Large Eddies in the Atmospheric Boundary Layer and Their Possible Role in the Formation of Cloud Rows[1]

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ABSTRACT

Turbulent shear flow generally contains large eddies which appear to be due to a shear instability of the profile of the mean flow. By analogy with laboratory experimental results, it is inferred that large eddies in the planetary boundary layer of the atmosphere should take the form of horizontal roll vortices with an orientation between that of the surface wind and that of the geostrophic flow. The vertical extent and intensity of the vertical motions associated with these roll vortices often may be sufficient to give rise to bands of clouds in the lower atmosphere. For an adiabatic boundary layer the spacing of cloud bands formed by this mechanism is tentatively predicted to be given by the relation $L=200U/\sin\phi$, where L is measured in meters, U is the geostrophic speed near the ground in meters per second and ϕ is latitude.

1. Introduction

We have become so accustomed to thinking of thermal convection as the primary convective mechanism that we often lose sight of the possibility that other dynamical processes may also give rise to vertical motions of sufficient scale and intensity to result in the formation of cloud. While we are all aware of other mechanisms, the tendency is to think first of thermal convection, perhaps because when condensation has occurred the release of latent heat enhances the thermal gradients and increases convective activity. However, while there is little argument that latent heat is exceedingly important once clouds have been formed, one may question to what extent the initiation and in particular the organized patterns of clouds are due directly to thermal processes.

As concrete examples of cloud producing mechanisms which are not due to thermal convection in the ordinary sense we may list: the forced flow over topography, the "equivalent mountain effect" introduced by Malkus and Stern (1953) to describe the effects of a local heat source in a generally stable atmosphere, and synoptic scale convective motions. This paper is concerned with an additional possibility which has received little serious consideration, namely that shear flow in the atmosphere may give rise to organized convective motions not associated with thermal processes. As one example of shear flow instability I suggest that the turbulent atmospheric boundary layer may generally contain "large eddies"[2] of sufficient vertical extent to cause the initiation of bands of clouds. It is argued that these large eddies are the result of an instability of the "normally turbulent" boundary layer.[3]

Boundary layer flows in rotating systems, whether laminar or turbulent, may be approximated by the spiral flow first introduced by Ekman (1905) as an approximation to the flow in the surface layer of the ocean due to wind stress. By analogy with laboratory experimental results on the stability of a laminar Ekman boundary layer (Faller, 1963), it is inferred that large eddies in a turbulent Ekman layer should take the form of stationary (or slowly moving) horizontal roll vortices oriented in the general direction of the wind, but at a small angle to the left of the geostrophic flow (Northern Hemisphere). Their horizontal wavelength should be 1 km or greater, dependent upon the geostrophic speed and upon latitude.

This general concept already has been advanced as an explanation of the spiral rain bands of hurricanes (Faller, 1961), but the arguments are extended here with additional experimental and theoretical material. This includes: observations and theory of turbulence in jets, wakes, and other non-rotating boundary layers (Sec. 4); comparison of the instability of Ekman flow with other boundary layer flows (Sec. 5); numerical integration of the dynamical equations to obtain the characteristics of the convective motions associated

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[2] The term "large eddies" has been used by Townsend in his book *The Structure of Turbulent Shear Flow* (1956) to describe a scale of motion large compared to other existing scales of turbulence.

[3] By the "normally turbulent" boundary layer is meant that flow which would exist in the absence of the large eddies that are under consideration.

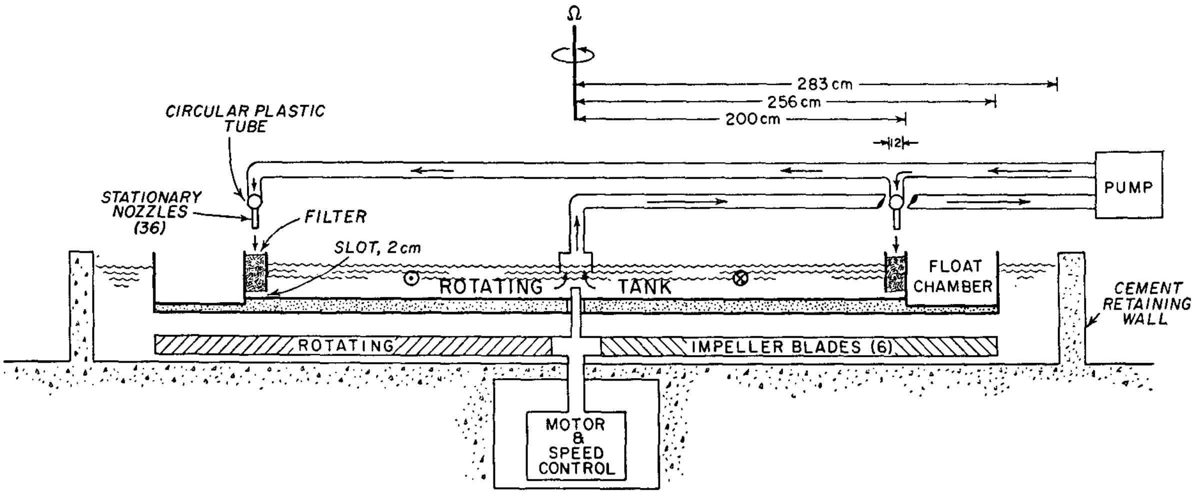

FIG. 1. A schematic diagram of the 4-meter diameter tank used at the Woods Hole Oceanographic Institution for the experimental studies.

with Ekman boundary layer instability (Sec. 6); and similarity arguments concerning the nature of the turbulent adiabatic boundary layer (Sec. 7). These topics are considered in turn after a brief discussion of the experimental basis of this study.

