Sea-Air Interaction: A Simplified Model

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ABSTRACT

In the long range prediction of the large scale motion of the atmosphere it is essential to prescribe realistic lower boundary conditions over oceans which incorporate the important features of the sea-air interaction. These features include the energy input to the atmosphere arising from the flux of latent heat of the water vapor evaporated from the sea as a result of the absorption of solar energy in the upper levels of the ocean. In addition, one must include the braking action of the wavy sea surface on the surface winds. For a simplified model incorporating these features a steady model is postulated where advection effects are neglected. The domain considered includes the atmospheric surface boundary layer and the upper levels of the sea down to a depth below the diurnal thermocline. In the surface boundary layer the shear stress and the heat flux are considered invariant, and the turbulent diffusivities proposed by Pandolfo are used. For the oceans we essentially use the model due to Munk and Anderson in which a simplification is carried out which decouples the energy equation from the momentum equations and permits a direct determination of the sensible heat flux in the ocean from the former equation. A sample calculation is carried out and a procedure is given to incorporate the results as a lower boundary condition for the initial value problem for the prediction of the large scale atmospheric motion.

1. Introduction

It is well known that global atmospheric motions are sustained by the radiation energy from the sun. This energy is added to the atmosphere primarily in the tropical latitudes through an intermediate process involving a sea-air interaction. Here the solar energy is first absorbed in the upper levels of the sea, and a significant portion of this energy is then used to evaporate the sea water. The resulting water vapor with its latent heat is then transported upward both by free and forced turbulent convection. In its upward flight the vapor is cooled, finally condensing to form the rainless cumuli in the tropics and the giant cumulonimbi in the equatorial trough. Upon condensing, the latent heat of the vapor is released and transferred to the ambient atmosphere as sensible heat. The resulting heating of the atmosphere generates pressure gradients that finally drive the atmospheric winds. Clearly, for long range predictions of the global circulations the above aspects of the sea-air interaction must be incorporated into the atmospheric model.

Mathematically, the prediction of the atmospheric circulations forms an initial value problem involving the unsteady, turbulent Navier-Stokes equations. For long range predictions (of the order of several weeks or longer) it will be meaningless to consider any domain less than that covering the entire globe. To render the solution unique one must prescribe appropriate boundary conditions on the dependent variables at the upper and lower boundaries of this domain, and these conditions must be known *a priori* for the entire prediction period. Of particular importance is the formulation of the lower boundary conditions over the oceans, since it is here that the above solar energy additions will be incorporated. Unfortunately, the mean sea surface cannot be taken as the lower boundary, since such essential quantities as the sea-surface water vapor flux cannot be determined beforehand without a simultaneous consideration of the upper ocean levels.

It will thus be the purpose of the present paper to consider in a simplified way the sea-air interaction in the domain including the atmospheric surface boundary layer and the upper layers of the sea down to a depth below the diurnal thermocline where sea conditions may be considered invariant; that is, from a height of the order of 10 m above the mean sea surface to a depth of the order of many tens of meters below it. The resultant sea-air interaction flow obtained will then serve as a lower boundary condition over the oceans for the prediction of the large scale motions.

2. Simplified model

It is clearly impossible to obtain a simple model applicable for all conceivable situations. The following model is proposed as a first attempt which hopefully will contain the essential effects. In the domain of interest we shall assume that the flow is quasi-steady and that advection is negligible. It will be assumed that the radiant energy is absorbed at the sea surface and that locally the net heat input (absorbed minus the reradiated) is a prescribed function of time. In general, this absorbed energy will be disposed of in several ways:

