7	5204-8	4506-9
	35	99-2-2	94-3-3	89-3-4	82-3-5	74-3-6	66-2-7	5700-8	4902-9
60	10	3103-1	3004-1	2805-1	2607-1	2408-1	2109-1	1811-1	1512 0
	15	4502-1	4304-2	4205-2	4007-3	3709-3	3410-3	3012-3	2613-3
	20	6102-2	5803-2	5604-3	5306-4	5007-5	4609-6	4111-6	3613-6
	25	7501-2	7302-3	6902-4	6504-5	6005-6	5506-6	4908-7	4410-7
	30	9100-2	8700-3	8300-4	77-1-5	7002-6	6403-7	5605-8	5007-9
	35	T4-2-2	T1-2-3	95-2-4	89-3-5	80-2-6	7200-7	6302-8	5404-9

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