The Structure of Clear-Air Turbulence Derived from "TOPCAT" Aircraft Measurements[1]

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ABSTRACT

Measurements of clear-air turbulence spectra conducted by a Canberra aircraft over Australia between July and October 1963 reveal the existence of a wavelength region from somewhat less than 1000 ft to 4000 ft, in which the atmosphere receives turbulent energy. It is suggested that this energy stems from gravitational shearing waves which break up into turbulent eddies below a critical wavelength. The energies of these turbulent eddies seem to be well represented by a proportionality to $k^{-5/3}$, characteristic of the inertial subrange of turbulence.

1. Introduction

Clear-air turbulence (CAT) still remains a major hazard of modern aviation, which may result in severe damage or even loss of aircraft (Reiter, 1963a, 1964a). Structural fatigue and passenger discomfort are factors which have to be taken into account even in less dramatic encounters with CAT. Our knowledge of this phenomenon has been summarized by the author and by others in previous publications (Reiter, 1960, 1961, 1962, 1963b, c, 1964b, c, d; Reiter and Hayman, 1962; Reiter and Nania, 1964; Clodman *et al.*, 1960; Endlich and McLean, 1964; Hildreth *et al.*, 1963; Panofsky and McLean, 1964; Pchelko, 1962; Pinus and Shmeter, 1962).

The present study is aimed to highlight certain implications of CAT with respect to atmospheric structural characteristics that might eventually lend themselves to exploration by radiowave propagation methods.

2. Energy spectra of CAT

It has been shown by Kolmogorov (1941), Obukhov (1941), Heisenberg (1948), and by others [see, e.g., Batchelor (1959), and Lumley and Panofsky (1964)] that the energy spectra of turbulence in the inertial subrange may be expressed as

$$E(k)=\alpha\epsilon^{2/3}k^{-5/3}, \qquad (1)$$

where $E(k)$ is the (wave-number dependent) kinetic energy of turbulent motion in the wave number range k to $k+dk$, α is a universal constant, and ϵ is the rate of dissipation of energy.

Within the inertial subrange the energy spectrum is independent of the kinematic viscosity. No production or viscous dissipation of turbulent energy is assumed to take place in this range. Turbulent energy is simply transferred to smaller and smaller eddies.

The validity of the "$-5/3$ law" has been well substantiated in low-level turbulence observations. Recent investigations of turbulence measured by aircraft have indicated, that such an "inertial subrange" of turbulence also exists in the free atmosphere, in clouds as well as in clear air.

Typical power spectra of turbulence in thunderstorms, in cumulus clouds, and in clear air at low levels were obtained by Rhyne and Steiner (1962). MacCready (1962, 1964) reports on spectra measured by sailplane. Another set of aircraft turbulence data has been presented by Shur (1962) for well and weakly defined jet streams, as well as for CAT over mountains, in cumulus and in cirrus clouds. Shur's power spectra pertain to vertical velocity components only, which were derived in a manner somewhat different from that of the above investigators. He makes use of aircraft accelerometer readings, deriving the spectra from the relationship

$$E_0(\omega)=E_i(\omega)|T(\omega)|^2, \qquad (2)$$

where $E_0(\omega)$ is the measured "output" spectrum function in terms of frequency, ω (e.g., the gust loads on the aircraft as determined from the accelerometer records), $E_i(\omega)$ is the "input" spectrum function (the spectral density of atmospheric vertical gusts causing the observed accelerations of the airplane), and $T(\omega)$ is the

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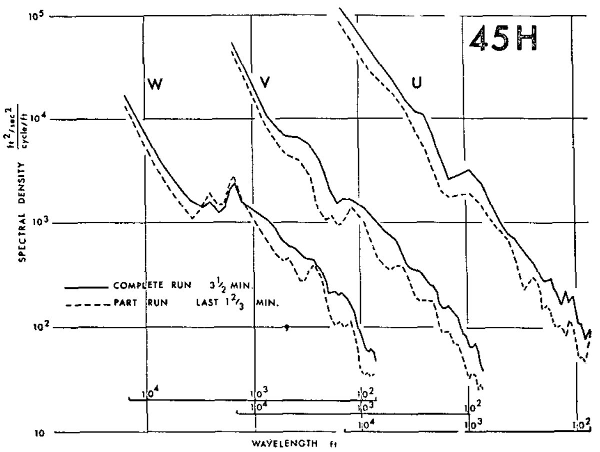

FIG. 1. Spectral densities of u-, v-, and w-components of gustiness, computed separately for last 1⅔ minutes and for total flight time (3½ minutes) of Run 45H (after Burns and Rider, 1965).

absolute value of the frequency response function of the aircraft, describing the rigid and elastic oscillation modes of the aircraft in response to sinusoidal gusts of various frequency.