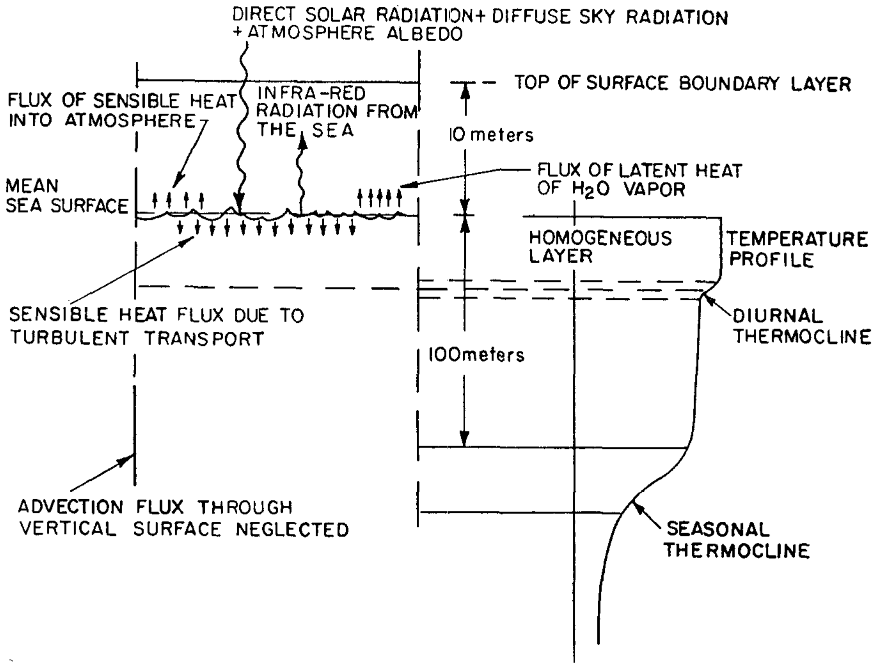

FIG. 1. Local energy balances.

first, by sensible heat flux from the sea surface into the interior of the atmosphere by turbulent transport; second, by sensible heat flux downward into the depth of the ocean by turbulent motions generated by the surface waves as well as by the shear of surface advection currents; and finally, by the evaporation of the sea water. (The energy balance is shown in Fig. 1.) We shall neglect here sensible heat flux downward by thermohaline convection, and, of course, a quasi-steady model will rule out as a possible heat sink the heat capacity of the sea water itself.

For the atmospheric surface boundary layer we shall postulate the following equations:

Conservation of momentum

$$(K_m U')' = (K_m V')' = 0 \tag{1}$$

Conservation of energy

$$(K_T T')' = 0 \tag{2}$$

Conservation of water vapor

$$(K_Q Q')' = 0 \tag{3}$$

For the upper levels of the sea we next postulate the equations:

Conservation of momentum

$$(k_m u')' + fv = (k_m v')' - fu = 0 \tag{4}$$

Conservation of energy

$$(k_t t')' = 0 \tag{5}$$

Conservation of salinity

$$(k_\sigma \sigma')' = 0 \tag{6}$$

Here z is the vertical coordinate, positive upward, with the origin at the mean sea surface; U, u the eastward component of the wind or sea current; V, v the northward component of the wind or sea current; T, t the temperature of the air or sea water; f the Coriolis parameter; and K_i, k_i the eddy diffusivities for air or water for the quantity denoted by the subscript i. The primes denote differentiation with respect to z.

The systems of equations for the atmosphere and the ocean are each an eighth-order differential system. We shall therefore require eight boundary conditions for each system. They are as follows:

At the upper boundary of the surface boundary layer at $z=D$,

$$\left.\begin{aligned} U = U_1 &= \text{constant} \\ V = V_1 &= \text{constant} \\ T = T_1 &= \text{constant} \\ \rho_A C_P K_T T' = H_0 &= \text{constant} \end{aligned}\right\}; \tag{7}$$

at the lower boundary $z=d$ (an appropriate point below the thermocline),

$$\left.\begin{aligned} u = u_1 &= \text{constant} \\ v = v_1 &= \text{constant} \\ t = t_1 &= \text{constant} \\ \sigma = \sigma_1 &= \text{constant} \end{aligned}\right\}; \tag{8}$$

and at the mean interface $z=0$,

$$\left.\begin{aligned} &k_m u' = K_M U' = \text{constant} = \tau_x \\ &k_m v' = K_m V' = \text{constant} = \tau_y \\ &Q = Q_0(T_0)\,(\text{fully saturated})^1 \\ &T = t \\ &U = V = 0 \\ &LK_Q Q' = S_0 - H_0 + h_0 \\ &\rho_w k_\sigma (\ln\sigma)' = \frac{|K_Q Q'|}{\{1 - |K_Q Q'|\}} \end{aligned}\right\}. \tag{9}$$

In (7), (8) and (9) C_P is the specific heat at constant pressure for air; ρ_A the density of air (assumed as constant); L the latent heat of evaporation (585 cal gm^{-1}); S_0 the net radiation flux at the mean sea surface; and h_0, H_0 the constant sensible heat flux in the sea or in the atmosphere, h_0 to be determined from the solution.