2. The laboratory experiments

Because the laboratory work is discussed in detail in an earlier paper (Faller, 1963) only those results pertinent to the ensuing discussion are reviewed here. Fig. 1 is a schematic diagram of the 4-meter diameter rotating tank which was formerly in use at the Woods Hole Oceanographic Institution for these studies. A circulation was generated by pumping water from the center and redistributing it uniformly around the rim. The equilibrium circulation consisted of a circular vortex with a shallow Ekman boundary-layer flow at the bottom where the required inward transport of water took place. The tangential speed U above the boundary layer, in response to a pumping rate S, varied inversely with radius r by the relation $U=S/\pi r\nu$ where ν is the kinematic viscosity. Except for small effects of the cylindrical geometry, the boundary-layer flow was that given by the equations

$$u=1-e^{-z}\cos z,\quad v=e^{-z}\sin z,\quad w=0. \tag{1}$$

These are the solution to the Navier-Stokes equations of motion for the boundary conditions $u=v=w=0$ at the bottom boundary $z=0$, and $\partial u/\partial z=\partial v/\partial z=w=0$ at $z=\infty$. Right handed Cartesian coordinates x', y', z' have been used, where x' is in the direction of the basic flow (tangential), y' is radially inward, and z' is vertically upward in the direction of the rotation vector $\mathbf{\Omega}$. z is a dimensionless vertical coordinate defined by $z=z'/D$, where $D=(\nu/\Omega)^{\frac{1}{2}}$ is a characteristic measure of the boundary-layer thickness. u and v, the component speeds in the x' and y' directions, respectively, have been made dimensionless by dividing u' and v' by U.

The pumping rate was adjusted so that the laminar Ekman flow became unstable at radii convenient for detailed observations (see Fig. 2). Streaks from crystals of potassium permanganate dye, introduced at the bottom near the rim, showed the pattern of circulation in the boundary layer. The dissolved dye at first spiraled toward the center with the flow near the bottom of the Ekman layer. In each case the thin layer of dye was observed to form a series of light and dark spiral bands within some critical radius. This pattern of bands indicated spiral regions of convergence and divergence near the bottom of the boundary layer and these have been interpreted as a series of horizontal vortices superimposed upon the basic boundary-layer flow.

These bands were found to originate at very nearly a constant Reynolds number defined as

$$\mathrm{Re}\equiv UD/\nu=U/(\Omega\nu)^{\frac{1}{2}}. \tag{2}$$

Variations in the critical value Re_c were caused by curvature of the flow in the cylindrical tank, but when the values of Re_c were extrapolated to the limit of zero curvature, the critical Reynolds number for linear Ekman flow was found to be $\mathrm{Re}_c=125$.

In addition, other characteristics of the instability were obtained. The average spacing of the bands was found to be $L=11D$, and the angles of orientation of the bands were mostly in the range $\epsilon=10-17$ degrees to the left of the tangential direction, with the average angle $\bar{\epsilon}=14.5$ degrees. Usually, the bands moved very slowly inward (normal to their axes) with respect to the rotating tank.

Other experimental evidence of instability of the same general character has been presented by Gregory, Stuart, and Walker (1955) for the flow over a rotating disc. They found the somewhat greater value $\mathrm{Re}_c \approx 400$, the average angle for stationary bands $\epsilon = 14.0$ degrees, and a band spacing $L = 21.5D$, approximately twice the value for Ekman instability.

3. An application to the atmosphere

From these experimental results, an analogy with the spiral rain bands of hurricanes was proposed (Faller, 1961). The concept of an average turbulent viscosity $\bar{\nu}_t$ was used to define a Reynolds number $\mathrm{Re}_t \equiv U/(\Omega\bar{\nu}_t)^{\frac{1}{2}}$ for the turbulent flow. Using data of Malkus and Riehl (1960) Re_t for the boundary layer of a mature hurricane was estimated to be $\mathrm{Re}_t = 650$. This value is in the range where by analogy one might expect a roll-vortex structure. As in the laminar experiments, instability would give rise to roll-vortices superimposed upon the main flow, but in the atmospheric case this structure would generate bands of clouds wherever the vertical motions carried moist air to the condensation level. After condensation, thermal effects might well dominate the continued convective process, but the relative importance of thermal and shear-flow effects would depend upon many factors.