During Project "TOPCAT" an additional set of CAT data has been obtained over southern Australia during the period from 21 July to 3 October 1963 (Burns and Rider, 1965; Mizon, 1964; Radok, 1964; Reiter, 1964e; Spillane, 1964). Measurements were made by a Canberra aircraft flying at altitudes of 26,000 to 33,000 ft (9–11 km). The airplane carried a differential pressure probe on a nose boom which measured separately small pressure fluctuations caused by atmospheric gusts in the u, v and w components. (Coordinates are given with reference to the motion of the aircraft.) In addition, acceleration, gyroscopic, and strain gauge measurements were available to allow a study of aircraft responses.

As described by Burns and Rider (1965) special care has been taken to eliminate contamination of the records by aircraft motions, such as the "Dutch roll." In spite of the restrictions inherent in the mathematical treatment of response characteristics, the atmospheric turbulence spectra obtained should, therefore, be considered quite reliable. Specifically, contamination of the records by the pilot's maneuvers was eliminated, thereby reducing a problem with which Shur (1962) was faced. This was done mainly by including corrections for recorded aircraft motions, both pilot and gust induced. Furthermore, the pilot was instructed to keep control movements to a minimum.

Corrective handling of the controls by the pilot would mainly affect the low-frequency end of the observed spectra. That any such effects were negligible is shown in Fig. 1 which contains spectra of the u, v and w components of Flight 45, Run H, computed for both the entire run (3½ minutes), and for the last half of the run (1⅔ minutes). There is no significant difference between the two groups of spectra, especially not in their long-wave end. Furthermore, it may be seen from this diagram that the spectrum analyses do not suffer significantly by the relative shortness of the sample.

Vertical incremental accelerations in the CAT patches for which power spectra were calculated were of the order of 0.5 to 0.8 g with airspeeds of approximately 740 ft sec^{-1} ($\sim$226 m sec^{-1}). CAT experienced by the Canberra aircraft was estimated to be moderate to severe. Table 1 (Burns and Rider, 1965) contains information on the research flights evaluated so far. Meteorological conditions for these flights are summarized in Table 2.

As may be seen from Table 1, the standard deviations of all three components of turbulence fluctuations in the wavelength region of 70 to 15,000 ft showed some variation. Nevertheless, the spectra reproduced in Figs. 2 to 4 have not been standardized because they were all characteristic of CAT near the "moderate" level. An exception is the set of data obtained at low levels which revealed considerably less turbulence energy, especially in the w-component, than the rest of the spectra.

Judging from subjective experience, CAT seems to consist mostly of "bumps" in a frequency range of more than one "bump per second." The airspeeds given in Table 1 would thus indicate CAT to occur in the wavelength range of $<$1000 ft. In agreement with this, the spectra, especially in the w-component (Fig. 4), show remarkable similarity at wavelengths $<$1000 ft. An

TABLE 1. Summary of traverses.

Date	Flight and run no.	Average barometric height (ft)	Heading	Average true airspeed (ft sec^{-1})	Duration of run (sec)	Wind (deg/kt)	Mean sq. gust velocity* (ft sec^{-1}) σu	σv	σw	Cross correlation coefficient between w and u
21/ 8/63	18.02 (Run B)	28,200	155°	734.1	37.5	270°/90	3.19	4.84	2.83	−0.188
21/ 8/63	18.04 (Run D)	29,100	260°	737.9	50	261°/90	5.09	6.25	3.39	0.289
4/ 9/63	27.06 (Run F)	26,690	280°	730.5	150	226°/94	2.95	3.45	2.78	0.069
12/ 9/63	33.05 (Run E)	32,040	255°	742.2	150	260°/90	4.02	4.00	2.60	−0.263
1/10/63	44.04 (Run D)	300	—	507.0	150	—	1.99	2.38	1.89	−0.022
3/10/63	45.08 (Run H)	28,450	086°	741.8	270	256°/43	4.36	2.93	2.39	−0.198
3/10/63	46.05 (Run E)	29,260	216°	744.4	100	278°/45	5.10	5.65	3.79	0.160

* Truncated values referring to wavelengths ranging from 70 ft to 15,000 ft.

TABLE 2. Meteorological conditions during research flights.