The radiation term S_0 in Eq. (9) is the net radiation flux, subtracting those portions reflected as well as reradiated. The incoming solar flux must be attenuated due to cloud cover, which, of course, must be stipulated.

In the interface conditions in Eq. (9) we have not required the continuity of the velocity vector across the interface as would be required by an averaging on a molecular scale on either side of the moving interface. This apparent discrepancy may be resolved by remembering that we have adopted a steady model for the unsteady neighborhood of the sea surface and have taken averages in the neighborhood of the mean sea surface. Thus, if we consider a point on the mean sea surface, it will be occupied a portion of the time by air and the remainder of the time by the sea water. If we now average the velocities for each medium over an appropriate time period, these averages quite obviously will not be equal, since the instantaneous velocities during the occupancy time for each medium will be governed by different flow equations. In the present model we shall therefore simply postulate a matching of the mean shear stress across the mean sea surface and consider the latter a contact surface across which velocity slip is permitted.[2]

Let us consider next the various eddy diffusivities. In contrast to an unstratified medium of ordinary fluid dynamics, in meteorological and oceanographic applications we shall have different expressions for the transport coefficients depending upon the vertical density distribution. In other words, turbulent eddy motions can be suppressed or enhanced by the vertical variation of the buoyancy forces.

[1] Diffusion rates are seldom great enough to invalidate this assumption.

[2] These same considerations would obviously call for a temperature jump across the mean sea surface, but for the lack of an appropriate expression for this jump we shall simply set the jump equal to zero.

There have been several different forms of the eddy diffusivities suggested for the atmosphere and in the present calculations we shall adopt a form suggested by Pandolfo (1966) based upon the results of Priestley (1959) and Monin and Obukhov (1954). For the atmosphere we shall consider only the lapse case since inversions are less frequently encountered over oceans, especially over the tropics. These expressions are as follows:

For the lapse-free convection case: $(z/L) < (Z_T/L) = -0.048$,

$$K_M = K_T (c/3)^{\frac{1}{2}} (z/L)^{-\frac{1}{6}} \tag{10}$$

$$K_T = K_Q = W^* C_P \rho_A k (z+z_0)(3/c)\,|z/L|^{\frac{1}{3}} \tag{11}$$

or the lapse-forced convection case: $0 \geq (z/L) \geq -0.048$,

$$K_M = \rho_A (z+z_0) k W^* [1 + (3z/L)]^{-1} \tag{12}$$

$$K_T = K_Q = K_M [1 + (3z/L)]^{-1} \tag{13}$$

In the above $C = 3k^{\frac{4}{3}}\,h^{-\frac{2}{3}}$ where $k=0.4$ (von Kármán's constant) and $h=1.32$ (Priestley's constant); W^* is the shear velocity given in terms of the surface shear stress τ by $\tau = \rho_A W^{*2}$; z_0 the aerodynamic roughness height due to the waves to be described below; and L the Monin-Obukhov length defined by

$$L = \frac{-u^{*3}}{k(g/\bar{T})\tilde{H}_0}, \tag{14}$$

where $\bar{T}$ is a representative temperature in the surface boundary layer, g the gravitational acceleration, and

$$\tilde{H}_0 = (C_P \rho_A)^{-1} H_0.$$

For the value of z_0, the aerodynamic roughness height, we shall use the expression proposed by Charnock (1955), i.e., $z_0 = W^{*2}/ag$, where $a \sim 81.1$ when length and time units of centimeters and seconds are used. There is some controversy over the proper value of z_0, but the above expression appears to be the most acceptable [see Roll (1965, p. 138)].

Clearly, for small values of z/L, i.e., near the mean sea surface, we shall have forced convection and Eqs. (10) and (11) will be pertinent. For larger values of z free convection will predominate. Thus, if $z_T = -0.048L < D$, a patching in the calculations will be required where the appropriate set of diffusivities must be inserted in the proper intervals.

