Disintegration of Pairs of Drops Raised to Equal and Opposite Potentials

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ABSTRACT

In order to assess quantitatively the role of drop disintegrations in producing the electrification of warm clouds, it is necessary to establish the electrohydrodynamical equations governing the stability of drops subjected to electrical forces. In the present paper a theoretical and experimental study is presented of the disintegration of drops raised to equal and opposite potentials.

In his theoretical treatment of the deformation and disintegration of individual water drops of undistorted radius R_0 raised to a potential V, Taylor assumed that the drop retained a spheroidal shape until the instability point was reached and that the equations of equilibrium between the stresses due to surface tension T, the potential V, and the difference between the external and internal pressures was satisfied at the poles and the equator. He showed that since there is no stationary value for V as the elongation a/b increases, the only stable condition is when the drop is stable and $V(\pi R_0 T)^{-\frac{1}{2}}<4$. Taylor's spheroidal assumption has been applied to the problem of the deformation and disintegration of pairs of drops raised to equal and opposite potentials. In this case directionality is imposed upon the problem by the attractive forces between the drops which provide a contribution, increasing with decreasing separation, to the outwardly-directed stresses in their surfaces. Stationary values of V were found to exist at values of $a/b>1$, and the corresponding values of $V(\pi R_0 T)^{-\frac{1}{2}}$ were less than 4.0 by a factor which increased rapidly as the initial separation was decreased. These critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ at the disintegration point ranged from Rayleigh's value of 4.0 at infinite separations to 3.117, 6.842×10^{-1}, 2.880×10^{-2} and 8.654×10^{-4} for initial separations of 10, 1, 0.1 and 0.01 radii, respectively. These values of $V(\pi R_0 T)^{-\frac{1}{2}}$ are slightly reduced for larger drops owing to the influence of the hydrostatic pressure difference between their vertical extremities.

These calculations were tested experimentally on suspended drops of water, aniline and benzene, and good agreement was obtained in all cases. High speed phototraphs indicated that the process of disintegration was similar to that observed by Taylor, with an extremely rapid transformation ($<10^{-3}$ sec) from an approximately spheroidal shape to a conical profile. Measurements taken from the photographs demonstrated that the radius of curvature and the elongation of a drop at the moment of disintegration agreed quite closely with the predicted values.

1. Introduction

Evidence has accumulated in recent years to show that intense electric fields sometimes exist inside clouds whose summits are always warmer than 0C. It is possible that the electrification of these warm clouds is primarily a consequence of the convective processes postulated by Vonnegut (1955). An alternative explanation is that the inductive mechanism propounded by Elster and Geitel (1885) and developed by Sartor (1961a,b) is dominant. One manifestation of this inductive process is the transfer of charge accompanying the disintegration under electrical forces of closely separated charged or uncharged drops polarized in the prevailing electric field of the cloud. To assess the meteorological significance of this mechanism of charge generation, it is essential to establish the criteria for disintegration of separated drops. Latham and Roxburgh (1966) calculated, and verified experimentally, values of the strength of the external field required to disintegrate an uncharged water drop as a function of its separation from an identical drop. In the following section a similar electrohydrodynamical treatment is presented of the stability of a pair of drops raised to equal and opposite potentials; this problem is related to the cloud physical situation of the interaction of cloud droplets carrying equal charges of opposite sign. The treatment employs the spheroidal assumption utilized successfully for single drops by Taylor (1964) and by Abbas *et al.* (1967) for isolated charged drops situated in an electric field.

Taylor treated theoretically the problems of the stability of an isolated drop of radius R_0 and surface tension T raised to a potential V and a similar drop situated in a uniform electric field of strength F. He assumed that as V or F increased the drop became elongated into a spheroidal shape which it retained until the disruption point was reached, and that throughout this deformation the equations of equilibrium between the stresses due to surface tension and the electrical forces, and the difference between the external and internal pressure were satisfied at the poles and at the equator. In the case of a drop situated in an electric field Taylor calculated that the onset of instability occurs for a particular value of the elongation when $F(R_0/T)^{\frac{1}{2}}=1.625$, which is in good agreement with the experiments of Nolan (1926) and Macky (1931) on water drops and of Wilson and Taylor (1925) on the bursting of soap bubbles situated in electric fields. However, in the case of an isolated drop raised to a potential V, Taylor found no stationary value for V as the deformation increased and he therefore deduced that the only stable equilibrium condition is when the drop is spherical and $V(\pi R_0 T)^{-\frac{1}{2}}<4$. This is Rayleigh's (1882) criterion. Taylor concluded that there can be no criterion of the type envisaged by Zeleny (1915) for isolated drops and that the instability which Zeleny (1917) observed when drops were already elongated at a definite value of $V(\pi R_0 T)^{-\frac{1}{2}}$ must be attributed to the fact that his drops were not in fact isolated, but were connected through the conducting fluid with the source of high potential, and were therefore in a distorted electric field.

However, if two drops are raised to equal and opposite potentials, each drop exerts upon the other an attractive force which is a maximum at the near points of the drops. In this case a stationary value of $V(\pi R_0 T)^{-\frac{1}{2}}$ exists for a finite elongation of the drops. This value is less than 4.0 and represents the criterion for disruption of the drops. As the initial separation of the drops is reduced, the mutual interaction is markedly enhanced and the critical value of the potential required to effect disintegration is correspondingly reduced. The situation is similar to that treated by Latham and Roxburgh who showed theoretically and experimentally that the strength of the electric field required to disintegrate one of a pair of drops whose line of centers is parallel to the field diminishes rapidly as the initial separation of the drops decreases.

A theoretical study is presented in Section 2 of the deformation and disintegration of pairs of liquid drops raised to equal and opposite potentials. The analysis utilizes the assumptions made by Taylor in his treatment of individual drops and those introduced by Latham and Roxburgh in their study of pairs of drops situated in an electric field. This is followed by a description of experiments designed to test the predictions emanating from this analysis together with an assessment of the quantitative validity of the spheroidal treatment in the light of the measured agreement between theory and experiment.