Date	Meteorological conditions	CAT
(Mizon, 1964)		
21/ 8/63	160 kt jet core between 28 and 30S, westward of 140E. Core at 250 mb. Strongest vertical shears near 300 mb.	Light to moderate CAT at 31°40′S and 140°27′E at 28,000–29,000 ft. Accelerations ≦0.5 g.
(Reiter, 1964e)		
4/ 9/63	Passage of jet stream and cold front over Flinders Range, east of Adelaide, should induce mountain-wave formation near sharply defined tropopause (27,000 ft).	Moderate to strong CAT along Flinders Range.
12/ 9/63	Strong jet core (balloon-measured winds ∼225 kt, maximum aircraft winds 133 kt) over Adelaide near 34,000 ft. Stable layer near 28,000 ft.	Thin cirrus observed north of Adelaide near 28,000 ft. Patch of moderate CAT first identified near 31°48′S, 137°02′E at 32,000 ft. Patch was marked repeatedly with smoke trail, and was followed for 45 minutes while drifting downstream with wind 80–100 miles. CAT 1000 ft above and below main level was noticeably less. Pilot reported wave formation on smoke trail. CAT patch had to be abandoned for lack of fuel.
1/10/63	Low level comparison with DC-3 measurements near Wagga-Wagga.	See Table 1.
3/10/63	Well developed confluence between two jet branches. Rapid turning of wind with height (196° to 260°) in layer 26,000 ft to 31,000 ft. Generally light winds below jet-stream velocities in this region.	Extensive light to moderate CAT near shear line between 28,000 and 29,000 ft. One haze horizon observed at flight level, another one higher up. Some smoke puffs released by aircraft remained well rounded, some spread out into thin, nearly-horizontal sheets indicating strong shearing of meso-structural layers.

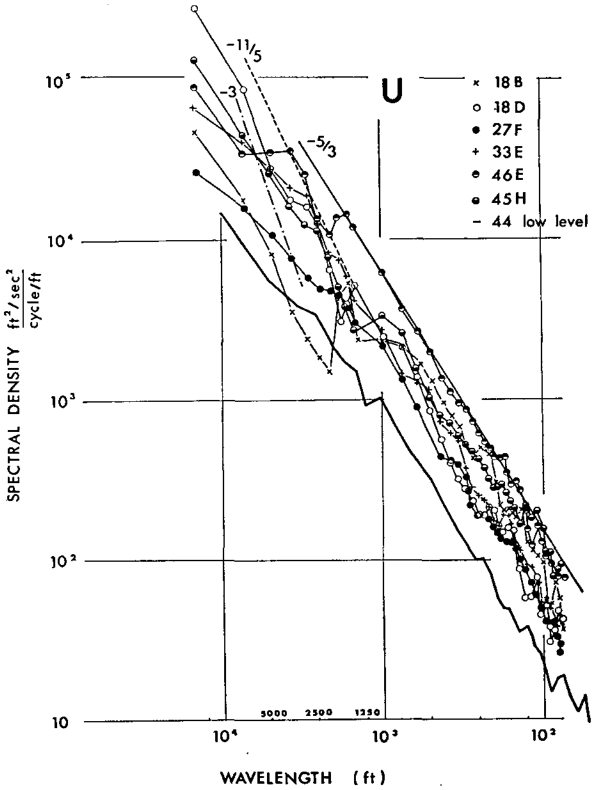

FIG 2. Spectra of gustiness in *u*-component (along flight direction).

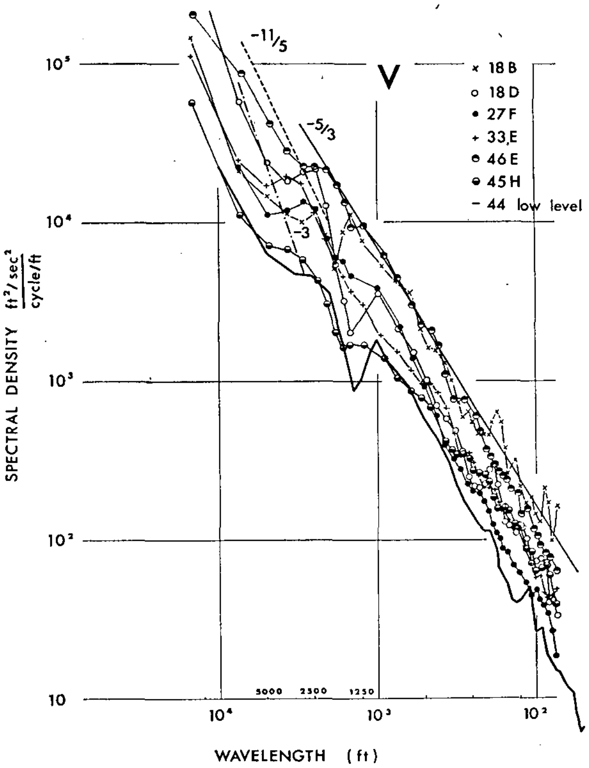

FIG. 3. Same as Fig. 2, except *v*-component (across flight direction).