For the oceans we shall adopt for the simplified model the form of the diffusivities due to Munk and Anderson (1948); that is,

$$k_m = A_0 [1 + 10\mathrm{Ri}]^{-\frac{1}{2}}, \tag{15}$$

$$k_t = A_0 \left[1 + \frac{10}{3}\mathrm{Ri}\right]^{-\frac{3}{2}}. \tag{16}$$

Here A_0 is the diffusivity for the neutrally stable case and is assumed to be solely a function of the wind velocity W_{10} at 10 m. This functional dependence is given in Munk and Anderson and is based upon observations. Ri is the Richardson number defined by

$$\mathrm{Ri}=\frac{-g(\ln\rho_w)'}{w'^2},$$

where $w^2=u^2+v^2$, and ρ_w is the density of the sea water. In general, $\rho_w=\rho_w(\sigma,t)$ so that

$$(\log\rho_w)'=\tilde{a}(\sigma,t)t', \qquad (17)$$

where the function $\tilde{a}(\sigma,t)$ is defined and tabulated in Munk and Anderson. As in the latter reference we shall for specific regions take $\tilde{a}=$constant. With this assumption we will not require a knowledge of the salinity distribution so that in the following we may omit further consideration of σ.

A more sophisticated form of the diffusivities for the upper layers of the oceans has been suggested by Dobroklonskii and Tsikunov [see Kitaigorodsky (1957)], which attempts to incorporate both the turbulent transport due to wave motion as well as that due to the shear of the advection currents. This form is perhaps too detailed for the present effort, but it is hoped that a comparison of the ocean mixing layer using the two forms can be made in the near future.

3. Solution for the atmospheric boundary layer

The problem is now to solve the coupled system of equations given by Eqs. (1)–(5), fulfilling the boundary and interface conditions of Eqs. (7)–(9). In the atmospheric surface boundary layer we have assumed that the flow direction remains invariant. We need therefore determine only the profile for the absolute magnitude W of the velocity vector, where $W^2=U^2+V^2$. Thus, using Eqs. (1)–(3) with the diffusivities from Eqs. (10)–(13), we obtain the result [see Pandolfo (1966)]:

$$\left.\begin{aligned} W(z)&=\frac{W^*}{k}\left[\ln\left(\frac{z+z_0}{z_0}\right)+\frac{3}{L}(z-z_0)\right]\\ T(z)-T_1&=\frac{\Theta^*}{k}\left[\ln\left(\frac{z+z_0}{z_0}\right)+\frac{6}{L}(z-z_0)+\frac{9}{2L^2}(z^2-z_0^2)\right]\end{aligned}\right\}, \qquad (18)$$

for $0\geq(z/L)\geq-0.048=z_T/L$. If $z_T/L=-0.048<D$ we must patch the following solution to Eq. (18) at $z/L=-0.048$ to be valid in the interval $-0.048L\leq z\leq D$. Thus,

$$\left.\begin{aligned} W(z)-W(z_T)&=\frac{6W^*}{k}\left(\frac{c}{3}\right)^{1/2}\left[\left|\frac{z}{L}\right|^{-1/6}-\left|\frac{z_T}{L}\right|^{-1/6}\right]\\ T(z)-T(z_T)&=\frac{\Theta^*}{k}\left[\ln\left(\frac{z}{z_T}\right)+\frac{6}{L}(z-z_T)+\frac{9}{2L^2}(z^2-z_T^2)\right]\end{aligned}\right\}, \qquad (19)$$

where

$$\Theta^*=-\frac{1}{u^*}\frac{H_0}{C_P\rho_A}.$$

Finally, at $z=D$ we must fulfill the condition $W(D)=(U_1^2+V_1^2)^{\frac{1}{2}}$. This will then determine the value of W^* or τ, the surface shear stress. Since Θ^* is prescribed, Eq. (17) will yield the sea surface temperature T_0.

For the specific humidity we obtain essentially the same profile as for T with $Q(z)$ and Q_0 replacing T and T_0 in Eqs. (18) and (19) and Θ^* substituted by the vapor flux Q^* at $z=0$, which is to be determined shortly from a consideration of the ocean layers.