2. Calculations of criteria for disintegration

The situation under consideration is illustrated in Fig. 1. Two drops of identical undistorted radii R_0 are raised to equal and opposite potentials $+V$ and $-V$. Following Taylor's approach to the problem of the deformation of an isolated drop raised to a potential V, we assume that the drops retain a spheroidal shape given by the equation $x^2a^{-2}+y^2b^{-2}=1$ until the onset of instability; a and b are, respectively, the semi-major and semi-minor axes and the eccentricity e is given by $(1-b^2a^{-2})^{\frac{1}{2}}$. The initial separation X_0 of the near points A and C of the undistorted drops in the absence of applied potentials is related to the smaller separation X when the drops are elongated along the line of centers in the presence of the applied potentials by the equation

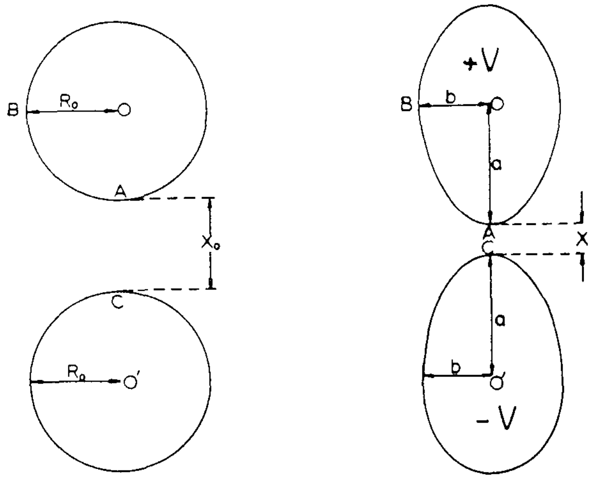

FIG. 1. The geometry of the problem.

$$X=X_0-2(a-R_0). \tag{1}$$

It is difficult to assign an accurate value for the strength of the electric field at the points A and C. For extremely large separations the mutual interaction of the drop charges is negligible and the field reduces, with a high degree of accuracy, to that given by Taylor for an isolated drop, namely $V/[b(1-e^2)^{\frac{1}{2}}I]$, where $I=0.5e^{-1}\ln[(1+e)/(1-e)]$. In view of the fact that the interaction between the drops increases markedly as the separation decreases, the potential required for disintegration decreases rapidly with a corresponding diminution in the value of $V/[b(1-e^2)^{\frac{1}{2}}I]$. For very close separations the value of V required for disintegration is exceedingly small and the average field between the drops reduces effectively to $2V/X$. Since several assumptions are present in the spheroidal treatment, it was considered justifiable to introduce the further assumption that the field at the points A and C is given by the sum of Taylor's expression for an isolated drop and the average field $2V/X$ between the drops: the preceding arguments demonstrate that this assumption is completely valid both for widely separated and closely separated drops. The normal electrical stress at the points A and C is therefore given by

$$(\widehat{nn})_a=\frac{1}{8\pi}\left(\frac{V}{b(1-e^2)^{\frac{1}{2}}I}+\frac{2V}{X}\right)^2. \tag{2}$$

Following Taylor's treatment we assume that the equation of equilibrium at A and C is given by

$$2ab^{-2}T-p=(\widehat{nn})_a, \tag{3}$$

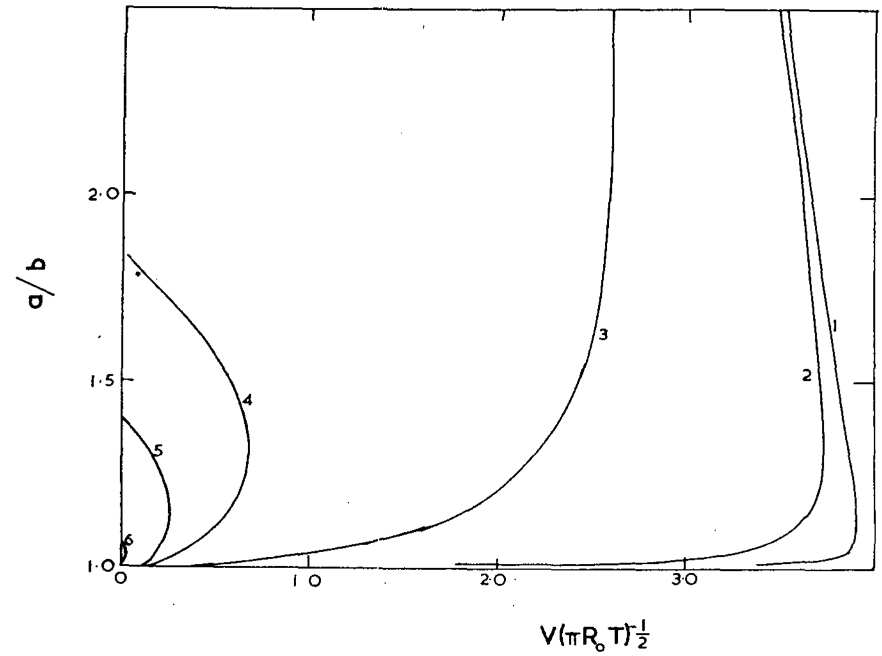

FIG. 2. The elongation a/b of small drops as a function of $V(\pi R_0 T)^{-\frac{1}{2}}$: curve 1, $X_0/R_0=1000$; 2, $X_0/R_0=100$; 3, $X_0/R_0=5.0$; 4, $X_0/R_0=1.0$; 5, $X_0/R_0=0.5$; 6, $X_0/R_0=0.1$.

where p is the difference between the internal and external pressures and T is the surface tension. The equation of equilibrium at the equatorial points B is given by

$$T(ba^{-2}+b^{-1})-p=(\widehat{nn})_0=\frac{1}{8\pi b^2}\frac{V^2}{I^2}. \quad (4)$$