Of particular interest for this analogy is the result found by Senn, Hiser and Low (1959) that individual clouds which originated in the spiral bands soon moved out of the bands in a more tangential direction and dissipated. This observation suggests: 1) that the bands are a quasi-permanent feature of the flow field which act as sources of vertical motion for the origin of individual clouds, but 2) as the clouds develop vertically they come into a more tangential flow and are advected into a descending branch of the spiral band structure.

4. The large eddies of turbulent shear flow

The first reported observations of the structure of turbulence near the free boundaries of a turbulent shear flow were made by Corssin and Kistler (1954) who found that a hot wire placed near the free boundary of a turbulent jet only intermittently indicated turbulence, and at other times showed smooth flow. Townsend (1956) interpreted intermittency in terms of a lateral advection of the boundary of the turbulence by large eddies with a scale comparable to the width of the profile of the mean flow. Grant (1958) made detailed observation of the structure of the large eddies and concluded that "these are the results of the instability of the turbulent shear stress." Although Townsend did not specifically refer to the large eddies as an instability, he considered at length the balance requirements for the large eddies, and he showed that: 1) they have a scale quite distinct from the general spectrum of turbulence; 2) they draw their energy from the energy of the mean shear flow; and 3) they are dissipated by the smaller scales of turbulence which act

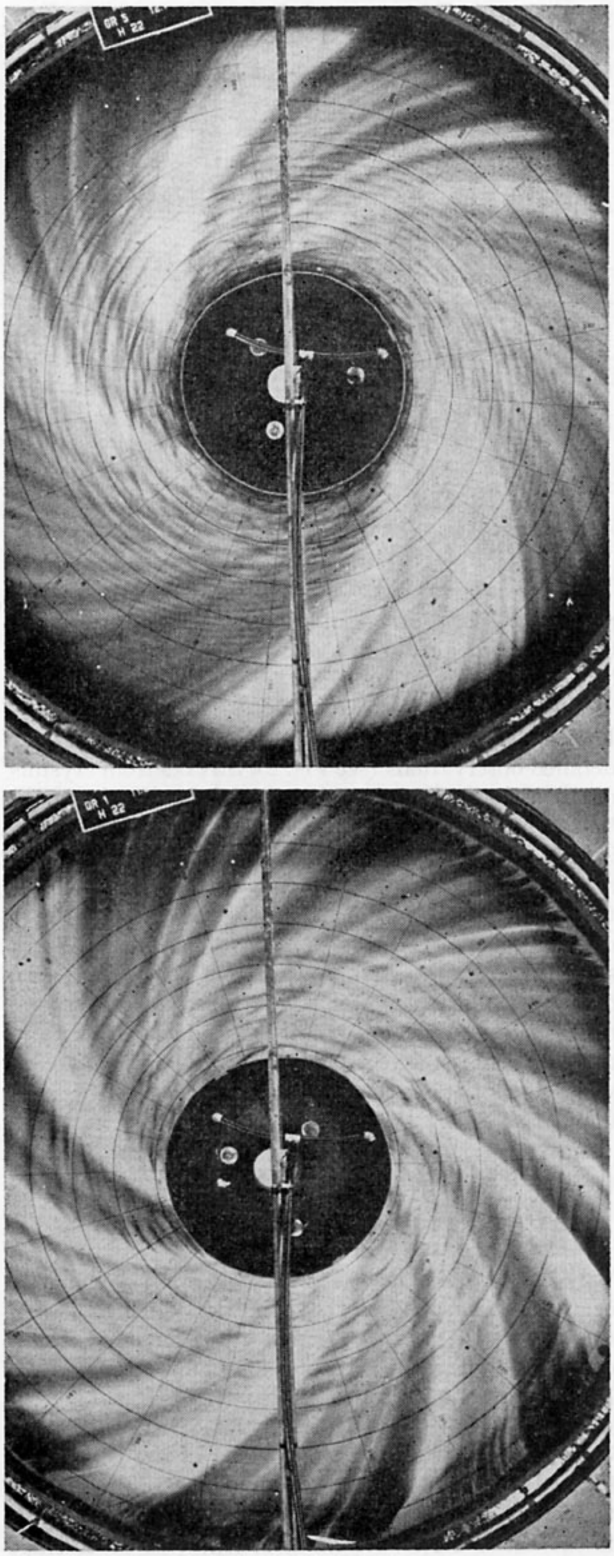

FIG. 2. Examples of the instability of laminar Ekman boundary-layer flow, illustrating the variation of band spacing with rotation rate: a) $\Omega = 0.0951\ \mathrm{sec}^{-1}$, $S = 753\ \mathrm{cm}^3\ \mathrm{sec}^{-1}$, $\nu = 1.13 \times 10^{-2}\ \mathrm{cm}^2\ \mathrm{sec}^{-1}$. b) $\Omega = 0.0272\ \mathrm{sec}^{-1}$, $S = 731\ \mathrm{cm}^3\ \mathrm{sec}^{-1}$, $\nu = 1.10 \times 10^{-2}\ \mathrm{cm}^2\ \mathrm{sec}^{-1}$. [For other examples see page 560 of the *Journal of Fluid Mechanics*, Vol. 15, 1963.]

as a turbulent viscosity. These conclusions are tantamount to regarding turbulent shear profiles as being unstable to a scale of motion comparable with the transverse dimension of the shear flow, much as a laminar shear flow may be unstable.