4. Solution for the upper levels of the sea

The upper levels of the ocean must next be considered, and it is solely for the purpose of determining the sensible heat flux h_0 which is required to determine the sea surface water vapor flux. For the special form of the diffusivities given in Eqs. (15) and (16), h_0 may be determined approximately without consideration of the complicating momentum equations.

This is due in part to the fact that for the simplified form of diffusivities used it can be shown [see Munk and Anderson (1948)] that at the depth z_t at which the maximum temperature gradient occurs (Ri=3/5), a minimum of the velocity gradient arises. Thus, the corresponding temperature and velocity gradients are found to be

$$t'(z_t)=3^{\frac{3}{4}}(-\tilde{h}_0/A_0), \qquad (20)$$

$$[w'(z_t)]^2=5\times3^{\frac{1}{4}}\tilde{a}g(-\tilde{h}_0/A_0). \qquad (21)$$

We can further find the temperature gradient at the mean sea surface from a knowledge of the surface shear stress and from the fact that the turbulent transport in the vicinity of the sea surface corresponds to $\mathrm{Ri}\sim O(|t'|\ll1)$, i.e.,

$$t'(0)=(-\tilde{h}_0/A_0)[1+5\tilde{a}g(A_0/\tau)^2(-\tilde{h}_0/A_0)]. \qquad (22)$$

Here the quantity in the bracket in Eq. (16) for k_t has been expanded, assuming that $|\mathrm{Ri}|\ll1$, and setting in the expression for Ri, $t'\sim(-\tilde{h}_0/A_0)$ and $w'=(\tau/A_0)$, where τ is the known surface shear.

With these preliminaries let us return to the energy equation given in (5) and recast it into an integral equation to set the stage for an iteration procedure. Thus, we write

$$t(z)-t(0)=(-\tilde{h}_0/A_0)\int_0^z[1+(10/3)\tilde{a}gt'w'^{-2}]^{\frac{3}{2}}d\bar{z}. \qquad (23)$$

As a zero approximation $t=t^{(0)}$, we set

$$t^{(0)}(z)-t(0)=(-\tilde{h}_0/A_0)z+\beta_0[\tanh\alpha(z-z_t)-\tanh\alpha(-z_t)], \qquad (24)$$

where z_t is defined by $t''(z_t)=0$ and α and β_0 are constants. $t^{(0)}(z)$ must now fulfill the slope requirements given in Eqs. (20) and (22) and the condition $t(d)=t_1$; these conditions now yield the equations

$$\alpha\beta_0=(3^{\frac{1}{4}}-1)(-\tilde{h}_0/A_0), \tag{25}$$

$$5\tilde{a}g(A_0/\tau)^2(-\tilde{h}_0/A_0)=\alpha\beta_0(A_0/-\tilde{h}_0)\operatorname{sech}^2\alpha(-z_t), \tag{26}$$

$$t(0)-t_1=(-\tilde{h}_0/A_0)(-d) +\beta_0[\tanh\alpha(-z_t)-\tanh\alpha(d-z_t)]. \tag{27}$$

In Eq. (24) for $t^{(0)}$ there occur four unknown parameters, $-\tilde{h}_0/A_0$, α, β_0 and z_t, so that despite the above three equations there still remains one undetermined parameter. Let us next form $t^{(0)\prime}$ from Eq. (25) and insert it into Eq. (23). For $w'(z)$ we next approximate it by setting it equal to its minimum value $w'(z_t)$ given in Eq. (21). This will insure an accurate representation in the vicinity of the thermocline where the effect of the Richardson number is the most significant. It will be approximate, however, above and below the thermocline, but here the transport is essentially neutral so that the above approximation will not have a deleterious effect.