Eliminating p from the equilibrium equations and substituting for $(\widehat{nn})_a$ from (2) we deduce that

$$T(2ab^{-2}-ba^{-2}-b^{-1})=\frac{V^2}{8\pi}\left(\frac{e^2}{b^2I^2(1-e^2)}+\frac{4}{bX(1-e^2)^{\frac{1}{2}}I}+\frac{4}{X^2}\right). \quad (5)$$

Eq. (5) can be rewritten, if we define $\alpha=1-e^2$, ($a=R_0\alpha^{-1/3}$, $b=R_0\alpha^{1/6}$) as

$$V(\pi R_0 T)^{-\frac{1}{2}}=8\alpha^{-\frac{7}{6}}(2-\alpha^{\frac{2}{3}}-\alpha^{\frac{1}{3}})^{\frac{1}{2}}/M(\alpha,X_0/R_0), \quad (6)$$

where

$$M^2(\alpha,X_0/R_0)=\left(\frac{1-\alpha}{\alpha^{\frac{4}{3}}}\right)\frac{1}{I^2}+\frac{4}{\alpha^{\frac{2}{3}}I\left[\dfrac{X_0}{R_0}-2(\alpha^{-\frac{1}{3}}-1)\right]}+\frac{4}{\left[\dfrac{X_0}{R_0}-2(\alpha^{-\frac{1}{3}}-1)\right]^2}.$$

TABLE 1. The critical potential V required for the disintegration of a water drop of undistorted radium $R_0=0.01$ cm separated from an identical drop raised to a potential $-V$ by a distance X_0. The value a/R_0 represents the elongation of the drop at the moment of disintegration.

X_0/R_0	$V(\pi R_0T)^{-\frac{1}{2}}$	V (volts)	a/R_0
0.5	2.810×10^{-1}	127.44	1.091
0.3	1.390×10^{-1}	63.06	1.051
0.1	2.840×10^{-2}	12.87	1.017
0.08	2.020×10^{-2}	9.24	1.013
0.05	9.981×10^{-3}	4.50	1.010
0.03	4.502×10^{-3}	2.04	1.0067
0.01	8.146×10^{-4}	0.369	1.0019

Eq. (6) can be solved numerically for any initial separation X_0/R_0 of the drops. Solutions of (6) for various selected values of X_0/R_0 are shown graphically in Fig. 2. It is seen that for large separations, where the mutual interactions of the drops are minimal, the curve of $V(\pi R_0T)^{-\frac{1}{2}}$ vs. a/b is approximately identical with that derived by Taylor for an isolated drop. For infinite separations there exists no stationary value of $V(\pi R_0T)^{-\frac{1}{2}}$ as a/b increases, and the only stable equilibrium condition occurs when the drop is spherical and $V(\pi R_0T)^{-\frac{1}{2}}<4$. This is Rayleigh's criterion. Therefore, as pointed out by Taylor, there can be no criterion of stability of the type envisaged by Zeleny for isolated drops. However, for pairs of drops having finite separations, a stationary value of the $V(\pi R_0T)^{-\frac{1}{2}}$ is always obtained from (6) with a corresponding value of $a/b>1$. This stationary value of $V(\pi R_0T)^{-\frac{1}{2}}$ is a consequence of the directionality imposed upon the problem by the presence of the second drop and is taken to correspond to the criterion for instability. As can be seen from Fig. (2), the critical values of $V(\pi R_0T)^{-\frac{1}{2}}$ decreases rapidly as the initial separation X_0/R_0 decreases, owing to the enhancement of the electrical stress at the points A and C produced by the increasing mutual interaction of the drops. The rapid diminution in the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ and the corresponding deformation a/R_0 with decreasing separation X_0/R_0 is shown in Table 1. It is seen that for an initial separation $X_0/R_0=10^{-2}$, the critical potential V to produce disintegration is about four orders of magnitude less than that predicted by the Rayleigh criterion. The applied potential required to disintegrate a drop can therefore be enormously reduced by the proximity of a second drop raised to an equal and opposite potential.

McDonald (1954) has shown that the contribution to the stresses acting upon a drop of the hydrostatic pressure difference between its vertical extremities becomes significant in comparison with the surface tension, electrical and internal pressure terms when the drop radius exceeds about 0.5 mm. The hydrostatic pressure difference term should therefore be considered in cloud physics problems involving the disintegration of raindrops. Latham and Roxburgh showed that the inclusion of this term into their equation for the disintegration criteria for a pair of uncharged drops separated in an electric field of strength F produced a significant reduction in the critical values of $F(R_0/T)^{\frac{1}{2}}$. They argued that the value for the hydrostatic pressure difference between the top and bottom of the upper drop will lie between $2g\rho a$ and $g\rho(R_0+a)$ where ρ is the density of the drop and g the acceleration due to gravity. The corresponding equations of equilibrium at the point A in Fig. 1 become, if we retain the spheroidal approximation,

$$2ab^{-2}T-p=(\widehat{nn})_a+2g\rho R_0\alpha^{-\frac{1}{3}},$$

and

$$2ab^{-2}T-p=(\widehat{nn})_a+g\rho R_0(1+\alpha^{-\frac{1}{3}}).$$

Since no contribution is made by the hydrostatic pressure difference term at the equatorial points B, the respective final equations become

$$V(\pi R_0 T)^{-\frac{1}{2}}=\frac{[8\alpha^{-\frac{2}{3}}\{(2-\alpha^{\frac{2}{3}}-\alpha^{\frac{1}{2}})-2g\rho R_0^2\alpha^{\frac{1}{3}}/T\}]^{\frac{1}{2}}}{M(\alpha,X_0/R_0)}, \quad (7)$$

and

$$V(\pi R_0 T)^{-\frac{1}{2}}=\frac{[8\alpha^{-\frac{2}{3}}\{(2-\alpha^{\frac{2}{3}}-\alpha^{\frac{1}{2}})-g\rho R_0^2\alpha^{\frac{2}{3}}(1+\alpha^{-\frac{1}{3}})/T\}]^{\frac{1}{2}}}{M(\alpha,X_0/R_0)}. \quad (8)$$

Fig. 3 shows that the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ deduced from Eqs. (7) and (8) are extremely insensitive to the selected value of the hydrostatic pressure difference term. It is therefore unnecessary to utilize in our calculations a more accurate expression for this term, which would merely provide a curve intermediate between the curves 1 and 2 in Fig. 3. In all subsequent calculations, critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ computed from Eq. (7) are utilized.