Far from their origin, two-dimensional turbulent jets and wakes are self-preserving when the flow is fully turbulent. Self-preserving implies that the dimensionless characteristics of the flow are independent of distance along the axis of the mean flow. Townsend derived this result directly by dimensional arguments, and he concluded that for self-preserving flows the turbulent Reynolds number is constant. With respect to the turbulent shear flow in the wake of a circular cylinder he made the following observation (footnote, p. 166):

"It is of interest that the "turbulent Reynolds number" of wake flow, $C_d(U_1d/\nu_t)=62$ is very close to the critical Reynolds number for the observed instability of a laminar wake, $C_d(U_1d/\nu)=64$."[4] The large eddies appear to be an integral part of the mechanism by which a constant turbulent Reynolds number is maintained, by virtue of their effect on the spread of the flow and their redistribution of the smaller-scale turbulence.

The analogy between the instability of laminar flows and that of turbulent flows is of great importance. There is no reason to conclude that this analogy is limited to turbulent wakes since the phenomenon of intermittency and the existence of large eddies have been observed for wakes, jets, and turbulent boundary-layer flow over a flat plate (Grant, 1958).

In a related study Tani (1962) showed that the Görtler vortices formed during laminar flow over a concave wall (Görtler, 1940) have their counterpart in turbulent flow. He investigated both laminar and turbulent flows (with artificially induced turbulence) and showed that the vortices corresponded approximately in wavelength and in the critical value of the Görtler parameter when a turbulent viscosity was used in the appropriate cases. Although the Görtler vortices are more analogous to thermal instability than to the inflectional instability of shear flow, Tani's results demonstrate the utility of the concept of a turbulent viscosity for parameterization of the effects of small scale turbulence.

5. A comparison of boundary-layer shear flows

To argue logically that the large eddies of two-dimensional shear flows should have their counterpart in the spiral boundary layers of rotating systems, it is necessary to point out the basic similarity in their mechanisms of instability. In a recent analytical study of the stability of Ekman flow, Barcilon (1964) found an equivalence between the Ekman stability problem and that of ordinary two-dimensional (2-*d*) boundary-layer flow. A similar result was found by Stuart (Gregory, Stuart and Walker, 1955) for the flow over a rotating disk. Barcilon in addition pointed out a similarity between the vertical profile of Ekman flow at various angles (Fig. 3) and that for non-rotating boundary-layer flow against a pressure gradient (see Schlichting, 1960, Fig. 12.4) in which the inflection point of the profile is an important factor for instability. Brown (1961) obtained numerical solutions for the neutral stability curves of the transverse component of flow for various 3−d boundary layers including that due to a rotating disk. Following a suggestion of N. Gregory he found empirical relations between a Reynolds number and shape parameters for several types of flow. These relations, one for the profile transverse to the basic flow and one for the "critical" profile, give $Re_c=133$ and $Re_c=110$, respectively, for laminar Ekman flow, in good agreement with the experimentally determined value, $Re_c=125$.

These studies indicate that many of the experimental and theoretical results pertaining to 2-d shear profiles may be carried over to rotating systems. Accordingly, it is reasonable to expect the formation of large eddies in turbulent Ekman flow.

Perhaps an important distinction between the non-rotating 2-d flows and Ekman flow is that in the former the thickness of the turbulent layer and the scale of the large eddies vary along the axis of the flow. Therefore in non-rotating jets, wakes, and similar shear flows, the dimensions of the large eddies are in a constant state of flux as they are advected with the flow. But in an Ekman layer, no such transients in the scales of individual eddies would seem to be required because the boundary layer has constant thickness. Accordingly, it is suggested that large eddies in turbulent Ekman flow should be fairly steady, more regular in spacing, and more easily identified than in the 2-d examples.

6. A numerical study of the characteristics of Ekman boundary-layer instability

In order to test and to extend the experimental results, numerical iterations of the time-dependent Navier-Stokes equations have been performed. The basic flow field and the boundary conditions were for the Ekman boundary layer described in Section 2. Following the experimental results, the instability was assumed to be 2-dimensional, and a stream function in the vertical plane normal to the axes of the vortices was defined. The vortices were assumed to be oriented at an angle ϵ to the direction of the geostrophic flow. The equations which were iterated consisted of a

[4] This definition of turbulent Reynolds number may be transformed to $Re_t=2U_1\theta/\nu_t$ where U_1 is the free stream speed, θ is a "displacement thickness" given by

$$\theta=\int_{-\infty}^{\infty}\frac{U}{U_1}\left(1-\frac{U}{U_1}\right)dy$$

and U is the mean flow at distance y from the center of the wake (Townsend, 1956, p. 132). The mean turbulent viscosity ν_t is not given explicitly, but it is related to θ through the profile of U much as the depth of the turbulent boundary layer in the atmosphere D_t must be related to some mean turbulent viscosity.