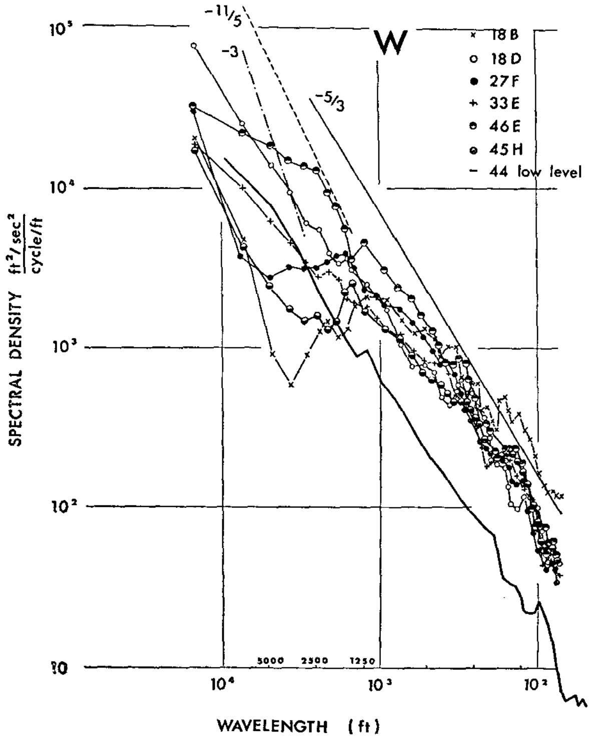

Fig. 4. Same as Fig. 2, except *w*-component.

exception, however is the spectrum obtained at 300 ft pressure altitude (Run 44). At wavelengths larger than ~2000 ft, individual spectra of the vertical velocity component vary considerably for individual runs. The *u* and *v* spectra also show some dispersion for wave lengths >2000 ft, although to a much less extent than is evident in the *w*-component.

Comparisons with the "$-5/3$" line show that *u*- and *v*-spectra seem to fit this theoretical value of turbulent energy distribution in the inertial subrange remarkably well. The fit, again, is best for wavelengths >2000 ft. On the average, the *w*-spectra for high-level CAT seem to be better approximated by a slope of $-\frac{4}{3}$, especially for wavelengths between approximately 200 and 1000 ft.

The most conspicuous feature in these *w*-spectra seems to be a "hump" at wavelength ≅2000 ft. Several spectra actually show a reversal of slope in the wavelength range adjacent to, and larger than, 2000 ft. Such an irregularity is only weakly expressed in the *u*-spectra, and moderately well discernible in the *v*-spectra.

The significance of this "hump" may better be judged from Fig. 5, which contains the *u*- and *w*-spectra for Flights No. 18B and 18D in a coordinate system with $\log k$ as abscissa and $kE(k)$ as ordinate. Transformation of spectral curves into this coordinate system is energy conserving, since

$$\int_{k_1}^{k_2} kE(k)d(\ln k)=\int_{k_1}^{k_2} E(k)dk. \tag{3}$$

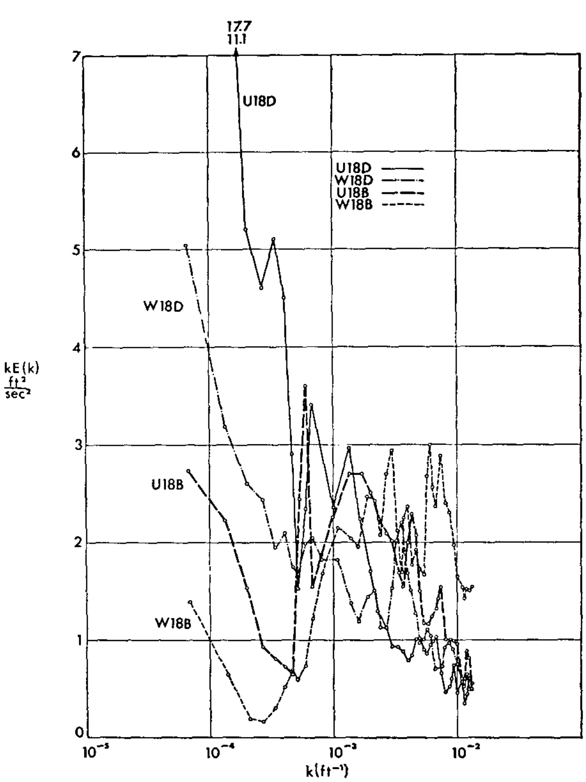

Fig. 5. Spectra of the *u*- and *w*-components of turbulence measured during flights 18 B and 18 D.