Eq. (23) with Eq. (24) now becomes

$$t^{(1)}(z)-t(0) =(-\tilde{h}_0/A_0)(1+\gamma)^{\frac{1}{4}}[G\{\alpha(z-z_t)\}-G(-\alpha z_t)], \tag{28}$$

where

$$\gamma=(10/3)\tilde{a}gw'^2(z_t)(-\tilde{h}_0/A_0),$$

$$G(\zeta)=\int^{\zeta}[1+\delta\operatorname{sech}^2\zeta]^{\frac{1}{4}}d\zeta, \tag{29}$$

with

$$\delta=\frac{(3^{\frac{1}{4}}-1)\gamma}{1+\gamma}. \tag{30}$$

The value of the parameter still outstanding will now be determined by requiring

$$t^{(0)}(d)=t^{(1)}(d)=t_1,$$

or

$$\alpha(-d)-(3^{\frac{1}{4}}-1)[\tanh\alpha(d-z_t)-\tanh\alpha(-z_t)] =(1+\gamma)^{\frac{1}{4}}[G\{-\alpha(d-z_t)\}-G(-\alpha z_t)]. \tag{31}$$

Thus, Eq. (28) with Eqs. (31), (27), (26) and (25) will now constitute the desired solution for the temperature profile and, therefore, for the determination of h_0 in terms of t_1, $t(0)$ and τ. Knowing h_0, we can finally determine the water vapor flux at the mean sea surface from Eq. (9).

5. Sample calculations

For the atmospheric boundary layer we set:

D (height of the surface boundary layer) $=10$ m
$C_p=0.24$ cal gm^{-1} (°C)$^{-1}$
$\rho_A=1.2\times10^{-3}$ gm cm^{-3}
$U_1=7.68$ m sec^{-1} (so as to obtain $\tau=1$ dyn cm^{-2})
$V_1=0$
$-W^*C_p\rho_A\Theta^*=H_0=0.50\times10^{-3}$ cal cm^{-2} sec^{-1}
$z_0=u^*/81.1g=0.105$ cm
L (Monin-Obukhov length) $=-97.1$ m
$z_T=-0.048L=4.65$ m (transition height from forced convection to free convection)
$T_1=15$C

The resulting velocity and temperature profiles may be calculated from Eqs. (17) and (18) and are shown in Fig. 2. The resulting surface temperature is found to be $T_0=16.60$C. The surface shear τ is 1 dyn cm^{-2}.

For the ocean we set:

$d=-50$ m (depth of the bottom boundary)
$f=0.729\times10^{-4}$ sec^{-1} (latitude, 30N)
$\tilde{a}=2\times10^{-4}$(°C)$^{-1}$ [from Munk and Anderson (1948)]
$t_1=16.18$C (t_1 chosen[3] such that $\tilde{h}_0=-2\times10^{-3}$ cal cm^{-2} sec^{-1}.
$A_0=155$ gm cm^{-1} sec^{-1} [Munk and Anderson (1948)]

Inserting these values into Eqs. (25), (26), (27) and (31) we obtain the following results:

$\tilde{h}_0=-2\times10^{-3}$ cal cm^{-2} sec^{-1}
$-z_t=26.8$ m
$\alpha=7.80\times10^{-4}$
$\beta_0=0.0695$

The function $G(\zeta)$ defined in Eq. (29) gives essentially the temperature profile and is shown in Fig. 3 for the sample case.

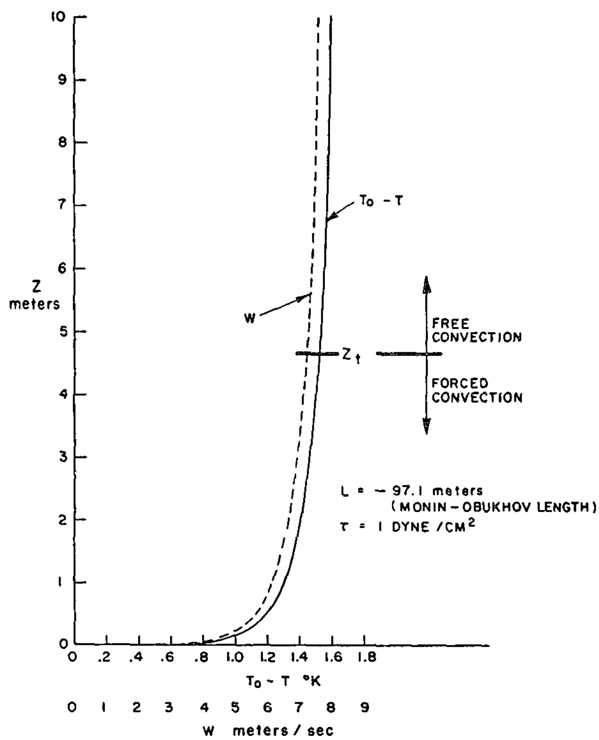

FIG. 2. Velocity and temperature profiles (atmosphere).

