Computations of Rain Formation by Coalescence

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(Manuscript received 28 July 1965)

ABSTRACT

Numerical integrations were made of the statistical equations describing the evolution of droplet distributions by coalescence. The results confirmed that increasing the droplet concentration, for the same liquid water content, greatly adds to the difficulty attending rain formation by coalescence. The spread of the initial distribution was not, on the other hand, an important parameter.

The computed times for rain formation were less than a half-hour when the cloud contained 50 droplets cm^{-3} and 1 gm m^{-3} liquid water. When compared with observations these times are a little too long, but rain which forms more quickly can be explained by more rapid growth in regions of higher liquid water content.

1. Introduction

To a considerable extent the larger scale atmospheric processes which cause clouds to form also determine their properties. However, some of the colloidal properties of the cloud are determined by microphysical parameters; among these is droplet concentration. It is, therefore, of considerable importance to know how much this microphysical parameter affects the cloud evolution and, particularly, the process of rain formation.

In a previous paper (Twomey, 1959) the writer pointed out that a cloud of small droplets, being collected less efficiently by a larger droplet falling through, was lower in "collectable water" and thereby found it much more difficult to produce rain. For the same liquid water content the average droplet size decreases with increase of droplet number, and so high droplet concentrations lead to less "collectable water." In addition, high droplet concentration means that a droplet must start its growth from a smaller initial size where droplet area, falling velocity and collection efficiency are all small. The difficulty of growth in this size region further retards the process of warm rain formation.

The primary purpose of the present paper is to put these arguments on quantitative grounds and to demonstrate how much effect the initial cloud drop distribution has. The collection efficiency data of Hocking (1959) were used up to 30 μ radius and those of Shafrir and Neiburger (1963) at larger sizes. But in order to be sure that the results obtained were not critically dependent on the collection efficiencies selected, the computations were carried out also with a constant collection efficiency ($E=1$) at all sizes. It will be shown that the choice of collection efficiency was not crucial.

In another paper (Twomey, 1964) the importance of Telford's (1955) statistical effect in the coagulation of a continuous distribution was discussed. The statistical equations were derived in that paper and the difference between statistical growth and the "continuous growth" approximation discussed.[1] The present paper is in a sense a continuation of the previous paper and the derivation and discussion of the latter will not be repeated here.

2. Computation of droplet evolution

The process of evolution of rain can conveniently be divided into 1) the condensation phase, during which droplet concentrations are determined and an initial spectrum of cloud droplets is formed, 2) the statistical phase, when a very small number of droplets (10–100 per cubic meter) grow very much faster than average and thereby attain rather quickly radii several times larger than the mean radius, and 3) the continuous growth phase, during which these few large droplets grow into raindrops by coalescence with the main part of the cloud droplets (which have as yet been affected negligibly by coalescence).

The distinction between phases 2) and 3) is not a physical one, for phase 2) really continues until the few favored drops attain raindrop size. However, once droplets have grown well outside the main portion of the spectrum, their subsequent growth can be described to a close approximation by the much simpler "continuous growth" equation in which fluctuations are neglected and averages only employed. Without this simplification, continuation of the numerical computations until raindrops are present in significant numbers would be excessively time-consuming.

[1] In the previous paper (Twomey, 1964), the equations were given incorrectly, the right side of the various equations being twice their correct value. However, the machine computations reported in that paper were carried out correctly, so that the results described therein do not need correction by a factor of 2.