Hence, the area under any portion of the spectrum curve represents the turbulent energy between the wave numbers k_1 and k_2 defining the segment of the curve.

During Flight 18B (conducted nearly normal to the wind direction), a minimum in turbulent energy in the *u*- and *w*-components was encountered at wavelengths of approximately 2000 to 5000 ft. A significantly larger amount of energy is contained in the flow at shorter wavelengths.

The spectrum curves of Flight 18D (conducted under head-wind conditions) do not show such a broad spectrum region of excessively large turbulent energy.

In Figs. 2 to 4 the reference lines of $-5/3$ slope have been entered at the same energy levels. The position of the *u*-, *v*- and *w*-spectra relative to this line are approximately the same to the right of the "hump" described above, i.e., for wavelengths <2000 ft. This suggests that the turbulence in this range is nearly isotropic. Hinze (1959, p. 167) and Pasquill (1962, p. 7) have shown that in isotropic turbulence the spectra of *v* and *w* should exceed those of *u* by a factor of $\frac{4}{3}$. Thus

$$E_2(k_1)=\frac{1}{2}\left[E_1(k_1)-k_1\frac{\partial E_1}{\partial k_1}(k_1)\right], \tag{4}$$

where $E_1(k_1)$ and $E_2(k_1)$ are the energy-spectrum functions of the longitudinal and traverse velocity fluctuations, respectively. We may now substitute from (1) for E_1 in the inertial subrange to obtain

$$E_2(k_1)=\tfrac{4}{3}E_1(k_1). \qquad (5)$$

Fig. 6 shows "smoothed" representations of the spectra measured during Flights 18B and 18D. During the headwind flight (18D) condition (5) is well fulfilled for wavelengths <300 ft. The v- and w-components of Flight 18B show an excess of turbulent energy over the u-component at small wavelengths larger than the one predicted by Eq. (5).

Table 3 contains an evaluation of the ratio E_v/E_u and E_w/E_u evaluated at $\lambda=200$ ft. Condition (5) on the average seems to be better fulfilled for the w- than for the v-component. If turbulence were indeed isotropic at $\lambda=200$ ft, these data would suggest that the absolute magnitude of $E_u(k)$ has been slightly underestimated.

For wavelengths >2000 ft the w-components show significant departures in turbulent energy from those of the u- and v-components, suggesting anisotropy of turbulence in this range. The inertial subrange, therefore, seems to be confined to waves <2000 ft (Burns and Rider, 1965), possibly to $\lambda<300$ ft (Fig. 6). The larger one of these two limits is in excellent agreement with Shur's (1962) findings, who also suggests 600 m as the upper border of the inertial subrange.

3. Interpretation of data

Shur (1962) reports on a consistent irregularity which he found in CAT power spectra. For wavelengths <600 m the spectra were well approximated by an exponent of $-1.7(\cong -5/3)$. For wavelengths larger than 700 to 800 m the exponents ranged between -3 and -3.2.

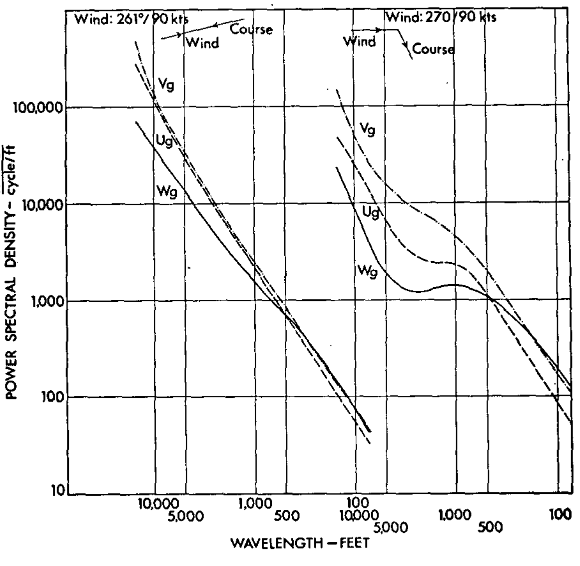

FIG. 6. "Smoothed" spectra of u-, v-, and w-components of turbulence measured during flights 18B and 18D (after Burns and Rider, 1965).

TABLE 3. Ratio of turbulent energies at $\lambda=200$ ft.