[3] Calculations are simplified considerably by prescribing h_0 and then determining the resulting value of t_1.

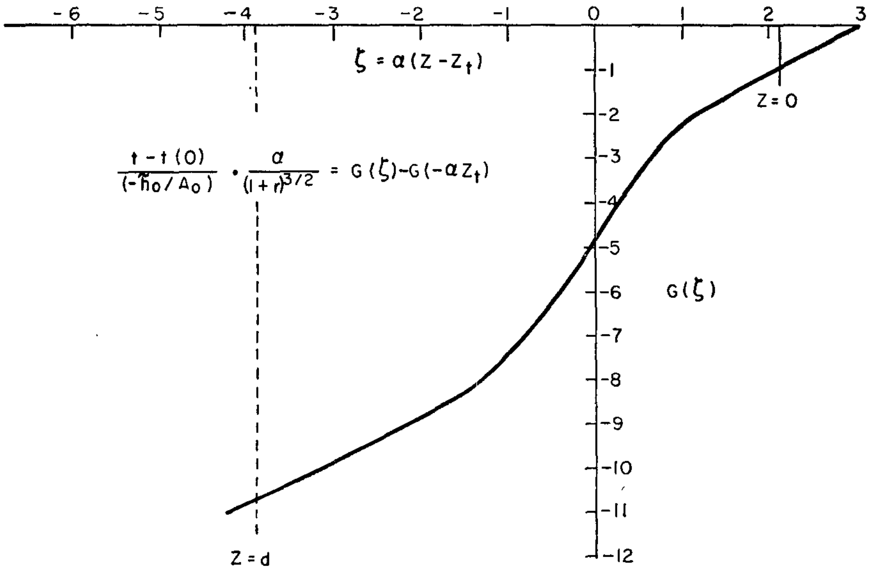

FIG. 3. Temperature profile (sea).

The location of the lower boundary used above is dictated by two opposing requirements. First, it must not be located so shallow that conditions are not effectively invariant over the prediction period. Second, it must not be located at too large a depth since the simplifying assumptions such as the quasi-steadiness, and zero advection become less valid with increasing depth. Moreover, the simplified diffusivities may also be expected to be less and less valid with increasing depth. For the present example we have simply taken the lower boundary at a depth of 50 m.

To find the rate of evaporation Q' from the sea surface from Eq. (9) we must next prescribe the net influx of radiant energy to the sea surface. The exact determination is very difficult, and although a meaningful approximate analysis can be made, we shall simply use for illustrative purposes a value of $S_0=0.54\times10^{-2}$ cal cm^{-2} sec^{-1}. We shall further assume that this amount is essentially absorbed at the ocean surface. Thus, a net flux of $0.54\times10^{-2}-[2\times10^{-3}+0.50\times10^{-3}]$ $=2.9\times10^{-3}$ cal cm^{-2} sec^{-1} is used to evaporate the sea water. Thus from Eq. (9) we have

$$Q^*=k_QQ'=(1/L)(S_0-H_0+h_0)$$

$$=\frac{2.90\times10^{-3}\text{ cal cm}^{-2}\text{ sec}^{-1}}{585\text{ cal gm}^{-1}}$$

$$Q^*=0.495\times10^{-5}\text{ gm cm}^{-2}\text{ sec}^{-1}.$$

The value of Q_0 is taken as the fully saturated value at the surface temperature $T=16.60$C and for a surface pressure of 1 atm. The water vapor profile will now be essentially that for the temperature where Q^* and Q_0 are to replace Θ^* and T_0.

6. Lower boundary conditions

In this simplified model for the sea-air interaction treated above we prescribed at a given location the velocity, temperature and sensible heat flux at the top of the atmospheric surface boundary layer taken nominally at 10 m. We were then able to determine the velocity and temperature profiles, and, in particular, the temperature at the mean sea surface. From this information together with the knowledge of the temperature at the lower boundary, we were then able to calculate the temperature profile in the sea, and from this the constant sensible heat flux. Knowing additionally the net radiative energy absorbed at the sea surface, we could then calculate the flux of water vapor at the sea surface. The water vapor density at the sea surface was determined by assuming the air to be saturated at the known surface pressure and temperature. Finally, the water vapor profile was then ascertained and, in particular, its concentration and flux at the top of the boundary layer were determined.