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It is, of course, possible to commence a numerical computation before the first (condensation) phase, e.g., by assuming an initial distribution of nuclei, and modeling the condensation process numerically. However, when such computations are carried out, they lead to cloud droplet distributions which are much narrower than are ever observed. It must be concluded that some mechanisms are at work in nature to produce an initial widening of the droplet spectrum (condensation alone acts to narrow the size distribution). It did not, therefore, seem realistic to include the condensation phase in the computations; what was done was to start with an initial droplet spectrum. Since it is neither practical nor realistic to start with a homogeneous group of droplets, some dispersion of the initial droplet distribution had to be adopted and some initial shape had to be assumed. Unpublished studies of Squires and the writer showed that the coefficient of dispersion $\sigma/\bar{r}$ (standard deviation of the radii σ divided by mean radius $\bar{r}$) in natural cloud samples ranged from about 0.15 to 0.50 with most values between 0.15 and 0.30 and none below 0.15. In clouds freshly formed in expansion cloud chambers, Pollak and Metnieks (1959) found that over a wide range of droplet concentration and mean size, the coefficient of dispersion was almost constant, ranging only between 0.14 and 0.18, while the droplet concentration ranged from 858 cm^{-3} to 42,400 cm^{-3}. Thus there appears to be considerable evidence that as clouds are formed they acquire an initial dispersion of this approximate magnitude. Even if no physical explanation is available for the apparent constancy of the ratio $\sigma/\bar{r}$, its inclusion seems the most realistic procedure.

The values 0.15, 0.25 and 0.50 were therefore chosen for the coefficient of dispersion and the values 50, 200 and 800 cm^{-3} for the droplet concentration giving nine initial distributions in all; liquid water content was taken at 1 gm m^{-3}. The initial distributions were Gaussian, but they were truncated when the ordinate fell below 0.05 cm^{-3} μ^{-1}.

3. Evolution of the spectrum, statistical phase

The equations used for the statistical phase can be derived by following the variations of the number of droplets in a specified size interval. They can be written in terms either of volume or of radius but are somewhat simpler in form if volume is chosen. The equation for the variation with time of the number of droplets $n(v)\Delta v$ with volume between v and $v+\Delta v$ is

$$\frac{d}{dt}n(v)=-n(v)\int_0^\infty K(v,u)n(u)du+\frac{1}{2}\int_0^v K(u,v-u)n(u)n(v-u)du, \quad (1)$$

if $K(v,u)$, the coagulation coefficient, gives the number of droplets of volume v coagulating with droplets of volume u when both species are present in unit concentration.

The computation for the statistical phase was a direct numerical application of Eq. (1) in which the spectrum was tabulated at up to a hundred points and $K(v,u)$ up to 100×100 points, the integrations in (1) being replaced by quadratures on this array of points. Thus (1) was replaced by a matrix approximation.

4. Evolution of the spectrum, continuous phase

During the statistical phase a long tail of very small height is evolved in the size spectrum, i.e., a few of statistically fortunate droplets grow much more quickly than their fellows. Once this process has provided a significant number (100–1000 per cubic meter in this context) of droplets several times larger than the majority, their further evolution can be accounted for quite accurately by the familiar "continuous growth" equation which considers the growth of droplets as a continuous process in which all droplets of volume v grow at a uniform, continuous rate. The volume collected in unit time by an average droplet of volume v is obviously $\int_0^v K(v,u)un(u)du$, hence the average droplet grows in volume at that rate. Larger droplets, however, are depleting the ranks by coalescence at the rate $n(v)\int_v^\infty K(u,v)n(u)du$. Therefore, the number of droplets with volume v varies in the (approximate) continuous growth model according to

$$\frac{d}{dt}n(v)=-n(v)\int_v^\infty K(u,v)n(u)du-\left[\frac{\partial n(v)}{\partial v}\right]_t\int_0^v K(u,v)un(u)du. \quad (2)$$

This (incorrect) formulation becomes approximately correct only when v is very much larger than average, i.e., if the integrations can be terminated at some point m without serious loss in accuracy and if v is much larger than m. For these conditions one may write

$$\int_m^{m+M} K(u,v)n(u)du<\epsilon\ll\int_0^m K(u,v)n(u)du,$$

which permits the approximation (for $v\gg m$) of

$$\frac{1}{2}\int_0^v K(u,v-u)n(u)n(v-u)du \cong\int_0^m K(u,v-u)n(u)n(v-u)du,$$

which is intuitively obvious and can be derived rigorously by applying the Schwartz inequality. Thus when