Flight no.	$\frac{E_v}{E_u}$	$\frac{E_w}{E_u}$
18 B	1.88	1.80
18 D	1.34	1.35
27 F	0.92	1.42
33 E	1.09	1.30
45 H	0.67	0.85
46 E	0.93	0.82
44 (low level)	1.02	0.71

Shur attributes the steeper slope of the spectral curve at long wavelengths to an additional dissipation effect, caused by negative buoyancy forces in a stable environment. His argument is that turbulent energy will not only be dissipated at the rate ϵ, determined by the transfer of turbulent energy to smaller and smaller eddies, but also by work against Archimedean forces. Turbulent energy at wavelengths smaller than the ones of the source range—not yet established by Shur's measurements—will therefore decay more rapidly than indicated by the $-5/3$ proportionality to k.

Theoretical treatment of this "buoyant subrange" in the case of stable stratification has been offered by Bolgiano (1959, 1962) (see also Lumley and Panofsky, 1964) who arrives at the proportionality

$$E(k)\propto k^{-11/5}. \qquad (6)$$

Isotropy is not expected to prevail in this range of the turbulent spectrum.

Although Bolgiano's and Shur's exponents of k in the "buoyant subrange" are not in agreement with each other, they both suggest a steeper slope of the spectrum curves at wavelengths larger than those of the inertial subrange. For comparison, both -3 and $-11/5$ slopes have been entered into Figs. 2 to 4. In the w-components (Fig. 4) the long wave portions of the spectra to the left of the "hump" seem to have a preference for a "-3 slope" only in Run No. 27F. Runs No. 18B and 45H seem to line up with a $-11/5$ slope in this region. Not too much significance should be attached to this statement, however, since there are too few data points available to establish the slope with confidence. The other runs actually seem to show a decrease of the slope of the spectrum curve in the w-component of long wavelengths. The spectra of the v-component seem to indicate slopes significantly steeper than $-5/3$ at long wavelengths, whereas the u-spectra follow the $-5/3$ slope, rather than a -3 or $-11/5$ slope. The absence of a clearly defined "buoyant subrange" should not be surprising, since its theoretical derivation takes into account the turbulence attenuating buoyancy forces only, and not the generating forces of vertical wind shear. This conclusion was formulated during discussions at the International Colloquium in Moscow.

It has been mentioned that some of the w-spectra show a significant "hump" near a wavelength of 2000 ft. For longer waves the *w*-component shows a marked deficiency in turbulent energy, suggesting that Archimedean forces in a stable environment inhibit the development of large eddy motions in a vertical plane. Even though the *w*-spectra reveal the "hump" most clearly, there is some indication in the *u*- and *v*-spectra as well of either a reduced slope in the spectrum curves at long waves or of a slight reduction in spectrum density as compared with an extrapolation of $E(k) \propto k^{-5/3}$ from the inertial subrange (see, for instance, Figs. 5 and 6).

This leads to the conclusion that there has been a source of turbulent energy at wavelengths between approximately 300 to 5000 ft in several of the encountered CAT cases.

At least for those runs for which the "hump" is sharply defined (see, for instance, the *u*-, *v*-, and *w*-components of Run 18B) one may argue that the energy sources lie in a relatively well defined wave motion. This is in view of the fact that the research aircraft was deliberately dispatched into thermally stable regions with vertical (directional) wind shear. One would exclude, therefore, (random) convective motions with a positive contribution of energy from buoyancy forces, as likely energy sources. It seems, rather, that stability and wind shears together would cause wave motion similar to gravity waves along a stable interface. As has already been shown by Helmholtz (1888, 1889, 1890) such waves become unstable and amplify exponentially if they are shorter than a critical wavelength which depends on wind shear and temperature gradient across the interface (see Haurwitz, 1941; Reiter, 1961, 1963b, c). Critical wavelengths of 300 to 4000 ft are entirely within the range of possibilities offered by observed atmospheric structure. Gravity waves of this wavelength range may be observed relatively frequently in cirrus, especially near the jet stream (Reiter and Hayman, 1962; Reiter and Nania, 1964).

Runs 18D, 33E, and 46E do not show a clearly defined "hump" in the *w*-spectra. These three runs were made under almost straight head-wind conditions.[2] The obvious dependence of the shape of the spectrum curves on the angle between course and wind direction may be illustrated by the examples of Runs 18B and D which were obtained from the same turbulence patch. Run D was measured on a course heading straight into the wind, Run B on a course 65 degrees across the wind. No "hump" was observed in D, a pronounced "hump," however, was present in B. This dependence of spectral density on flight direction supports the conclusion that a wave phenomenon of specific orientation with respect to the mean-wind direction was responsible for the observed energy input at wavelengths corresponding to the "hump."

