# Mass and Available Energy in Growing Convective Clouds

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## 1. Introduction

While comparing growth rates of cumulus and cumulonimbus clouds, as revealed in the literature, the author was struck by the consistent increase of growth rate as a function of the size of the cloud. This suggested that the rate of cloud growth could be approximated by some simple power function of cloud size. The purpose of this note is to describe the use of available data to evaluate such a power function which is then employed to compute an order-of-magnitude estimate of the rate at which variously sized convective clouds grow. Also, a simplified approach is used to approximate the order of magnitude of the energy available in the clouds for growth.

A problem arises when one tries to specify a quantity to represent the "size" of a cloud. One might consider such quantities as diameter, cross-sectional area, height, volume, mass, etc. Mass or volume seem to be better suited for the present purpose than the one- and two-dimensional quantities. Of these two, *mass* was then arbitrarily chosen to represent cloud size.

The power functions that were used to obtain estimates of the growth rate and available energy are

$$\frac{dM}{dt}=aM^{b}, \tag{1}$$

$$E=cM^{d}, \tag{2}$$

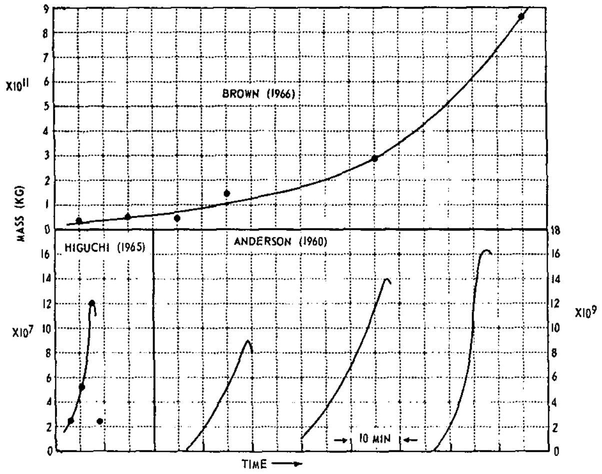

FIG. 1. Mass measurements in small cumulus (Higuchi, 1965), cumulus congestus (Anderson, 1960; Brown, 1966), and cumulonimbus (Brown, 1966). Mass scale for Anderson's data is given at right side.

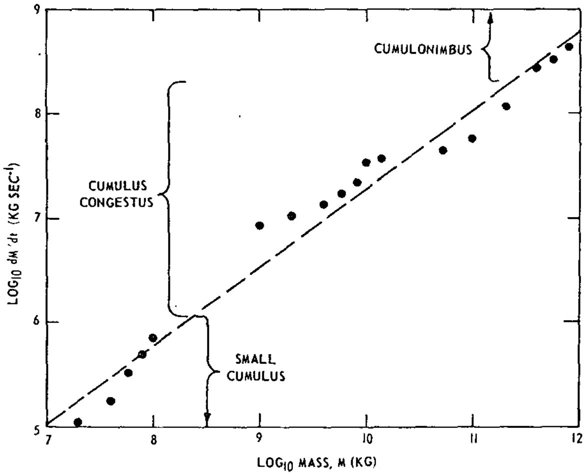

FIG. 2. Growth rate as a function of mass of convective cloud. Dashed line is least-squares fit to data points.

where $M$ is cloud mass (kg), $E$ available energy (ergs), $t$ time (sec), and $a$, $b$, $c$, and $d$ are parameters to be determined from empirical data.

## 2. Cloud mass data

In recent years there have been a sufficient number of photogrammetric mass measurements of convective clouds to make it worthwhile to evaluate the parameters in Eqs. (1) and (2). Anderson (1960), using stereo-pair photographs from the ground and the assumption of axial symmetry, obtained volume and growth-rate measurements of cumulus clouds developing over a mountain range near Tucson, Ariz. More recently, Higuchi (1965) used aerial photographs and a half-ellipsoidal shape assumption to obtain three-dimensional measurements of small cumuli forming over flat ground near Wagga Wagga, New South Wales, Australia. Brown (1966), with both surface stereo-pair and aerial photographs, computed the mass and growth rate of orographic cumulus congestus clouds which grew into a cumulonimbus near Flagstaff, Ariz. The fact that all three studies were made in semi-arid regions should not affect the general nature of the results, which will be to the nearest order of magnitude at best.