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$v \gg m$, (1) becomes

$$\frac{d}{dt}n(v) \cong -n(v)\int_v^\infty K(u,v)n(u)du$$

$$+\int_0^m [K(u, v-u)n(u)n(v-u)-K(u,v)n(u)n(v)]du,$$

which reduces to (2) if the Taylor expansion of $K(u,v-u)n(v-u)$ is truncated after the two terms $K(u,v)n(v)-uK(u,v)[\partial n(v)/\partial v]$. Thus for smooth enough kernals and large values of v, (1) may be replaced by (2) and indeed by

$$\frac{d}{dt}n(v) = -\left[\frac{\partial n(v)}{\partial v}\right]_t \int_0^v K(u,v)un(u)du, \qquad (3)$$

since the first term in (2) is also negligible for large v.

5. Numerical procedure

The numerical computation of cloud evolution from given initial distributions and with collection efficiencies specified for various combinations of radii in the form $E(r_i,r_j)$, $i=1, 2\cdots N$; $j=1, 2\cdots N$, consisted of:

1) Computation of the coagulation coefficient for droplets of radii r_i coagulating with droplets of radii r_j. The coagulation coefficient K_{ij} is given by the expression

$$K_{ij}=\pi r_i^2[U(r_i)-U(r_j)]E(r_i,r_j), \quad r_i>r_j,$$

if $U(\rho)$ is the falling velocity of a droplet of radius ρ. For $\rho<40\,\mu$, Stokes law was used to obtain $U(\rho)$ and when $\rho\geq 40\,\mu$, Gunn and Kinzer's (1949) data were applied.

2) The droplet distribution description by a N-dimensional vector representing the array $n(r_1)$, $n(r_2)\cdots n(r_N)$ where integration from $r=0$ to $r=r_m$ is replaced by a quadrature so that

$$\int_0^{r_m} K(r_i,r_j)f(r_j)dr_j \cong \sum_{j=1}^{m} k_{ij}f(r_j).$$

3) Proceeding in small steps in time with the quadrature approximation to (1) being used to integrate Eq. (1) by a straightforward predictor-corrector procedure.

4) Termination of the integration of (1) when the distribution had evolved to the stage where there were 100 drops m^{-3} above 50 μ radius; for growth from 50 to 1000 μ the approximate continuous growth Eq. (3) was employed.

The tabular points $r_j (j=1, 2\cdots N)$ were spaced 1 μ apart up to 20 μ; thereafter the spacing was increased as r_j increased, being 2 μ in the interval 30–40 μ and 10 μ above 100 μ, for example. N was usually 80, occasionally 100.

6. Preliminary tests

To test the adequacy and consistency quite a number of preliminary tests were run. A number of these will be briefly mentioned here.

a) *Time intervals.* The computations are quite time-consuming. Even with a high speed computer it was necessary to use initial time steps of the order of 3–10 sec and to lengthen the time step gradually as the calculation proceeded, so that the final time step might be 1 min after following an hour's cloud growth.

The sufficiency of the time steps was tested by halving the time steps and repeating one of a set of computations. The agreement between the data obtained in both ways was always extremely close (Table 1 shows the results of one such test). One could, therefore, be satisfied that the time steps were not excessive.

b) *Effect of initial spectral width.* One of the primary purposes of the investigation was to examine the effect of droplet concentration on rain formation. Since the initial distributions were postulated, in shape and dispersion, it was important to be sure that the choice of initial shape or dispersion was not critical. A series of tests showed the effect of shape to be negligible when the total number, total mass and dispersion were held constant; indeed, even the effect of varying the dispersion (i.e., the width of the original distribution) was surprisingly slight.