4. Conclusions

Project TOPCAT measurements of CAT power spectra over Australia suggest the following model of CAT formation which, in essence, confirms earlier hypotheses (Reiter, 1960, 1963b, 1964e).

In stable layers of the upper troposphere and stratosphere, which contain sufficient vertical wind shear (measured in terms of *vector* wind shear), long wave perturbations tend to be anisotropic, showing a significant suppression of vertical perturbation components of motion because of the stabilizing action of negative buoyancy forces.

CAT may be expected in such a stable environment, if critical wavelengths (below which the flow becomes unstable) are still above the ones equivalent to the CAT response frequencies by the aircraft. In the region of this critical wavelength a significant amount of turbulent energy is made available to the flow through the vertical shearing stresses which counteract the effects of thermal stability. This may be expressed by the stability criterion derived from gravitational shearing waves,

$$R = \Delta\rho - \frac{k\bar{\rho}}{2g}(\Delta\bar{u})^2 \begin{matrix} <0 \text{ unstable} \\ >0 \text{ stable} \end{matrix}, \tag{7}$$

which bears similarity to Richardson's criterion (Reiter, 1961, 1963c). $\Delta\rho$ is the density difference, $\Delta\bar{u}$ the wind shear across the interface, and k the wave number. As the latter increases, instability may be reached. Effects of shear on the formation of CAT through Richardson's criterion have recently been evaluated by Panofsky and McLean (1964).

At wavelengths shorter than the critical one the flow breaks down into isotropic turbulence. The CAT "bumps" themselves seem to be contained within this inertial subrange, although their cause should be sought in the energy initially released in a gravity wave-like phenomenon. The relatively high frequency of CAT observations over mountains and hills (Clodman *et al.*, 1960) bears this out, too.

The apparent longevity of some CAT patches over Australia (see Table 2, case of 12/9/63) suggests a rather stable mesostructure of the atmosphere to be present at times, which continuously supplies energy to the shorter waves in the inertial subrange. It may be of interest to note that Run 33E (on the same date) shows a "hump" only in the *v*-spectrum. The *w*-spectrum suggests an energy supply acting rather continuously at waves >400 ft.

Although valuable evidence corroborating the above conclusions has been collected during Project TOPCAT, the lack of exact data on vertical temperature gradients and wind shears in the CAT regions still is deplorable. The authors share the opinion expressed by Shur (1962) that careful measurements of supporting meteorological parameters are urgently needed.

Furthermore, it would be of great value, if the

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[2] Directions in Run 46E, made only a few hours after Run 45H, are considered with respect to directions of the strongest of the two winds measured across a shearing layer.

"smooth" regions surrounding a CAT patch were surveyed carefully, and power spectra were obtained from such less "exciting" measurement runs as well. It would be most interesting to investigate, for instance, whether the long-wave parts of the spectra remain the same inside and outside a CAT region so that the energy increment supplied by the gravity-type wave formation would have to be held entirely responsible for CAT observed in a thermally stable environment. If this were the case, interface regions that might possibly harbour such wave formation, could be explored by various sensing techniques, such as backscatter of electromagnetic waves.

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REFERENCES

Batchelor, G. K., 1959: *The Theory of Homogeneous Turbulence.* Cambridge, University Press, 197 pp.

Bolgiano, R., Jr., 1959: Turbulent spectra in a stably stratified atmosphere. *J. Geophys. Res.*, **64**, 2226.

——, 1962: Structure of turbulence in stratified media. *J. Geophys. Res.*, **67**, 3015–3028.

Burns, A., and C. K. Rider, 1965: Project TOPCAT power spectral measurements of clear air turbulence associated with jet streams. Royal Aircraft Establishment. Technical Rep. No. 65210, 11 pp.

Clodman, J., G. M. Morgan and J. T. Ball, 1960: High level turbulence. New York University, Research Division. Final Report, Contract No. AF 19(604)-5208, 84 pp.

Endlich, R. M., and G. S. McLean, 1964: Studies of the climatology of winds, temperature and turbulence in jet streams. Stanford Research Institute. Final Report, AFCRL-64-834.

Haurwitz, B., 1941: *Dynamic Meteorology.* New York, McGraw-Hill, 365 pp.

Heisenberg, W., 1948: On the theory of statistical and isotropic turbulence. *Proc. Royal Soc.*, A., **195**, 402.

Helmholtz, H. von, 1888, 1889: Über atmosphärische Bewegungen, I. und II. Sitz. -Ber. Akad. Wiss. Berlin.

