Comments on the "General Heat and Mass Exchange of Spherical Hailstones"

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In his paper on the heat and mass exchange from spherical hailstones, List (1963) assumes that the expressions relating the Nusselt number (Nu) and the Prandtl (Pr) and Reynolds (Re) numbers for forced convection heat transfer, and the analogue Nusselt number (Nu′) and the Schmidt (Sc) and Reynolds numbers for forced convection mass transfer given by Ranz and Marshall (1952 a, b), are valid for values of the Reynolds number up to 10^5. This assumption is incorrect and gives rise to the misimpression that the rates of transfer of heat and mass from the surface of a spherical hailstone are appreciably higher than those from a smooth sphere, the difference being presumed to be due to the roughness of the hailstone's surface.

The expressions given by Ranz and Marshall are

$$\mathrm{Nu}=2.0+0.60\,\mathrm{Pr}^{\frac{1}{3}}\,\mathrm{Re}^{\frac{1}{2}} \tag{1}$$

and

$$\mathrm{Nu}'=2.0+0.60\,\mathrm{Sc}^{\frac{1}{3}}\,\mathrm{Re}^{\frac{1}{2}}. \tag{2}$$

These were deduced from data obtained over the Reynolds number range 0 to 200. Comparing their data with those of other workers, Ranz and Marshall state that the above equations may be extrapolated to five times beyond the experimental range of Reynolds numbers, i.e., to Reynolds numbers up to 1000. However, most hailstone growth takes place in the Reynolds number range from about 10^3 to 10^5 and, as seen in Fig. 1, the actual values of Nu and Nu′ depart markedly from those given by equations (1) and (2) as the Reynolds numbers increase.

In an earlier paper, List (1960) has described icing tunnel experiments in which he determined the rates of heat and mass transfer from accreting ice spheres at Reynolds numbers of about 10^4. His results may be expressed in the form

$$\mathrm{Nu}=0.60\theta\,\mathrm{Pr}^{\frac{1}{3}}\,\mathrm{Re}^{\frac{1}{2}} \tag{3}$$

and

$$\mathrm{Nu}'=0.60\theta\,\mathrm{Sc}^{\frac{1}{3}}\,\mathrm{Re}^{\frac{1}{2}}, \tag{4}$$

where θ is a "corrective factor" by which Ranz and Marshall's values have to be increased. (The omission of the constant 2.0 in (3) and (4) does not give rise to serious error and is convenient for computational reasons). Unfortunately List does not evaluate θ directly but relates it to the efficiency of catch, E, of the accreting sphere, viz., $\theta/E=1.48$. However, approximate values of E for the conditions of List's experiments may be deduced using those obtained theoretically by Langmuir and Blodgett (1946) and these give a value of 0.74 ± 0.07 for 0.60θ. This result compares very well with the corresponding value of 0.76 obtained from an investigation of the heat and mass transfer from melting ice spheres over the same Reynolds number range (Macklin, 1963).

The points plotted in Fig. 1 show that, although theory predicts that the rates of heat and mass transfer across a laminar boundary layer depend on the half-power of the Reynolds number (see, e.g., Eckert and Drake, 1959), the overall rates of heat and mass transfer from a bluff body depend on a somewhat higher power of Re. The reason for this is the contribution to the transfer processes by the turbulent flow in the wake. In a recent study of forced convection heat transfer from cylinders to liquids, Perkins and Leppert (1962) found that their results were best correlated by expressing the Nusselt number as the sum of two terms, the first depending on $\mathrm{Re}^{0.5}$ and the second on $\mathrm{Re}^{0.67}$. The first term corresponds to the heat transfer across the laminar boundary layer on the front surface of the sphere and the second to the heat transfer in the wake;

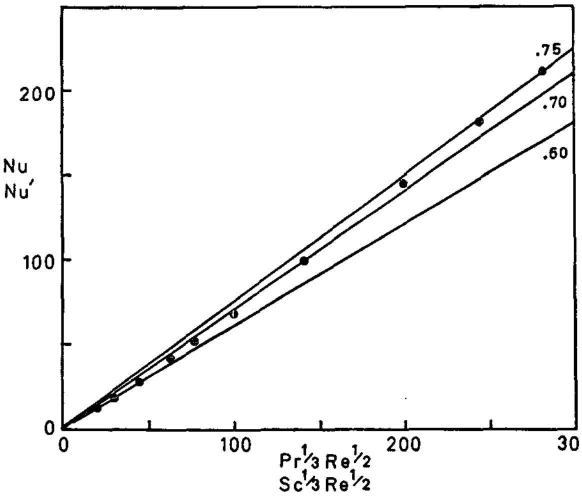

FIG. 1. The points plotted in this figure were read from the curve in Fig. 12 of Ranz and Marshall's (1953b) paper correlating the results of a number of experimental determinations of the heat and mass transfer from spheres. The slopes of lines fitting the points over various Reynolds number ranges are indicated, and it is seen that equations (1) and (2) cannot be used reliably for values of the abscissa beyond about 40. For the conditions encountered in hail clouds the Prandtl and Schmidt numbers have virtually constant values of 0.71 and 0.60, respectively (i.e., $\mathrm{Pr}^{\frac{1}{3}}=0.89$ and $\mathrm{Sc}^{\frac{1}{3}}=0.84$).

at higher Reynolds number the second term becomes increasingly more important. In treating the growth of hailstones it is convenient to use the $\mathrm{Re}^{0.5}$ law but this means choosing a value for the "constant" of proportionality which corresponds to the appropriate Reynolds number range, viz., about 0.70 to 0.75.

The above considerations show that the rates of heat and mass transfer from a spherical hailstone are not appreciably higher than those from a smooth sphere (possibly some 5 per cent) so that the surface roughness of the stone is not a particularly important parameter in this regard. It should be noted, however, that this conclusion applies strictly to hailstones growing near the critical condition as in List's (1960) experiments, i.e., to stones with a surface temperature near 0C. As shown by Browning, Ludlam and Macklin (1963), this appears to be the case for most of the period of growth.

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