To see how these data are now to be patched to the prediction of the large-scale atmospheric motion, let us recall the initial value problem for this calculation. The dependent variables will be essentially the three components of velocity, pressure, temperature, air density, and the water vapor density. At the initial time one prescribes within the domain of interest the distribution of all of these dependent variables. At the top boundary, say at a location where the pressure is 10 mb, one usually prescribes the values of the velocity vector (or its vertical gradient), the density, and the temperature, but not the water vapor density. The conditions at the lower boundary, which we may take nominally at $z=0$, will require the prescription of the velocity vector, pressure, air density, temperature, water vapor density and its normal flux. All of these boundary conditions must be known *a priori* for the entire duration of the prediction period. The conditions at the upper boundary may be taken to be invariant with time. For the time-dependent conditions at the lower boundary we shall use the quasi-steady model of the sea-air interaction proposed in the present paper in the following manner.

The prediction procedure is now a process of marching in time in steps of Δt from the known initial values of the dependent variables, using the finite difference analog of the unsteady flow equations. At each time step all lattice points will be computed in a standard way except for the points on the lower boundary and the set of points just above the boundary. The value of the velocity vector at a given boundary point and its neighboring point just above it will now be determined by an iterative process such that at the boundary the velocity vector and its gradient will agree with the corresponding values at the top of the surface boundary layer obtained from the sea-air interaction model. Here the velocity components for the large scale calculations may be approximated by a quadratic polynomial, assuming we are using a second-order difference scheme.[4] Similarly, the temperature and its gradient, and the water vapor density and its flux at the top of the boundary layer will be used to determine the temperature and the water vapor density at the boundary lattice point and its neighbor just above. Here again we use a quadratic polynomial to express T and Q. In this way the bottom two layers of lattice points will be determined at each time step from the sea-air interaction model.

7. Concluding remarks

The present simplified model is intended to serve as a first step in the incorporation of a realistic sea-air interaction into the lower boundary condition. It is, of course, based upon some limiting assumptions. First of all we have assumed a quasi-steady model. For this assumption to be valid the flow in the interaction domain must adjust itself to temporal changes in the boundary conditions in a time short compared to times characterizing, for example, the variation of the surface winds or the solar flux. Slowest adjustments take place in the oceans where large inertial effects are present because of the high heat capacity of water. To check whether the turbulent transport due to surface waves and advection shear is sufficient to overcome this inertia must await an unsteady treatment using simplified models as proposed by Turner and Kraus (1965).

The present simplified model depends essentially on the existence of a shear minimum at the ocean depth where a maximum in the temperature gradient exists. Whether this is indeed a result generally applicable and is not a degeneration due to the Munk-Anderson diffusivities can be only answered by controlled experiments. There are, of course, more sophisticated forms of the diffusivities, but there is still insufficient evidence to warrant using them at this time instead of those due to Munk and Anderson.

There are presently in use a number of semi-empirical formulae for the surface evaporation rate of the sea water solely in terms of atmospheric conditions near the surface. If such formulae were valid, there would, of course, be no necessity to consider simultaneously the underlying seas. Clearly, the situation is more complex than this, since the evaporation rate will depend upon the amount of the solar energy absorbed at the sea surface that is convected into the sea as well as that, of course, which is transported into the interior of atmosphere. The latter quantities do depend upon the state of flow off the sea surface.

In future considerations it would be of interest to incorporate the present rudimentary sea-air interaction model into the calculation of the large scale motions to determine when and to what extent its use starts to modify the flow in comparison to that calculated using a lower boundary condition which does not incorporate sea water evaporation.

[4] It may be recalled that a given difference scheme for a spatial derivative is equivalent to expanding the dependent variable in question in a given polynomial in the appropriate space variable. Thus, if we have a second-order difference scheme, this will be equivalent to expanding the given dependent variable in terms of a quadratic expression.

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