——, 1890: Die Energie der Wogen und des Windes. Sitz. -Ber. Akad. Wiss. Berlin.

Hildreth, W. W., Jr., *et al.*, 1963: High altitude clear-air turbulence. Lockheed-California Co., Technical Documentary Report No. ASD-TDR-63-440.

Hinze, J. O., 1959: *Turbulence.* New York, McGraw-Hill, 586 pp.

Kolmogorov, A. N., 1941: The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. *Comptes Rendus* (*Doklady*), Acad. Sci. U.R.S.S., **30**, 301–305.

Lumley, J. L., and H. A. Panofsky, 1964: *The Structure of Atmospheric Turbulence.* Interscience Publishers, 239 pp.

MacCready, P. B., Jr., 1962: Turbulence measurements by sailplane. *J. Geophys. Res.*, **67**, 1041–1050.

——, 1964: Standardization of gustiness values from aircraft. *J. Appl. Meteor.*, **3**, 439–449.

Mizon, E. A., 1964: Forecasting aspects of the first phase of the TOPCAT trials. Project TOPCAT Meteorological Reports, University of Melbourne, Meteorology Department.

Obukhov, A. M., 1941: On the spectral energy distribution of a turbulent flow. *Akademiya nauk, USSR. Izvestiya. Ser. geograf. i geofiz.*, No. 5.

Panofsky, H. A., and J. C. McLean, Jr., 1964: Physical mechanism of clear-air turbulence. Research Report to U. S. Weather Bureau, Department of Meteorology, Pennsylvania State University.

Pasquill, F., 1962: *Atmospheric Diffusion.* London, Van Nostrand, 297 pp.

Pchelko, I. G., 1962: Aero synoptic conditions of airplane bumpiness. Gidromet. Moscow.

Pinus, N. S., and J. M. Shmeter, 1962: Atmospheric turbulence affecting aircraft bumping. Gidromet, Moscow.

Radok, U., 1964: Preface. Project TOPCAT, Meteorological Reports, University of Melbourne, Meteorology Department.

Reiter, E. R., 1960: Turbulenz im wolkenfreien Raum. (Clear-Air Turbulence). *Berichte des Deutschen Wetterdienstes,* No. 61, 42 pp.

——, 1961: *Meteorologie der Strahlströme* (Jet Streams). Vienna, Springer-Verlag, 473 pp.

——, 1962: On the nature of clear-air turbulence. *Aerospace Engineering*, **21**, 39–46.

——, 1963a: A case study of severe clear-air turbulence. *Archiv. Meteorol., Geophysik., Bioklimatol.*, Ser. A., **13**, 379–389.

——, 1963b: Nature and observation of high-level turbulence especially in clear air. Colorado State University, Atmospheric Science Tech. Paper No. 41, and U. S. Navy Weather Research Facility, Report No. NWRF 15-1262-071.

——, 1963c: *Jet-Stream Meteorology.* Chicago, University of Chicago Press, 515 pp.

——, 1964a: CAT and SCAT. *Astronautics and Aeronautics*, **2**, 60–65.

——, 1964b: Jet streams and turbulence. *Australian Meteor. Mag.*, No. 45, 13–33.

——, 1964c: Nature and observation of high-level turbulence especially in clear air. *J. of Aircraft*, **1**, 94–96.

——, 1964d: Progress in clear-air turbulence research and forecasting. American Institute of Aeronautics and Astronautics, Paper No. 64–311, given at Annual Meeting, Washington, D. C., 29 June–2 July.

——, 1964e: Clear-air turbulence models and forecasting for Project TOPCAT, second phase. Project TOPCAT, Meteorological Reports, University of Melbourne, Meteorology Department.

——, and Hayman, 1962: The nature of clear-air turbulence. Colorado State University, Atmospheric Science Tech. Report No. 28.

——, and A. Nania, 1964: Jet-stream structure and clear-air turbulence. *J. Appl. Meteor.*, **3**, 247–260.

Rhyne, R. H., and R. Steiner, 1962: Turbulence and precipitation problems associated with operation of supersonic transports. Paper presented at 4th Conference on Applied Meteorology, Hampton, Va., 10–14 September 1962.

Shur, G. N., 1962: Experimental investigations of the energy spectrum of atmospheric turbulence. Tsentral'naya aerologicheskaya observatoriya. *Trudy*, No. 43, 79–90.

Spillane, K. T., 1964: A survey of the subtropical jet stream and clear-air turbulence models. Project TOPCAT, Meteorological Reports, University of Melbourne, Meteorology Department.