An example is shown in Fig. 1, which illustrates the evolution with time of three cloud models differing only in dispersion. Each cloud contained 1 gm m^{-3} liquid water and 50 droplets cm^{-3}, but the coefficients of dispersion $\sigma/\bar{r}$ were, respectively, 0.15, 0.25 and 0.50. The portion of the spectrum which was initially devoid of

Table 1. Comparison of two numerical integrations of the coagulation equations with different time steps and the same initial distribution.

Radius (μ)	Droplets (cm^{-3} μ^{-1}) Time=2228 sec Steps: 70	Droplets (cm^{-3} μ^{-1}) Time=2209 sec Steps: 45
0	0	0
1	25.97	25.97
2	32.98	32.98
3	49.34	49.34
4	64.13	64.15
5	142.45	142.49
6	150.12	150.15
7	120.64	120.51
8	79.21	79.25
9	41.71	41.70
10	19.04	19.03
11	8.26	8.25
12	3.36	3.36
13	1.28	1.28
14	0.446	0.445
15	0.144	0.143
16	0.0414	0.0412
17	0.0109	0.0109
18	0.00259	0.00258
19	0.000548	0.000546
20	0.000106	0.000105
21.25	0.000013	0.000012
22.5	0.000002	0.000002

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droplets is seen to have become populated in very much the same way, irrespective of the width of the initial distribution.

This was an unexpected and quite important result, for intuitively one is inclined to suppose that a wider drop distribution would be markedly more effective with respect to coagulation. However, it must be borne in mind that a considerable number of coalescences are needed to make a droplet several times the average radius so that a droplet which attains that size is likely to have coalesced with larger than average droplets about as frequently as with droplets smaller than average. The production of such large drops, therefore, tends to be predominately controlled by average size and number, provided the distribution is not initially so narrow that the coagulation coefficient is restricted to excessively small values. The narrowest distribution ($\sigma/\bar{r}=0.15$) still gives a ratio of the order of 0.85/1.15 ≈ 0.75 between typical smaller than average and larger than average droplets. Thus, the velocity differences between the coagulating droplets will be of the order of half the terminal velocity of the larger with the collection efficiencies being quite close to the maximum for a radius ratio of 0.75.

The conclusion that varying the spectrum width did not greatly alter the production of large drops (in the "tail" of the distribution) did not, of course, hold if the collection efficiencies were identically zero for almost all drops in the narrow spectrum, while drops in the wider spectrum reached a size such that coalescence could occur.

In view of these results, further discussion will be limited to three cloud models:

I) Cloud with 50 droplets cm^{-3}, $\sigma/\bar{r}=0.15$.
II) Cloud with 200 droplets cm^{-3}, $\sigma/\bar{r}=0.15$.

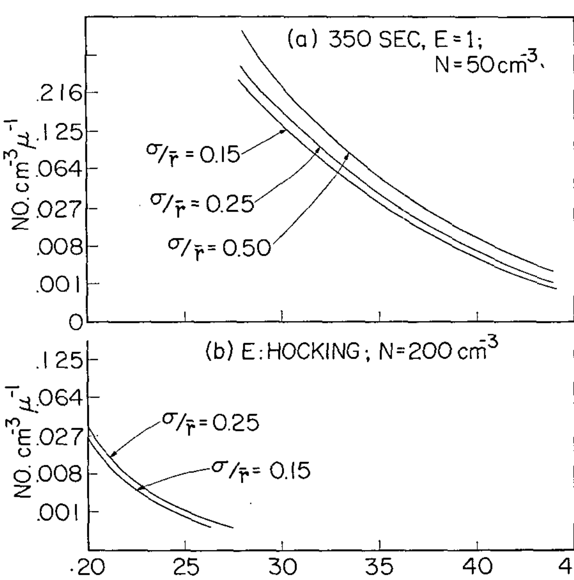

FIG. 1. Examples of evolution from initial spectra differing only in the width of the spectrum. σ is the standard deviation of drop radius, $\bar{r}$ the mean radius.