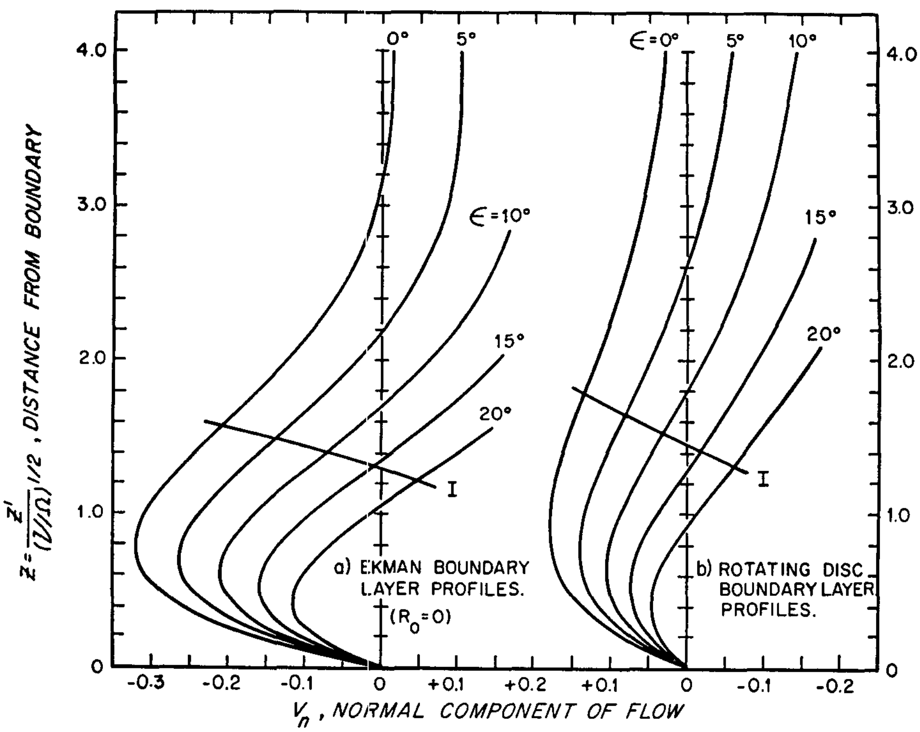

FIG. 3. Components of boundary-layer flow normal to the direction at angle ϵ with the tangential direction. The trace of the inflection point is indicated by I.

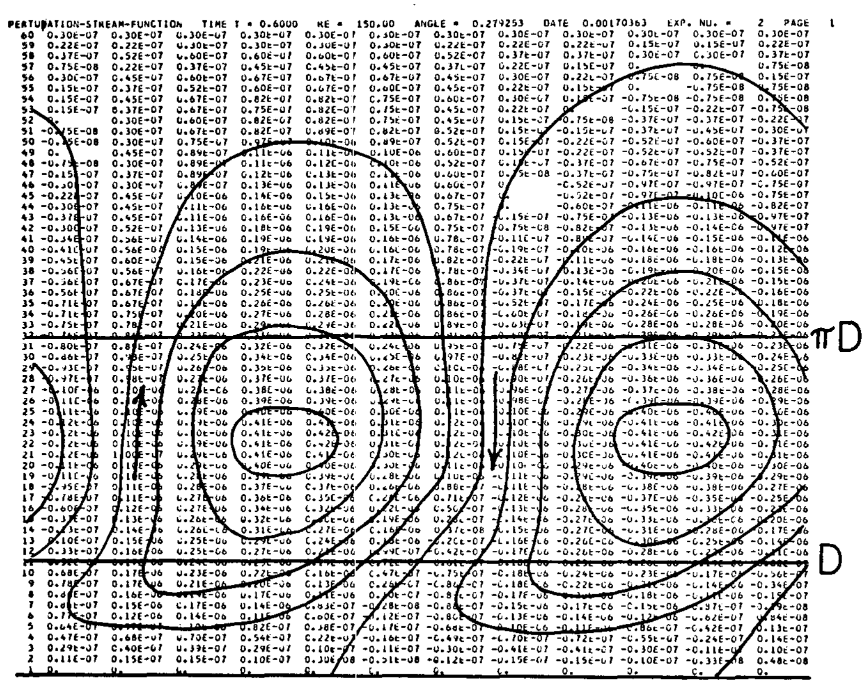

FIG. 4. The perturbation stream function for an amplifying disturbance to the laminar Ekman boundary layer. This pattern was obtained by numerical iteration of the dynamical equations for the wavelength $L=11D$, the angle $\epsilon=16$ degrees, and Re=150. The vertical exaggeration is approximately 1.5. The vertical spacing of grid points is $0.1D$, and the horizontal spacing is $0.92D$.

vorticity equation in the vertical plane and an equation for the flow normal to that plane, and the flow was assumed to be horizontally periodic with a basic wave length L. The exact Ekman solution was used as the initial flow field, and perturbations to this flow were introduced either from the round-off errors of the computer (IBM 7090) or as random numbers applied at each grid point. Iteration of the equations was performed for several values of ϵ, L, and Re, and the results for Re_c and the optimum values of ϵ and L closely agree with those found experimentally.