The increase of cloud mass with time for the clouds studied by Anderson, Brown, and Higuchi is presented in Fig. 1. The curves for Anderson's and Higuchi's data are drawn only up to the point where the mass started to decrease with time. Anderson's and Higuchi's volume measurements were converted to mass units by employing the mean environmental density from cloud base to cloud top. Data points for Anderson's measurements were so close together (one-half to one minute) that only the resulting curves are given; his paper contains data for other cumuli.

Fig. 1 reveals that the rate of mass increase in a growing cloud is a function of the size of the cloud. This relationship is presented quantitatively in Fig. 2, where the $dM/dt$ data points were obtained by differentiating the curves in Fig. 1. Means of the differentiated values for the three Anderson curves were plotted in the figure so as not to bias the placement of the dashed line, which was computed using the least-squares principle. The data tend to substantiate the proposal that $dM/dt$ varies as a simple power function of cloud size. The parameters determined by the dashed line are $a=0.60$ kg $^{0.25}$ sec$^{-1}$ and $b=0.75$; substituting them into Eq. (1) results in

$$\frac{dM}{dt}=0.6M^{3/4}. \tag{3}$$

It may be noted that the data plotted in Fig. 2 are representative only of those portions of the curves in Fig. 1 for which both the first and second derivatives of mass with respect to time are greater than zero. The first derivative must be positive in order for a cloud to be growing. The specification that the second derivative also must be positive excludes all clouds except those which are in the most active stages of growth; to include clouds whose rates of growth have started to decrease would result in ambiguous growth rate values. Therefore, before being able to determine the parameters in Eq. (1) it was necessary to make sure that only maximum values of $dM/dt$ for each cloud size were used.

## 3. Computation of available energy

Now it would be of interest to estimate the total amount of energy that has been made available to a

TABLE 1. Mean total water content and latent heat for the three types of clouds.

| Cloud type | $\bar{\chi}$ (gm kg$^{-1}$) | $L$ (erg gm$^{-1}$) |
|---|---|---|
| Small cumulus | 0.5 | $2.5\times10^{10}$ |
| Cumulus congestus | 1.5 | $2.6\times10^{10}$ |
| Cumulonimbus | 2.5 | $2.7\times10^{10}$ |

TABLE 2. Summary of growth rate and energy values for cumulus and cumulonimbus clouds.

| Cloud type | $dM/dt$ (kg sec$^{-1}$) | Energy (ergs) |
|---|---|---|
| Small cumulus ($\leq10^{8}$ kg) | $\leq10^{6}$ | $\leq10^{18}$ |
| Cumulus congestus ($10^{9}$–$10^{11}$ kg) | $10^{7}$–$10^{8}$ | $10^{19}$–$10^{22}$ |
| Cumulonimbus ($>10^{11}$ kg) | $>10^{8}$ | $>10^{22}$ |

cloud up to the time at which it has attained a mass $M$. An order-of-magnitude answer can be found if we consider the energy to be represented by the latent heat released during the formation of liquid- and solid-water particles within the cloud. Therefore, the energy available for cloud growth can be computed approximately from

$$E=\bar{\chi}LM, \tag{4}$$

where $E$ is energy (ergs), $\bar{\chi}$ the mean water content (both liquid and solid) in the cloud (gm kg$^{-1}$), $L$ the latent heat released (erg gm$^{-1}$) and $M$ the total mass of the cloud (kg).