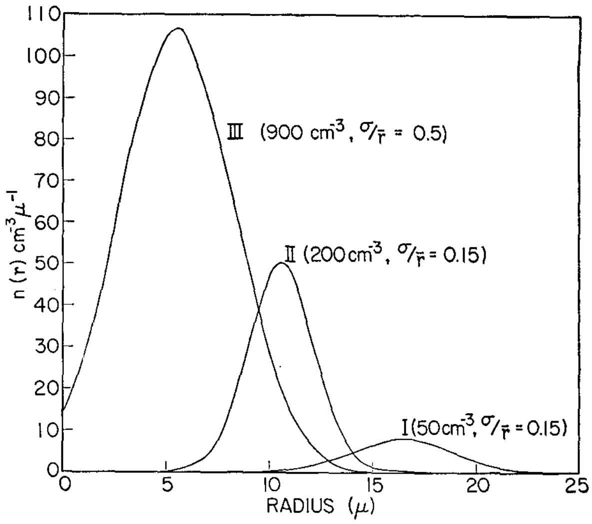

FIG. 2. Initial distributions in cloud models I, II and III.[2]

When Hocking's collection efficiency was used, coalescence was made to occur in this cloud model by continuing the spectrum out to 20 μ with ordinate 0.04 cm^{-3} μ^{-1} and dropping it linearly to zero at 21 μ.

III) Cloud with 800 droplets cm^{-3}, $\sigma/\bar{r}=0.50$ and extended to 21 μ in the same way as II.

These three initial distributions are shown in Fig. 2.

7. Results

a) *Effect of droplet number on cloud coagulation.* The raw results of the computations were a large number of tabulations of the distribution $n(r)$ at various times. These tabulations could be plotted as distributions or as cumulative numbers. Fig. 3 shows a set of such curves for cloud model II; the curves for other initial distributions were all similar to this in that they were close to, but not quite, straight lines on this scale, indicating that in the tail portion the distributions might be described by a relationship of the form $n(r,t) \approx a(t)e^{-rb(t)}$. In any case the presentation of the data is greatly simplified and little is obscured if the size corresponding to a fixed ordinate value is plotted against time, e.g., if one examines the growth of the 100 largest drops in each cubic meter (more strictly, the increase with time of the smallest of these 100 largest drops). In this way sets of graphs such as are shown in Fig. 3 can be reduced to single curves and several of these can be included in one figure. The value of 100 m^{-3} (0.0001 cm^{-3}) was used because this is roughly the number of drops in moderate rain.

Figs. 4 and 5 show how the 100 m^{-3} largest drops evolved in the three cloud models They show quite conclusively the very great increase in coalescence growth which results from increasing the initial average droplet size, even though droplet concentration is

[2] Model III in Fig. 2 should show 800 droplets cm^{-3}.

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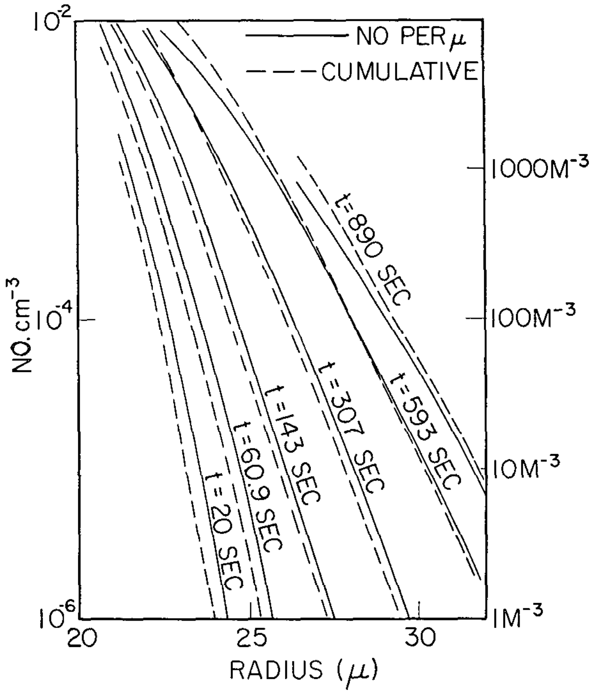