Fig. 4 shows that the hydrostatic pressure difference term produces a diminution in the predicted values of

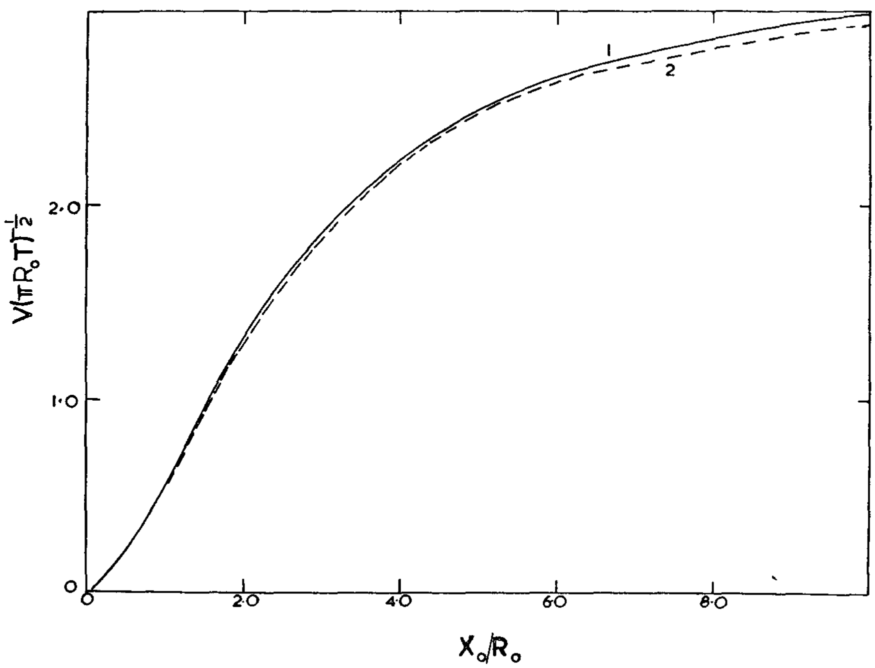

FIG. 3. The variation of the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ with drop separation X_0/R_0 for two definitions of the hydrostatic pressure difference term H with $R_0=0.1$ cm: curve 1, $H=g\rho(R_0+a)$; 2, $H=2g\rho a$.

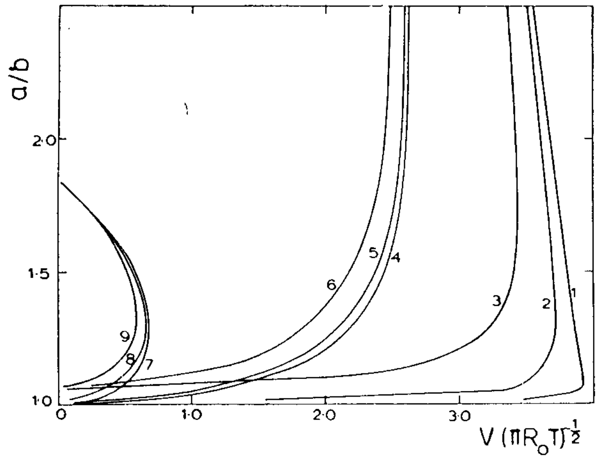

FIG. 4. The elongation a/b of water drops of undistorted radius R_0 as a function of $V(\pi R_0 T)^{-\frac{1}{2}}$: curve 1, $X_0/R_0=1000$, $R_0=0$; 2, $X_0/R_0=1000$, $R_0=0.05$ cm; 3, $X_0/R_0=1000$, $R_0=0.1$ cm; 4, $X_0/R_0=5.0$, $R_0=0$; 5, $X_0/R_0=5.0$, $R_0=0.05$ cm; 6, $X_0/R_0=5.0$, $R_0=0.1$ cm; 7, $X_0/R_0=1.0$, $R_0=0$; 8, $X_0/R_0=1.0$, $R_0=0.05$ cm; 9, $X_0/R_0=1.0$, $R_0=0.1$ cm.

$V(\pi R_0 T)^{-\frac{1}{2}}$ required to produce disintegration of a drop situated vertically above a second drop of equal volume which is raised to an equal and opposite potential. This diminution becomes more important as the drop radii increase but comparison of Figs. 2 and 4 indicates that the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ are generally much more sensitive to the initial separation of the drops than to the hydrostatic pressure difference term. However, the solutions of (7) and (8) demonstrate that for isolated drops of finite radius a stationary value of $V(\pi R_0 T)^{-\frac{1}{2}}$ exists with a value lower than that given by the Rayleigh criterion by an amount which increases with increasing R_0.

Many approximations have been made in this analysis of the criteria for disintegration of pairs of drops raised to equal and opposite potentials All the approximations made by Taylor in his treatment of the disintegration of an isolated drop raised to a potential V have been retained in the present study; his primary assumptions were that the drop retained a spherical shape until the moment of disintegration and that the equilibrium conditions are satisfied at the poles and at the equator. As can be seen from Fig. 1, the drop will not retain a spheroidal shape, but will be preferentially elongated over the regions where the electrical stresses are greatest; namely between the near surfaces. In addition, the shapes of the two drops will be different since the hydrostatic pressure difference term reinforces the electrical stress at the point A in Fig. 1, but equals zero at the point C. However, Fig. 3 together with the excellent agreement between Taylor's calculations and experiments on individual drops provide reasonable circumstantial evidence for the possibility that the errors emanating from this analysis are not serious. Perhaps the strongest evidence for this contention is the extent of the consistency observed by Latham and Roxburgh to exist between the experimental results and theoretical predictions of the criteria for disintegration of uncharged drops separated in an electric field. This basically similar two-body problem contains the same assumptions as those utilized in the present study. A description is now presented of experiments designed to test these theoretical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ for pairs of drops of various liquids raised to equal and opposite potentials.

