Quasi-Biennial Cycles in Angular Momentum Transports at 500 mb

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ABSTRACT

Geostrophic angular momentum transport at 500 mb has been computed as a function of latitude and zonal wave number for a 10-yr period. The record has been inspected for evidence of periodic variations at latitudes 17.5 N, 27.5 N, 42.5 N, and 57.5 N. A quasi-biennial cycle is most prominent in the case of the planetary waves, with wave numbers 1 and 3 out of phase with wave number 2. The cyclone waves display a phase shift with time that implies a varying scheme of interaction with the long waves.

Monthly average values of the 500-mb mean zonal wind, based on the same 10-yr record, do not indicate any strong periodicity.

1. Introduction

Recent diagnostic studies have shown that the biennial cycle is not confined to the tropical stratosphere. Angell and Korshover (1964), Landsberg (1962), Wallace and Newell (1966) and Labitzke (1965) have all presented evidence that the cyclic tendency extends to middle and high latitudes in both the troposphere and lower stratosphere (100–30 mb).

In view of the results of Newell (1963), Oort (1965) and Miller (1967) that indicate the lower stratosphere to be dynamically interrelated with the troposphere, and the suggestion by Newell (1964) that the quasi-biennial oscillation may therefore originate within the troposphere, it is of interest to consider the possibility of a quasi-biennial oscillation in the dynamics of the troposphere.

We present here the results of a study of the horizontal angular momentum transports at 500 mb for the Northern Hemisphere. The meaning and importance of this quantity in relation to the momentum and energy budget of the Northern Hemisphere have been demonstrated by Starr (1953) and Saltzman and Teweles (1964).

The daily 500-mb Northern Hemisphere analyses of the National Meteorological Center for the 10-yr period 1 April 1955 through 31 March 1965 have been subjected to zonal harmonic analysis (Saltzman and Fleisher, 1960; Van Mieghem, 1960). Harmonic coefficients of the geostrophic wind components were determined, and angular momentum transports computed in the wave number domain. Monthly mean values of the latter quantities form the basis for the present study.

2. Computational procedure

The meridional flux of relative angular momentum across a latitude wall per unit time is given by

\[ a \cos \phi \int_0^{2\pi} \int_0^\phi \frac{d\phi}{g} \int_0^\phi \cos \phi d\phi \int_0^\phi \sin \phi d\phi \int_0^\phi \cos \phi d\phi \int_0^\phi \sin \phi d\phi \]

where

\[ a = \text{distance from the center of the earth}, \]
\[ \phi = \text{latitude}, \]
\[ \lambda = \text{longitude}, \]
\[ P = \text{pressure}, \]
\[ u = \text{eastward component of the wind}, \]
\[ v = \text{northward component of the wind}, \]
\[ g = \text{acceleration due to gravity}. \]

The flux per unit pressure difference is then given by

\[ \frac{a \cos \phi}{g} \int_0^{2\pi} \int_0^\phi \cos \phi d\phi \int_0^\phi \sin \phi d\phi \int_0^\phi \cos \phi d\phi \int_0^\phi \sin \phi d\phi \]

If we now expand \( u \) and \( v \) in terms of zonal harmonics, where \( n = \text{wave number}, \ i.e., \)

\[ u = \sum_{n=-\infty}^{\infty} U(n) e^{i\lambda}, \]
\[ v = \sum_{n=-\infty}^{\infty} V(n) e^{i\lambda}, \]

then (2) becomes

\[ \frac{a^2 \cos^2 \phi}{g} \int_0^{2\pi} \sum_{n=-\infty}^{\infty} U(n) e^{i\lambda} \sum_{m=-\infty}^{\infty} V(m) e^{i\lambda} d\lambda \]

\[ = \frac{2\pi a^2 \cos^2 \phi}{g} \sum_{n=-\infty}^{\infty} U(n) V(-n), \]

(3)
and the flux of relative angular momentum per unit time per unit pressure difference by the "eddies" 

\[ T_m(\phi) = \frac{2\pi a^2 \cos^2 \phi}{g} \sum_{n=1}^\infty [U(n) V(-n) + U(-n) V(n)]. \]  

Daily values of \( T_m(n,\phi) \) were computed for wave numbers 1 through 15 at every 5 deg of latitude from 17.5N to 77.5N, and averaged to obtain monthly means. The results are presented for wave numbers 1 through 6 at 17.5N, 27.5N, 42.5N, and 57.5N. These latitudes were chosen as representative of tropical, subtropical, middle, and high latitudes, with the lowest latitude being of particular interest in the investigation of oscillatory phenomena. The truncation at wave number 6 was suggested by our results and those of Van Mieghem (1960) and Saltzman and Teweles (1964). Little additional information is available at the higher wave numbers; furthermore, the physical significance of these shorter waves appears to be limited to that of a shape parameter.

Since the amplitude of the annual cycle was expected to be much greater than that of the quasi-biennial, a statistical filter was applied to the time series. The filter, consisting of a 12-month running mean minus a 24-month running mean, has the frequency response shown in Fig. 1. This particular filter was selected specifically to ensure the relative enhancement of any quasi-biennial oscillation, and comparison of our results with the 12-month running mean indicate this to be the case.