Fig. 3. The evolution with time of the droplet distribution in cloud model II, with Hocking's collection efficiencies. Only the tails of the distributions are shown.

reduced so as to maintain liquid water constant. Fig. 4 was computed for constant collection efficiency ($E=1$) regardless of size, while Fig. 5 was based on the Hocking (1959) and Shafrir and Neiburger (1963) collection efficiencies. Comparison of the two sets of curves shows that the differences in the rate of production of large drops in clouds with fewer larger droplets (maritime type) as compared with clouds with many

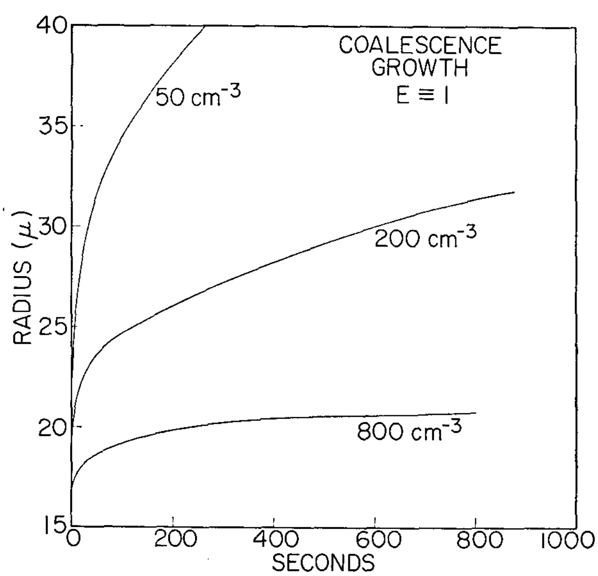

Fig. 4. The growth of the 100 largest droplets per cubic meter with constant (unit) collection efficiency.

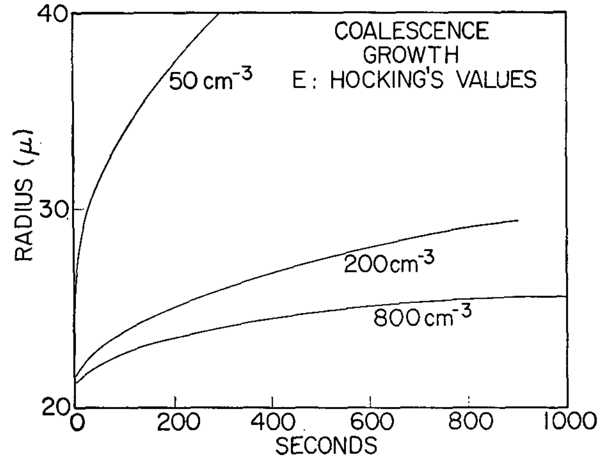

Fig. 5. The growth of the 100 largest droplets per cubic meter with Hocking's and Shafrir and Neiburger's collection efficiencies.

but smaller droplets (continental type) *is not a result of any particular assumptions about collection efficiency.* Specifically, it is enhanced by, rather than exclusively dependent upon, the decrease to zero of Hocking's collection efficiency when the collector drop is smaller than 18 μ radius. Small droplets coagulate poorly because their cross sections and falling velocities are small, apart altogether from the effect of collection efficiency.

The times taken for fine drizzle drops ($r=100\,\mu$) and moderate raindrops ($r=400\,\mu$) to be produced in numbers of 100 m^{-3} are tabulated in Table 2. The table shows very clearly the differences between the several cloud models. Cloud model I, with the fewest and largest droplets was the only one capable of producing rain by coalescence within a half-hour or so. It must be emphasized that in the case of II and III, droplet growth was deliberately facilitated by extending the spectrum to beyond 20 μ, and in the case of III, by adopting the improbably large value 0.5 for $\sigma/\bar{r}$. The above table therefore *underestimates* the increase in difficulty attending growth when droplet concentration is high.