Fig. 4 shows the perturbation stream function in the vertical plane normal to the vortices for an amplifying disturbance of length $L=11D$, at the angle $\epsilon=16$ degrees, and for Re=150. Nearly identical streamline patterns have been obtained for Re=300 and Re=1000. This figure shows that an unstable wave has its center of circulation and its maximum vertical velocity at $2.1D$, a substantial height above the inflection point of the Ekman profile (Fig. 3). The grid for this numerical experiment extended vertically to only $6D$, but a similar experiment to $12D$ showed no significant difference. The example shown is for a small amplitude, amplifying wave, but it is typical of patterns even when the perturbation interacts with the basic flow. Further details of the numerical computations will be presented in a subsequent paper, but the results to date confirm the roll-vortex structure inferred from the dye patterns of the experiments, and give the vertical extent of the vortex motions.

7. Similarity in the turbulent atmospheric boundary layer

In this section it is shown that the turbulent Reynolds number in an adiabatic boundary layer is approximately constant for a wide range of conditions. With some simplifications, boundary-layer flow in a rotating system may be described by the relation

$$\mathbf{v}'=f_1(U,\Omega,\nu,z',z_0'), \tag{3}$$

where $\mathbf{v}'$ is the time-averaged value of the horizontal velocity, U is a prescribed geostrophic speed above the boundary layer, z_0' is the roughness length, and the other symbols are as defined previously. Spatial and temporal variations of U and curvature of the flow may sometimes be important but they do not seriously alter the succeeding arguments, and as long as turbulence is principally a result of shear flow in the boundary layer, we may at first omit consideration of density gradients.

From the 6 variables in (3) we may form 4 dimensionless parameters, since there are but two basic dimensions involved, length and time. Expression (3) may then be rewritten

$$\mathbf{v}=f_2(\mathrm{Re},z,\mathrm{Ro}_0) \tag{4}$$

where Re is given by (2), $z=z'/(\nu/\Omega)^{\frac{1}{2}}$, and $\mathrm{Ro}_0=U/2\Omega z_0'$, the "surface Rossby number" as defined by Lettau (1958). For steady laminar Ekman flow with a smooth boundary ($\mathrm{Ro}_0\rightarrow\infty$), $\mathbf{v}$ is independent of Re and of Ro_0 as in Eq. (1). However, from the experimental results $\mathbf{v}$ depends upon Re for Re>125, and for Re>1000 the flow appears to have little or no organization and may be described as turbulent. For a typical value of Re for the atmosphere, take $U=10$ m sec^{-1}, $\nu=0.15$ cm^2 sec^{-1}, and $\Omega=0.5\times10^{-4}$ sec^{-1}. These numbers give $\mathrm{Re}=3.5\times10^5$, a value sufficiently high for the flow to to be termed "fully developed" turbulence. In such a case it may be assumed that the total shear stress at any level except near $z=0$ is determined principally by the Reynolds stress and that molecular viscosity has little influence upon the profile of the mean flow. Therefore, for sufficiently large Re we may replace ν in Eq. (2) by a turbulent viscosity ν_t and write

$$\mathbf{v}'=f_3(\Omega,U,\nu_t,z',z_0'). \tag{5}$$

Furthermore, the largest scales of turbulence, those which contribute most to ν_t, are not themselves directly affected by molecular processes except near $z=0$, and we may write

$$\nu_t=g_1(U,\Omega,z',z_0'). \tag{6}$$

By eliminating ν_t from (5) and (6) one may obtain

$$\mathbf{v}=f_4(z_t,\mathrm{Ro}_0) \tag{7}$$

which is to say that ν_t is implicitly determined by the total flow. In (7) $z_t=1/K_0[(\Omega/U)z']$ is a new dimensionless vertical scale, and K_0 is a constant to be determined below so that a turbulent scale height $D_{t0}=K_0U/\Omega$ bears approximately the same geometrical relation to the profile of flow in the turbulent case as does D in laminar flow.

In order to speak of a turbulent Reynolds number, defined here as

$$\mathrm{Re}_t\equiv\frac{U}{(\Omega\bar{\nu}_t)^{\frac{1}{2}}}, \tag{8}$$

it is necessary to determine a vertically averaged turbulent viscosity $\bar{\nu}_t$.

For the purposes of this paper I define $\bar{\nu}_t$ by

$$\bar{\nu}_t=\frac{1}{Z_t}\int_0^{Z_t}\nu_t dz_t, \tag{9}$$

where Z_t is a fixed value of z_t taken near the top of the boundary layer. From (6) it follows that