3. The drop disintegration experiments

The apparatus used to study the deformation and disintegration of pairs of separated drops raised to equal and opposite potentials is shown in Fig. 5. A drop obtained from a hypodermic syringe graduated to 10^{-4} cm^3 was suspended from the base of a thin Teflon rod which was rigidly clamped at its top. Drops of radii ranging from 0.025 to 0.17 cm were utilized. A second drop resided upon the top of a similar rod, firmly held in a precision micromanipulator which permitted sensitive adjustment of the drops until their line of centers was vertical. Copper wires passing centrally through the Teflon rods provided electrical connections between the drops and two continuously variable high-voltage supplies which could be used in synchronization to raise the potentials of the upper and lower drops to equal numerical values of up to +15 and −15 kV, respectively. The potentials could be measured to within ±1% by means of two sensitive electrostatic voltmeters. The initial separation X_0 of the near surfaces of the drops could be adjusted with a high degree of precision using the fine control of the micromanipulator and could be measured to within $\pm 10^{-3}$ cm by means of a vernier eyepiece attached to a telescope mounted on a cathetometer. X_0 was varied in these experiments from 5×10^{-3} to 5.0 cm. Photographs of the deformation and eventual disruption of the drops were taken by means of a high-speed camera of maximum framing rate 2500 sec^{-1}. An

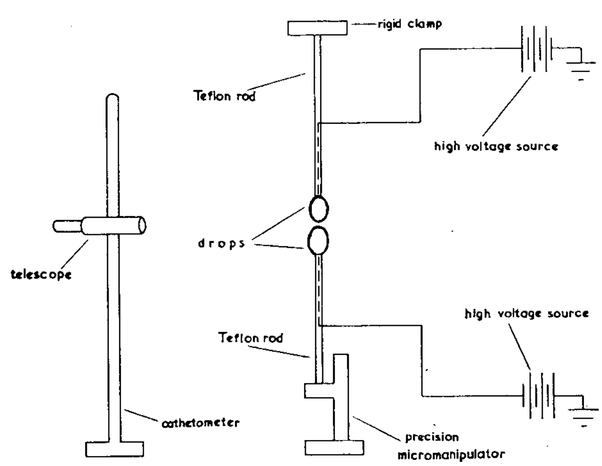

FIG. 5. Schematic diagram of the apparatus for studying the disintegration of drops raised to equal and opposite potentials.

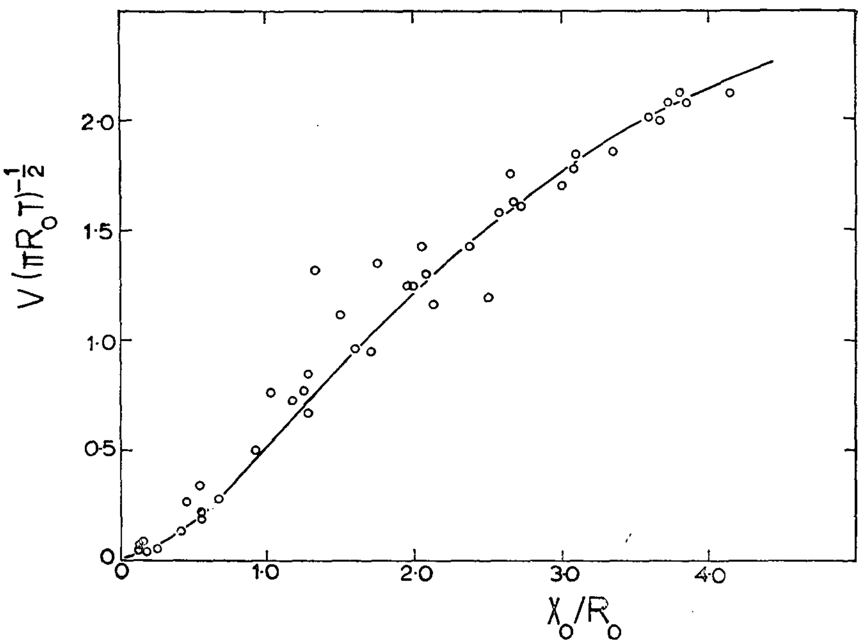

FIG. 6. The experimentally determined relation between the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ and X_0/R_0 for water drops separated by small distances with $R_0=0.134$ cm: solid line, theoretical curve; open circles, experimental readings.

electrical circuit provided synchronous actuation of the camera and the high voltage supplies, which were connected in series with a dual-trace pen recorder, thereby enabling the magnitude of the applied potentials to be determined at any state in the photographic sequence. Drops of water, aniline and benzene were utilized in order to ascertain the effect of surface tension and density upon the cirtical values of $V(\pi R_0 T)^{-\frac{1}{2}}$. Since it proved impossible to mount a benzene drop on the lower Teflon rod, the disruption criteria for benzene were studied using a benzene drop on the upper rod and a water drop on the lower one. This modification was not considered to introduce serious errors since in all the experiments with pairs of drops of equal volume the drop which disintegrated was the one located on the upper rod.