If three strictly comparable initial spectra are compared, using unit collection efficiency, the results of Table 3 ensue. (Similar results for Hocking's collection efficiencies are shown in parentheses, but they depend critically on the cut-off near 17 μ.) One can therefore conclude that coalescence rain formation is at best extremely slow when high droplet concentrations are present.

Although the present paper is concerned with warm rain, it is worth pointing out that coalescence is needed to produce anything much more than drizzle even when the Bergeron process operates. Inspection of the Shafrir and Neiburger (1963) data shows that collection efficiency drops off rapidly when the ratio of the droplet radii falls below about 0.05. Hence, even if the Bergeron mechanism circumvents the slow growth through the

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TABLE 2. Production time of drizzle and rain.

Cloud	Droplets (cm^{-3})	Time for 100 m^{-3} drizzle drops	Time for 100 m^{-3} raindrops
I	50	14 min	23 min
II	200	39 min	51 min
III	800	63 min	77 min

TABLE 3. Production time for drizzle and rain for Gaussian spectra, $\sigma/\bar{r}=0.15$ and unit collection efficiency.

No. droplets (cm^{-3})	Time for 100 m^{-3} drizzle drops	Time for 100 m^{-3} raindrops
50	15 min (13 min)	27 min (24 min)
200	25 min (52 min)	39 min (65 min)
800	40 min (∞)	52 min (∞)

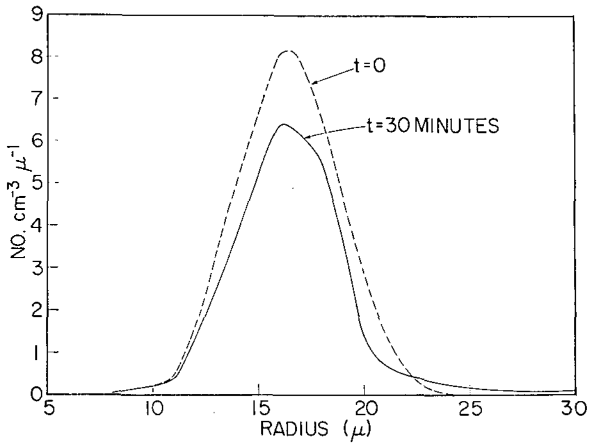

FIG. 6. The overall change in the initial spectrum after 30 min (cloud model I).

20–50 μ region, the growth of drops of radius 100 μ and up will be considerably faster in a cloud with 50 droplets cm^{-3} ($\bar{r}\approx 17\,\mu$ for 1 gm m^{-3} liquid water) than with 800 droplets cm^{-3} ($\bar{r}\approx 7\,\mu$ for 1 gm m^{-3}). Hence, droplet concentration is important even if a cloud extends well above the freezing level.

b) *Effect of coagulation on the droplet spectrum.* It is worth noting that the evolution of drizzle and rain droplets was found to take place without a collapse of the initial cloud spectrum. This is revealed in Fig. 6 which shows the extent to which the distribution in cloud I had been altered after 30 min. It is apparent that the initial distribution is not significantly altered, apart from a reduction in total number (from 50 cm^{-3} to 41 cm^{-3}) and the development of an extended tail. This point is important, because it means that the process of rain formation by coalescence is not one to which asymptotic solutions or "self-preserving" solutions can meaningfully be applied. These solutions are useful only when a process has evolved to a point where the distribution is dominated by the kinetics of the process and has "forgotten" the initial conditions. Golovin (1963), for example, has stated that "the choice of the initial distribution does not essentially influence the form of the spectrum for reasonably large times." While this may be mathematically true, it is certain that a time of 30–60 min is not a "reasonably large time" in this context and that after such times the initial form of the distribution is still very relevant. Asymptotic process-determined forms clearly cannot develop until each of the droplets present has undergone a number of coagulations. Since the coagulation coefficients of cloud droplets are typically of the order of 10^{-5} to 10^{-6} cm^3 sec^{-1}, the fraction of droplets undergoing a coagulation is perhaps 0.001 sec^{-1}. Under such conditions one would not expect an asymptotic form to begin to appear for at least many thousands of seconds.