$$\nu_t=\frac{U^2}{\Omega}g_2(z_t,\mathrm{Ro}_0), \tag{10}$$

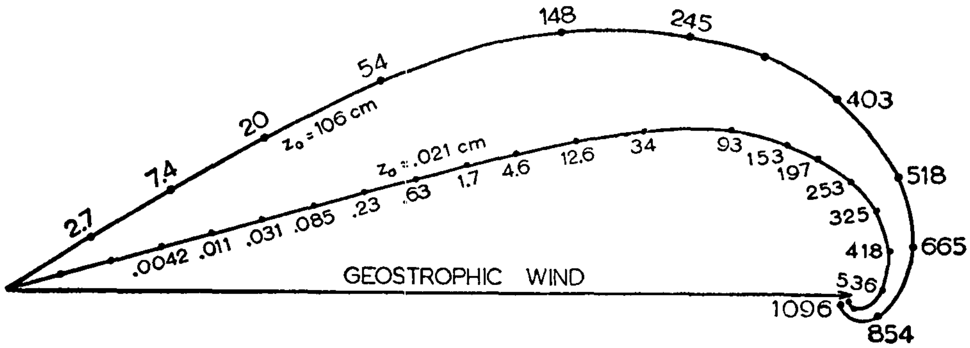

FIG. 5. Wind hodographs for rough and smooth surfaces corresponding to $U/2\Omega\sin\phi=10^7$ cm, after Blackadar (1962). Heights are in meters.

but with the provisional assumption that Ro_0 is not a significant parameter for the value of $\bar{\nu}_t$ it follows that

$$\bar{\nu}_t = K_1^2 \frac{U^2}{\Omega}, \tag{11}$$

where

$$K_1^2(Z_t) = 1/Z_t \int_0^{Z_t} g_2(z_t)dz_t.$$

When (11) is introduced into (8) we find $Re_t = 1/K_1$, so that within the assumption that Ro_0 is not a significant parameter, Re_t is constant and $\bar{\nu}_t$ varies as the square of the geostrophic speed.

By analogy with the laminar flow $D_t \equiv (\bar{\nu}_t/\Omega)^{\frac{1}{2}}$, and it follows using (11) that $D_t = K_1 U/\Omega$. If the constant K_0, introduced above for the definition of a turbulent scale height D_{t0}, is now identified with K_1, it follows that $D_{t0} = D_t$.

To examine the dependence of Re_t upon Ro_0, consider the model adiabatic wind profiles of Blackadar (1962) for which specific spiral profiles are displayed in Fig. 5. From assumptions about the form of the vertical variation of mixing length and using empirical data as a guide, Blackadar has obtained these spirals for the conditions $U=10$ m sec^{-1}, $z_0'=0.021$ cm and $z_0'=106$ cm. The corresponding values of Ro_0 are 5×10^8 and 1×10^5, respectively.

We expect that the flow will be more nearly independent of Ro_0 for small z_0', therefore the profile for $Ro_0 = 5\times10^8$ is used as a standard for the selection of a value of Z_t. From Fig. 5, Z_t has been determined as the height at which the cross component of flow first vanishes. This is approximately $Z_t' = D_t Z_t = 600$ m. From Blackadar's Fig. 6 for the distribution of turbulent viscosity with height, the average value of ν_t to 600 m is $\bar{\nu}_t = 2.5\times10^4$ cm^2 sec^{-1} and it follows that $D_t = 225$ *m*, $z_t = 2.67$, and from (8), $Re_t = 890$. With the same Z_t used, the computed values for $Ro_0 = 1\times10^5$ are given in Table 1. There it is apparent that Re_t changes little for a large range of Ro_0.

The arbitrariness in the method of determination of $\bar{\nu}_t$ (and therefore D_t and Re_t) should be noted. This arises because the turbulent profiles of flow are not exactly the same as that of laminar Ekman flow. $\bar{\nu}_t$ could be determined also through the definition of a displacement thickness similar to that of Townsend (footnote 4). Still another method would be to select as the characteristic scale height D_{t0}, that height which divides the cross-stream component into equal transports above and below D_{t0} since this condition is satisfied by D for the laminar flow. Consistent application of these methods to the Blackadar profiles give values of Re_t comparable to those of Table 1. Perhaps of greater importance for inflectional instability is the change of shape of the velocity profile with Ro_0, well illustrated in Fig. 5. In each profile the logarithmic portion extends to approximately 20 m, but above that height the spirals appear to be of similar shape. As an alternate definition of Re_t which may be more appropriate than the definition (8) for comparison with the inflectional instability of Ekman flow, consider $Re_t^* \equiv U^*/(\Omega\bar{\nu}_t)^{\frac{1}{2}}$ where U^* is the total shear above the logarithmic layer. The values of U^* and Re_t^* as obtained from Fig. 5 are included in Table 1. Thus the values of Re_t depend upon the method of comparison of the laminar and turbulent flows, but in all examples Re_t does not vary rapidly with Ro_0 and the values are in the range where one might expect an organized roll-vortex structure.