Because the above procedure does not permit assessment of the relative amplitude of the oscillation, the data were subjected twice to band-pass filtering (Brier, 1961). The first filter was centered at about 24 months and the second at about 12 months. A measure of the contribution of the periodicity being studied to the total variability is given by the percentage of variance of the unfiltered data accounted for by the filtered data. The reductions of variance for the 24-month and 12-month filters are presented in sections (a) and (b), respectively, of Table 1. The ratio of (a) and (b) is given in section (c). In general, the results indicate that for 17.5N and the higher latitudes the quasi-biennial oscillation tends

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**Table 1.** Percentage reduction of variance of the relative amplitudes for wave numbers 1 through 6, of monthly average angular momentum transport at 500 mb for various latitudes using a 24-month band-pass filter (a) and a 12-month band-pass filter (b). Ratios of (a) to (b) are given in (c).

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Fig. 2. 12-month minus 24-month running mean of monthly average angular momentum transport (10^9 erg mb^-1) at 500 mb at latitudes 17.5°N, 27.5°N, 42.5°N, and 57.5°N for wave number 1.

Fig. 3. Same as Fig. 2 for wave number 2.
Fig. 4. Same as Fig. 2 for wave number 3.

Fig. 5. Same as Fig. 2 for wave number 4.
Fig. 6. Same as Fig. 2 for wave number 5.

Fig. 7. Same as Fig. 2 for wave number 6.
to be more important than the annual cycle, while from 22.5N into mid-latitudes the latter predominates.

It may be noted at this point that a companion paper, outlining in detail the seasonal variation of the 500-mb angular momentum transports by wave number, is in preparation by the authors.

3. Results

It has long been recognized that individual cycles of the quasi-biennial oscillation in zonal wind component are extremely variable in length (Reed, 1965), and this feature is apparent in the momentum calculations. Because of this “erratic periodicity,” we decided not to perform a temporal harmonic analysis, but simply to present the results of the computations and point out some of the more interesting features. The results for wave numbers 1 through 6 are presented in Figs. 2–7.

Wave number 1. An approximate 2-yr period is shown quite clearly with maximum fluxes at 42.5N occurring in the winters of the odd-numbered years. The results at 27.5N and 57.5N are less regular than the mid-latitude results, but in general agree in phase with that region. At 17.5N, however, a phase reversal is quite evident, with the maximum transports occurring in the winters of the even-numbered years.

Wave number 2. A quasi-biennial oscillation is apparent, but surprisingly in phase opposition to wave 1 at 42.5N, with the maximum transports occurring in the winters of the even-numbered years. The patterns for the lower and higher latitudes in general coincide with the mid-latitude results, although a phase shift appears at 27.5N in 1962.

Wave number 3. The trace for 42.5N contains more short-period variability than the previous two wave numbers, but on the whole conforms in phase with wave number 1. The high latitude agrees with the mid-latitude results, but 27.5N presents a somewhat confused picture. The early years at 17.5N show a rather low amplitude, somewhat irregular pattern, but beginning in 1962 the amplitude increases quite dramatically, in phase with wave number 2.

The cyclic structure of waves 4, 5, and 6 is not as pronounced as that of the planetary waves, but one feature stands out. The transports at mid-latitudes by waves 5 and 6 display phase agreement with wave 1 until about 1960–1961, where a phase shift occurs that brings the fluxes into phase with wave 2. The data record, however, is of insufficient length to establish the permanence of this feature.

Fig. 8 shows the 12-month running mean minus the 24-month running mean of the monthly averaged mean zonal wind component (n=0) for the same time period, April 1955 to March 1965. One is not immediately impressed by any marked, continuous cycle in the winds, but examination of the overall shape of the curves indicates a phase shift with latitude. It is possible that our treatment is not sensitive enough to bring out a small-amplitude variation and that use of a more sophisticated technique is required.

![Fig. 8. 12-month minus 24-month running mean of monthly average mean zonal wind (cm sec⁻¹) at 500-mb at latitudes 17.5N, 37.5N, and 52.5N.](image-url)
4. Concluding remarks

There is no ready explanation as to why wave number 2 should be opposite in phase to waves 1 and 3, and it is clear that further investigation is required. At the same time, it is easy to see the confusion that would result from consideration of the total angular momentum transports $\sum_{m=1}^{n} T_m(n,\phi)$, since the contribution of the individual wave numbers would tend to cancel, resulting in a non-periodic variation.

The 1960–1961 phase shift in the cyclone waves (wave numbers 5 and 6) is also difficult to explain. Although there have been changes in map analysis procedure from time to time, none is likely to account for this feature. We must, then, admit the possibility that the cyclone waves vary in their modes of interaction with the planetary waves.

In addition, it is interesting to note that Wallace and Newell (1966) found the total angular momentum transports by transient eddies in the 50- to 15-mb region from the period from May 1958 to April 1963 to have a marked quasi-biennial oscillation with a phase relationship similar to that of wave number 2 in the present study. The relationship of their relative angular momentum transport calculations to the tropical zonal wind is shown by Reed's (1965) analysis of Canton Island (2°46'S, 171°43'W) winds. It is apparent that when the Northern Hemisphere relative angular momentum transports increase in strength above 30 mb, the Canton Island westerlies are reduced to easterlies. Wallace and Newell did not, however, detect any strong periodicity at 100 mb. In view of our results, this may be a consequence of the fact that they considered only the total transient eddy transports.

At the same time, it is significant that Saltzman and Teweles (1964) found wave number 2 at 500 mb to be important in the conversion of available potential energy to kinetic energy. This suggests that the quasi-biennial oscillation is directly influenced by the continent-ocean structure and its related thermal effect.

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