It should again be repeated that the spread of the initial spectrum does not affect the conclusions. If the three spectral widths referred to earlier were retained, Figs. 4–5, for example, would contain nine curves, but these would be three sets of three and each set would lie in a narrow band about each of the curves in the present figures.

c) *Effect of variation of liquid water.* The data of Table 2 seem to account satisfactorily for the differences in behavior of clouds differing in droplet concentration but otherwise similar (i.e., maritime versus continental types). On the other hand, the times required for rain to form are somewhat long; in warm oceanic air, clouds are often seen to rain within 15 min or so of formation, and there are no grounds for believing that such clouds are too different from the cloud models discussed here.

Fig. 7 suggests a possible explanation. This figure shows the development of large droplets in three clouds

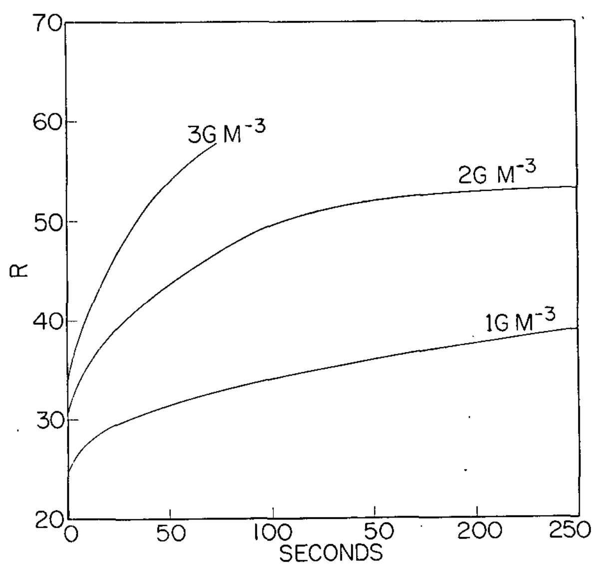

FIG. 7. The effect of varying the liquid water content showing the variation with time of R such that there are 100 m^{-3} droplets with radius $\geq R$.

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similar to model I but containing respectively 1, 2 and 3 gm m^{-3} of liquid water. It is seen that there is a dramatic increase in the production of large droplets at the higher liquid water content. Although an *average* liquid water content of 3 gm m^{-3} is quite improbable, *local* regions within a cloud may easily have 3 gm m^{-3} or more. If such regions persist even for a minute, a considerable number of large droplets will be evolved. The 100 m^{-3} largest droplets after 30 sec in a 3 gm m^{-3} region and thereafter growing in the average (1 gm m^{-3}) environment could produce drizzle in $6\frac{1}{2}$ min and large raindrops in just under 20 min. One can, therefore, account in this way for the observations, qualitatively at least. A fuller discussion must await better statistical information about the fluctuations in liquid water within clouds, or at least a fuller description of the number, size and lifetime of the wetter regions within clouds.

8. Conclusions

1) There are marked differences between the ease with which warm rain can be produced by clouds with relatively few larger droplets and by clouds with more (and proportionately smaller) droplets. Clouds with 50 droplets cm^{-3} were computed to produce rain in 20–30 min whereas clouds with 800 cm^{-3} required well over an hour even if the initial spectrum was extended to include a few large droplets.

2) The spread of the initial spectrum was only of secondary importance in determining how easily a cloud rained, but since the smallest coefficient of variation used was 0.15, it is not suggested that a sufficiently homogeneous cloud will not experience difficulty.

3) The computed times appear somewhat longer than observations suggest, but faster growth will occur in regions where the liquid water is high. It is likely that if the fluctuations in liquid water, as well as those in droplet coalescences, are taken into consideration, one can account quantitatively for observed rates of rain formation.

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