The experimental procedure was to mount two drops of pre-selected volume on the Teflon rods and, using the micromanipulator, to bring them into a predetermined separation which was measured by means of the vernier microscope. The high-speed camera was then switched on and the positive and negative potentials were raised quickly in unison until disintegration of one of the drops occurred. This procedure was repeated several times with drops of identical size and initial separation, and the entire sequence of events was then repeated for different values of these parameters and for different liquids. The magnitudes of the potentials at the moment of disintegration could be determined from the photographic and chart records; as a check on the results obtained in this way they were also determined in a separate experiment by increasing the potentials much more slowly than was feasible when the high-speed camera was being used and making visual observations of the disintegration point. The photographs also provided values of the radius of curvature and the magnitude of the elongation, expressed as the ratio a/R_0, of the drops at the disruption point. Values for both these parameters could be computed theoretically. In the case of benzene drops subsidiary experiments were performed in which the rates of evaporation of benzene drops of various sizes were measured in order to apply a correction for the slight decrease of radius which occurred with this volatile liquid in the period between the formation of the drops and their disintegration.

It is seen from Fig. 6 that, for separations X_0/R_0 less than 4.0, excellent agreement exists between the theoretical curve of $V(\pi R_0 T)^{-\frac{1}{2}}$ vs. X_0/R_0 for water drops and that obtained experimentally. Fig. 7 shows that for wider separations a discrepancy occurs between the two curves with the measured values of the disintegration voltage falling below the computed values by an amount which attains a maximum value of about 15% at separations X_0/R_0 around 10.0 and decreases gradually of zero for higher separations. It is interesting to note that significant deviations from the theoretical curve were found exclusively over an intermediate range of X_0/R_0 as was also observed in the experiments of Latham and Roxburgh. The deviations obtained in the present study can therefore probably be attributed to the same cause as those operating in the earlier experiments; namely, that for small separations, the degree of deformation of the drops is small. Therefore, the spheroidal approximation is valid for large separations

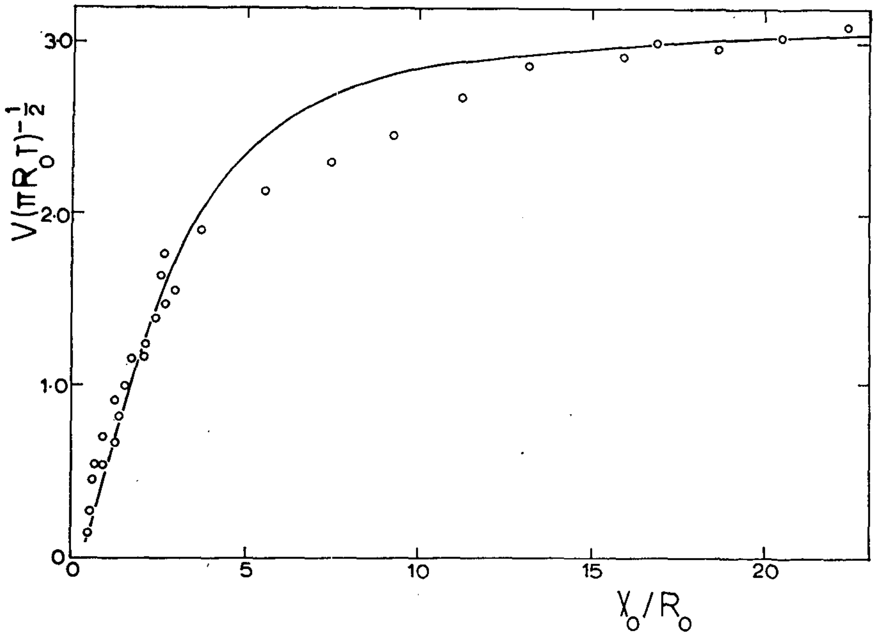

FIG. 7. The experimentally determined relation between the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ and X_0/R_0 for water drops with $R_0=0.134$ cm: solid line, theoretical curve; open circles, experimental readings.

and the influence of the second drop is negligible, the drop being effectively isolated, and the problem reduces to that treated by Taylor, which has been shown to be explicable in terms of the spheroidal assumption. However, for intermediate separations, where the mutual interaction of the drops is appreciable but where the initial separations are sufficiently large to permit significant elongation, the drops deform asymmetrically and the accuracy of the spheroidal approximation is reduced. However, the overall conclusion to be drawn from Figs. 6 and 7 is that for water drops good agreement exists between the theoretical and measured

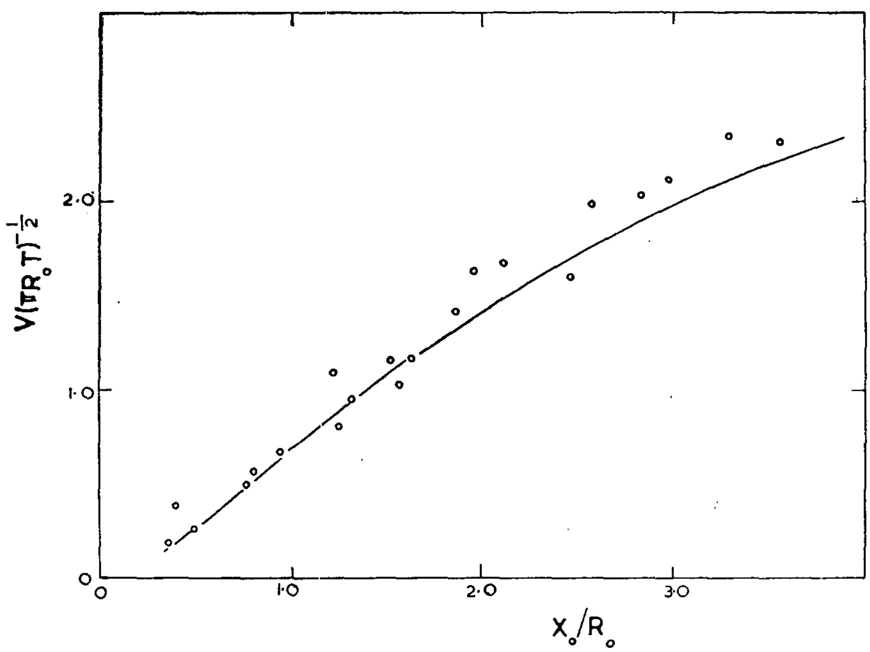

FIG. 8. The experimentally determined relation between the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ and X_0/R_0 for aniline drops with $R_0=0.106$ cm: solid line, theoretical curve; open circles, experimental readings.