The mass data for evaluating Eq. (4) were the same as those used in Fig. 2. The values of total water content and latent heat used are listed in Table 1. In determining the magnitude of $L$, it was assumed that the small cumulus was all water, that the cumulus congestus was 90% water and 10% ice, and that the cumulonimbus was one-third ice. The listed values of $\bar{\chi}$ and $L$ are quite arbitrary, but they are reasonable enough for the order-of-magnitude answers for which we are striving.

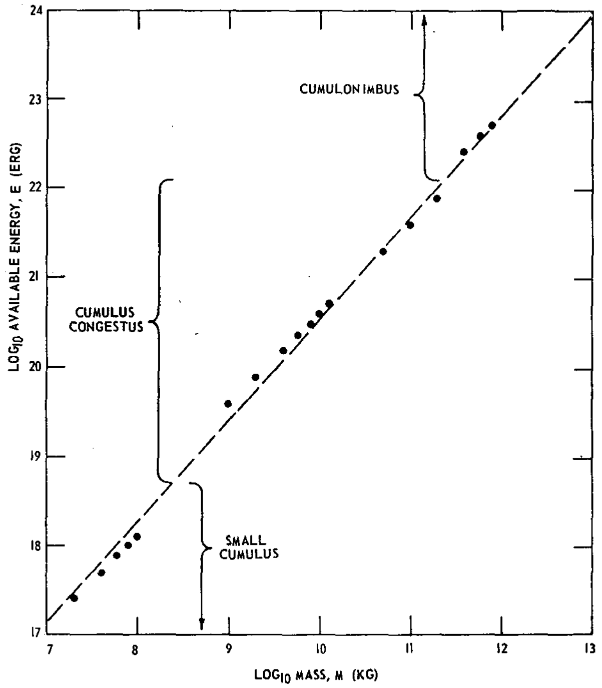

FIG. 3. Available energy as a function of mass of convective cloud. Dashed line is least-squares fit to data points.

The energy data are presented in Fig. 3. Again using the least-squares approach, the parameters in Eq. (2) are $c=1.4\times10^{9}$ ergs kg$^{-1.14}$ and $d=1.14$, such that the equation of the dashed line is

$$E=1.4\times10^{9}M^{1.14}. \tag{5}$$

All of the data points would have fallen on the dashed line if continuously increasing values of water content (as a function of mass) had been used instead of a mean value for each cloud type.

## 4. Concluding comments

Table 2 summarizes the growth rate and available energy information that were presented in Figs. 2 and 3. By extending his data, Anderson (1960) speculated that the growth rate for small cumuli could be as low as $10^{4}$ kg sec$^{-1}$, for cumuli congesti about $10^{8}$ kg sec$^{-1}$, and for a thunderstorm cell the rate may well reach $10^{9}$ kg sec$^{-1}$. Measurements of clouds, both larger and smaller than those he studied, confirm his speculations.

As a quantitative check of the energy computed above, a study made by Braham (1952) provided the needed data. Using Thunderstorm Project data (U. S. Weather Bureau, 1949), Braham investigated the water and energy budgets of a typical Ohio thunderstorm. He equated thunderstorm energy available for performing work against its environment with the resulting increase of internal and potential energy in the environment. The total energy available to the average thunderstorm cell (mass approximately $10^{11}$ to $10^{12}$ kg) was found to be of the order of $10^{22}$ ergs. Fig. 3 indicates that the simpler approach used in this paper suggests energy values of the order of $10^{22}$ to $10^{23}$ ergs. The realistic results obtained by using this approach were anticipated by Braham when he found that the latent heat released in forming the computed total water content of his thunderstorm also was of the order of $10^{22}$ ergs.

The purpose of this note has been to combine photogrammetric cloud measurements in such a way that the growth rate and available energy of any growing convective cloud (i.e., both first and second derivatives of mass with respect to time must be positive) can be estimated to an approximate order of magnitude. In addition to the purely academic aspects of this investigation, the resulting information can be of value when trying to confirm findings of theoretical cloud dynamics studies. Thus, it would be possible for a researcher to compare the growth rate or energy of a model cloud with that for an actual convective cloud at the same stage of development.

## REFERENCES

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