TABLE 1. Evaluation of the turbulent Reynolds number from data of Blackadar (1962) for the geostrophic speed $U=10$ m sec^{-1} and at 43 degrees latitude. U^* denotes the shear above the logarithmic boundary layer and Re_t^* is the corresponding turbulent Reynolds number.

z_0' (cm)	Ro_0	Z_t	D_t (m)	Z_t' (m)	$\bar{\nu}_t$ (cm^2 sec^{-1})	Re_t	U^* (m sec^{-1})	Re_t^*
0.021	5×10^8	2.67	225	600	2.5×10^4	890	3.45	307
106.0	1×10^5	2.67	300	800	4.5×10^4	670	7.25	485

8. Discussion

From experimental studies, large eddies are a rather general characteristic of 2-dimensional turbulent shear flows. Inasmuch as boundary-layer instability in rotating systems is similar to that of 2-dimensional flows, it may be expected that the turbulent boundary layer of the atmosphere will have some form of large-eddy structure. It is suggested, in accord with the findings of Tani (1962), that the large eddies of turbulent flow will have characteristics similar to those of the vortex motions which occur when the corresponding type of laminar flow becomes unstable. It follows by analogy with the laminar experiments and the numerical integrations that an adiabatic atmospheric boundary layer should contain large eddies in the form of roll vortices. The roll-vortex structure should occur for a wide range of conditions inasmuch as Re_t is approximately constant.

To estimate the horizontal scale and vertical extent of the roll vortices, recourse must be made to the laminar experiments. The ratio $L/D=11$ gives a first estimate of wavelength, although this ratio may depend very much upon the profile of flow. From the discussion in Section 7 it follows that on the earth $L=(U/\sin\phi)\times 200$ where L is measured in meters and U in meters per second. The height H to which appreciable vertical motion should extend is approximately $4D_t$ (see Fig. 4). The circulation speeds that such vortices might attain in the atmosphere is a matter of speculation. However, from preliminary numerical results it would appear that vertical velocities might easily reach $w=0.1U$. For the specific values $U=10$ m sec^{-1}, and $\phi=43$ degrees, these considerations give average values $D_t=262$ m, $L=2.9$ km, $H=1040$ m and vertical speeds of 1 m sec^{-1}. Each of these values would be proportional to the speed of the geostrophic wind, and the dimensions L and H would be inversely proportional to $\sin\phi$.

The probable influences of thermal stratification are not immediately obvious. Thermal stability most likely would inhibit roll vortex formation; for although Re_t would be increased by the reduction in intensity of the small scale turbulence, there would probably be a greater increase in the critical value of Re_t, so that no large eddies would form. On the other hand, for sufficiently large U an initially stable stratification might be destroyed by small-scale turbulence so that the subsequent development of large eddies was permitted.

In slightly unstable conditions the growth of large eddies would be enhanced by buoyancy forces. The vertical transport of heat by the large eddies could at times be sufficient to render organized thermal convective heat transport unnecessary, except close to the ground. Accordingly, the large eddy structure might organize buoyant elements so as to amplify the intensity of the existing large eddies, never permitting the establishment of a scale of motion characteristic of thermal convection itself. From expansion of the above argument, in a situation where there was a gradual elimination of a stable stratification by turbulence and by heating from below (and with a significant geostrophic flow), the large eddies would form as soon as neutral stratification was attained. Thereafter the pattern of flow would follow that of the large eddies unless the thermal sources were sufficiently intense. The relative importance of shear flow and thermal effects would depend upon the relative strengths of the geostrophic flow and the local thermal sources in a manner as yet undetermined.

It is important to note that these roll-vortex motions should not be limited to situations where clouds are formed, but should be a general feature of turbulent planetary boundary layers wherever the stratification permits their existence. By analogy with the experimental results, they should not generally move faster than $0.1U$ and may often be stationary. Accordingly, their minimum period for a fixed observer would be $T=10L/U=2000/\sin\phi$ sec, independent of the speed of flow. Therefore, it is unlikely that observers at a single fixed location would distinguish these circulations unless larger-scale atmospheric motions were unusually steady and diurnal effects were minimal. However, their presence would affect the representativeness of surface observations, in particular those of the average wind and turbulence in the boundary layer.

Another specific application is to the Langmuir circulation cells in the surface layer of the ocean (Langmuir, 1938). These cells create bands of convergence which are indicated by any floating material which may be present, for example, *Sargassum*, foam, or materials purposely spread on the ocean surface. It has recently been found that the spacing of lines of *Sargassum* is approximately proportional to wind speed measured near the surface (Faller and Woodcock, 1964), and that these lines of convergence are oriented at a systematic angle averaging 13 degrees to the right of the mean wind stress (Faller, 1964). Inasmuch as these observations are in accord with the analogy proposed in this paper, they lend support to the thesis that horizontal roll vortices due to shear-flow instability are a general feature of turbulent planetary boundary layers.

In addition to the possibility of large-eddies in boundary-layer flows, shear flow in the free atmosphere may exhibit similar phenomena. With somewhat different boundary conditions, the criteria for instability are not as clear, but the existence of an inflection point in the wind profile with a near-adiabatic lapse rate may be sufficient. The dimensional arguments of Section 5 are not directly relevant to the free atmosphere because the shear flow there will be controlled by many factors including the variation of the pressure gradient with altitude, rather than simply by the presence of a rigid boundary. It is difficult to point to specific examples where shear-flow instability is clearly present. However, the general banded structure of the atmosphere (Kuettner, 1959), recently illustrated most graphically by TIROS photographs, shows the presence of many organized scales of motion in which shear-flow instability as well as thermal instability may play an important role.

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