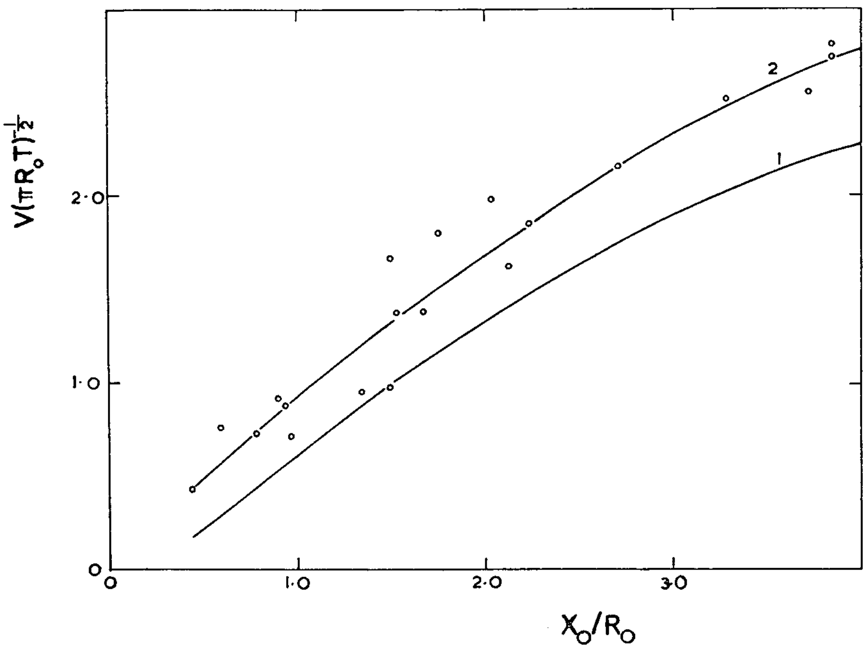

FIG. 9. The experimentally determined relation between the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ and X_0/R_0 for benzene drops with $R_0=0.134$ cm: curve 1, experimental; 2 theoretical.

values of $V(\pi R_0 T)^{-\frac{1}{2}}$ over the entire range studied experimentally. Fig. 8 demonstrates that excellent agreement also exists between theory and experiment when aniline drops were used. In the case of benzene drops Fig. 9 indicates that although the shape of the experimental curve was similar to that predicted theoretically, the numerical measured values of $V(\pi R_0 T)^{-\frac{1}{2}}$ were consistently about 15% above those computed from Eq. (7). This discrepancy is almost certainly a consequence of the fact that since benzene drops could not be mounted upon the lower Teflon rod, it was necessary to use drops of water, which have a higher surface tension and will therefore not deform to the same extent as benzene drops when subjected to an identical electrical stress. It can therefore be concluded from these experiments with drops of different liquids that the critical

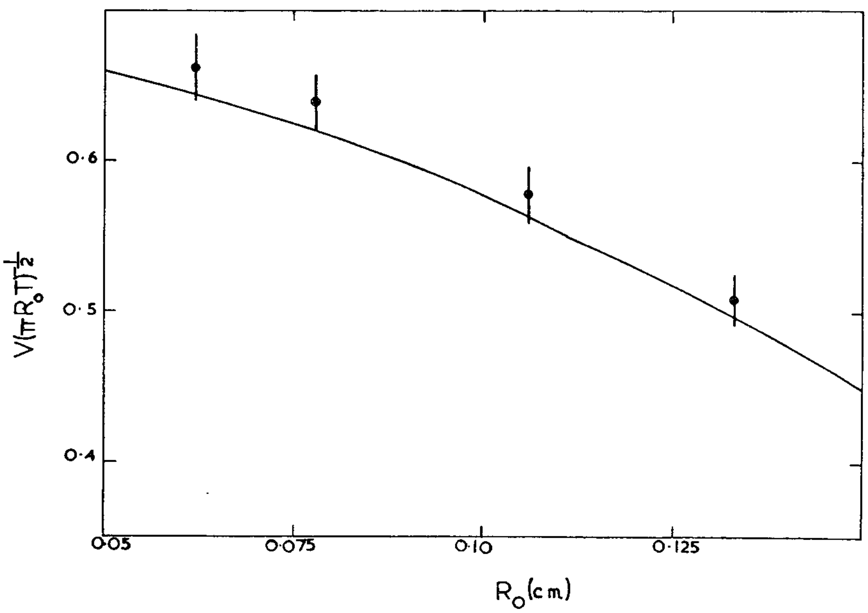

FIG. 10. The experimentally determined relation between the critical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ and R_0 for water drops with $X_0/R_0=1.0$: solid line, theoretical curve; solid circles, experimental readings.

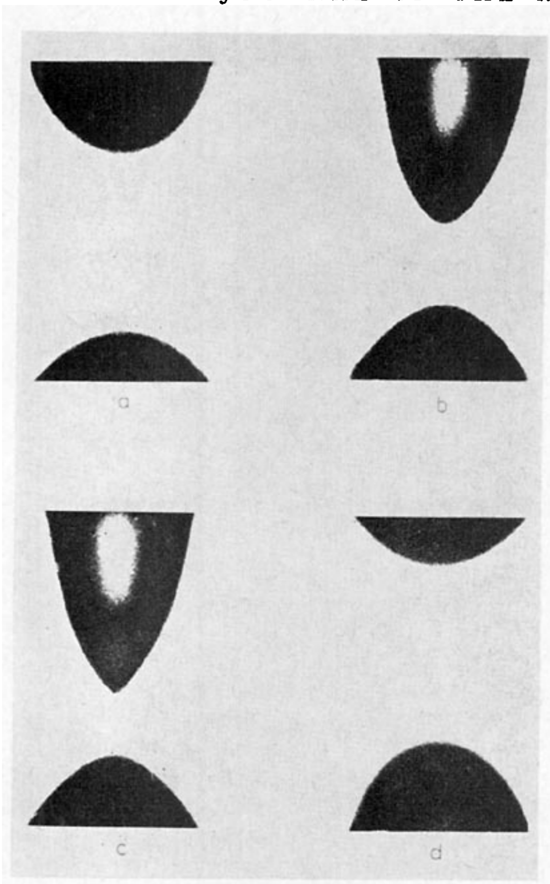

FIG. 11. Stages in the disintegration of a water drop raised to a potential V and separated from an identical drop raised to an equal and opposite potential with $R_0=0.106$ cm and $X_0/R_0=3.0$: a., $t=0$; b., $t=0.7500$ sec; c., $t=0.7505$ sec; d., $t=0.8800$ sec.

measured values of $V(\pi R_0 T)^{-\frac{1}{2}}$ vary with surface tension T in approximately the manner predicted in the foregoing calculations.

It is seen from Fig. 10 that, for a constant initial separation X_0/R_0, the measured values of $V(\pi R_0/T)^{-\frac{1}{2}}$ required for disintegration of water drops decrease slowly as the undistorted radius R_0 increases by a factor of about 3. The experimental and theoretical curves are in close agreement and it can be seen from comparison of Fig. 10 with Figs. 6 and 7 that the effect of R_0 upon the disintegration criteria is small in comparison with the effect of the initial separation X_0/R_0, except for extremely large separations where the numerical contribution to the excess stress at the point A in Fig. 1 is supplied primarily by the hydrostatic pressure difference term since the electrical interaction forces are negligible. Curves of $V(\pi R_0 T)^{-\frac{1}{2}}$ vs. R_0 obtained with aniline and benzene drops were also in reasonable agreement with the theoretical curves deduced from Eq. (7).

The high-speed photographs showed that the process of disintegration is similar to that observed by Taylor and by Latham and Roxburgh, with an extremely rapid transformation (less than 10^{-3} sec) from an approximately spheroidal shape to a conical profile. Comparison of photographs of the drops taken before and after disintegration demonstrates that mass is transferred from the disrupting drop to its companion drop. These features of the disintegration process are illustrated in Fig. 11. Values of the radius of curvature R_c of a drop at the moment of disintegration could also be measured from the photographs and compared with those computed theoretically. Although considerable scatter was

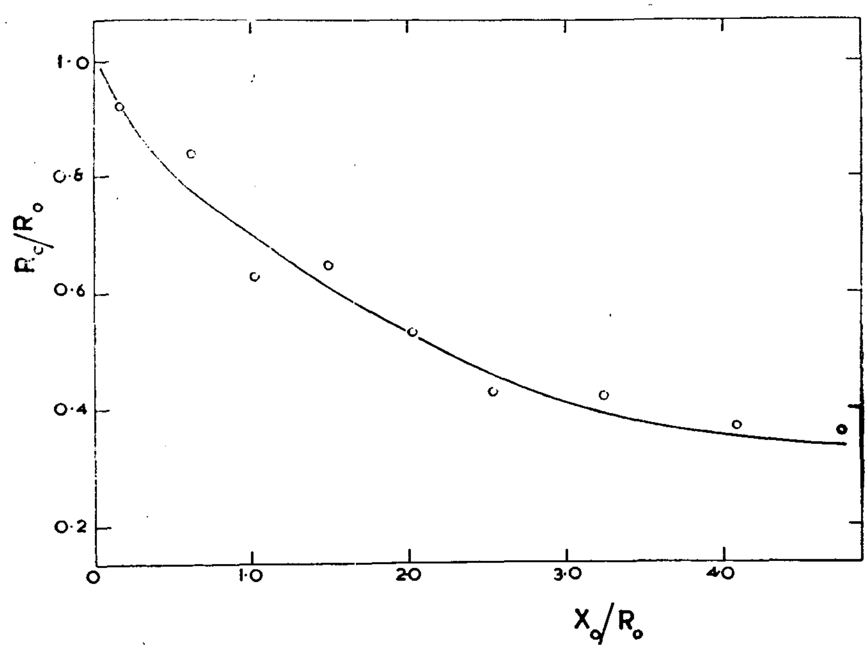

FIG. 12. The experimentally determined relation between the values of R_c/R_0 at the disintegration point and the initial separation X_0/R_0 for $R_0=0.05$ cm.

observed in the measured values of R_c, it can be seen from Fig. 12 that moderately good agreement was found to exist between the theoretical and average measured values of R_c over a wide range of separations. Reasonable agreement was also found to occur between the computed values of elongation of the drops at the disintegration point and those measured from the high-speed photographs.

As described in the preceding section many assumptions which are not strictly justifiable have been made in the formulation of the equations from which the theoretical values of $V(\pi R_0 T)^{-\frac{1}{2}}$ required for disintegration have been derived. An extremely accurate solution of this two-body problem would require a prohibitively large amount of computer time. In the absence of an exact solution, the good agreement between theory and experiment obtained in the present study suggests that the accuracy of the spheroidal assumption is sufficiently great to permit the solutions of Eqs. (7) to be utilized in most relevant practical situations. This is particularly true of the application of this study to cloud physics where, as shown by Latham and Roxburgh, numerical values defining the shapes and disintegration criteria are required for the solution of several current problems in which optimum accuracy is not essential.

It is impossible solely on the basis of the present study to assess the importance of drop disintegrations in generating electric fields within clouds, Measurements of the charge transfer accompanying the disruptive events were not made, aerodynamic forces were excluded from consideration, and it is probable that a large percentage of disintegrations are succeeded by coalescence. However, it is manifestly apparent that the pronounced intensification of the electric field between a closely-separated pair of drops can produce disintegration and consequent charge transfer, even when the external electrical charges or fields are weak.

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